8.2, 8.4, 8.8, 8.16

8.2. Find (a) \( \gamma \) and (b) \( \theta \) and (c) \( \theta \).

\[ M_0 \]

\[ A \]

\[ B \]

\[ M_x \]

\[ M_y \]

\[ x \]

\[ y \]

Soln: It's a statically determinate problem.

- Solve for moment in AB cut at any point between AB, and draw the FBD of the right cut:

\[ M_b = M_0 \]

\[ M_x = M_0 \]

- Part a.

\[ EI \frac{d^2 y}{dx^2} = M_x = M_0 \]

\[ EI \frac{dy}{dx} = M_0 x + C_1 \]

\[ EI \gamma = \frac{1}{2} M_0 x^2 + C_1 x + C_2 \]

Plugging B.C.3 for AB:

At point A \( (x=0) \): \( y=0 \), \( \frac{dy}{dx}=0 \)

\[ \begin{align*}
\theta &= \frac{dy}{dx} = M_0 (0) + C_1 = 0 \\
0 &= \frac{1}{2} M_0 (0)^2 + C_1 (0) + C_2 = 0 \\
C_1 &= 0 \quad C_2 = 0
\end{align*} \]

\[ \Rightarrow \]

\[ y = \frac{1}{EI} \left( \frac{1}{2} M_0 x^2 \right) \quad \text{for AB} \]

8.4. Solve for the beam below for the questions at 8.2.

\[ w \]

\[ A \]

\[ B \]

\[ M \]

\[ x \]

Soln:

- Solve for the moment in the beam.

Cutting at any point between AB and draw the FBD for the left part:

\[ y = \frac{1}{EI} \left( -\frac{1}{2} w x^2 + \frac{1}{6} w l^4 \right) \]

(b) from (a).

\[ \frac{dy}{dx} = \frac{1}{EI} \left( -\frac{1}{6} w x^3 + \frac{1}{6} w l^3 \right) \]

\[ \Rightarrow \]

\[ \frac{dy}{dx} \bigg|_{x=0} = \frac{w l^3}{6EI} \quad \theta_A = \frac{w l^3}{6EI} \]
8.8 Solve (b) and (c) for 8.4. Given \( AB \) is \( W \times 35 \), \( u_o = 3 \text{ kips/ft} \), \( L = 12 \text{ ft} \), \( E = 29 \times 10^6 \text{ psi} \)

Solution:
From 8.4 we have
\[
\begin{align*}
\frac{dy}{dx} & = -\frac{W}{24EI} (x^4 - 4L^3x + 4L^4) \\
\end{align*}
\]

\[ \frac{dy}{dx} \bigg|_{x=0} = \frac{W}{6EI} (x^3 - x^3) \]

(b) \[ \frac{dy}{dx} \bigg|_{x=0} = \frac{40113}{EI} \]

From appendix C:
\[ I = Jx = 285 \text{ in}^4 \]

\[ \frac{dy}{dx} \bigg|_{x=0} = \frac{(3 \text{ kips/ft})(12 \times 3)^3}{(29 \times 10^6 \text{ psi})(285 \text{ in}^4)} \]

\[ = \frac{(5 \times 10^3 \text{ kips})(12 \times 12 \times 12)^2}{(29 \times 10^6 \text{ psi})(285 \text{ in}^4)} \]

\[ \theta = 0.334 \times 10^{-3} \text{ rad} \]

(c)
\[ \frac{dy}{dx} \bigg|_{x=0} = -\frac{W}{8EI} \]

\[ = -\frac{(12 \times 3)^4}{8(29 \times 10^6 \text{ psi})(285 \text{ in}^4)} \]

\[ = -0.1804 \text{ in} \]

\[ y = -0.1804 \text{ in at free edge} \]

8.16. Find (a) \( M \) for \( AB \)
(b) \( \theta \) at \( A \)
(c) \( B \) at \( B \)

Soln:
- Solve for reactions at first

\[ \begin{align*}
\sum M_A &= 0 \\
B_y (L) - W (2L) (L) &= 0 \\
B_y &= 2WL \\
\sum F_x &= 0 \\
A_y &= 0 \\
\end{align*} \]

- Solve for moment in \( AB \): 
  - At any point between \( AB \), draw the FBD of the left part:

\[ y = \frac{W}{24EI} (x^4 - L^3x) \]

(a).
\[ \theta = \frac{40113}{24EI} (x^3 - L^3) \]

(b). From above:
\[ \frac{dy}{dx} = \frac{1}{EI} (-\frac{1}{6}wx^2 + \frac{1}{24}WL^3) \]
\[ = -\frac{W}{24EI} (4x^3 - L^3) \]

\[ \theta = \frac{40113}{24EI} \text{ at } A \]
Cont 8:6

(c): At B, \( \theta = 0 \)

\[ \theta = -\frac{w}{24EI} (4L^3 - L^3) \]

\[ \theta = -\frac{wL^3}{8EI} \text{ at } B \]

5. Compare the deflection at for a beam support a weight \( P \) mid away from the end: for case (a) and case (b)

Case (a) supported as cantilever beam

Cont. 5:

Solu:

First we solve it in a regular method

Case (a):

- Finding moment in the beam. cut at any point between AB and draw the FBD of the right part

\[ + \underbrace{M_x = 0} \Rightarrow \quad P(L-x) + M = 0 \]

\[ M = -P(L-x) \]

- Solve for elastic curve

\[ EI \frac{d^2y}{dx^2} = M = -P(L-x) \]

\[ \Rightarrow \quad EI \frac{dy}{dx} = -PLx + \frac{1}{2} x^2 + C_1 \]

\[ \Rightarrow \quad EI \ y = \frac{1}{2EI} (-P L x^2 + \frac{1}{6} (L-x)^3 + C_1 x + C_2) \]

- Plug in B.C.S at \( x = 0 \): \( \theta = 0, y = 0 \)

\[ y|_{x=0} = C_2 = 0 \]

\[ \theta|_{x=0} = C_1 = 0 \]

\[ y = -\frac{PLx^2}{6EI} \quad (3L-x) \]

when \( x = L \):

\[ y = -\frac{PL^2}{6EI} \quad \text{for } x = L \]

Case (b) both ends are clamped.

Cont. 5:

It's symmetric about the center, so we only need to study AB.

- Solve for \( M \) in AB. Cut at any point between AB and draw the FBD of the left part

\[ + \underbrace{M_x = 0} \Rightarrow \quad M = M_A - R_A x = 0 \]

\[ M = M_A + R_A x \]

- Solve for reactions \( R_A \) and \( M_A \)

Case (b)

From symmetry:

\[ R_A = R_B = \frac{P}{2} \]

\[ M_A = M_B = M_0 \]

- Solve for elastic curve equation for AB

\[ EI \frac{d^2y}{dx^2} = M = M_0 + \frac{1}{2} P x \]

\[ \Rightarrow \quad EI \frac{dy}{dx} = M_0 x + \frac{1}{6} P x^3 + C_1 \]

\[ \Rightarrow \quad EI \ y = \frac{1}{2EI} M_0 x^2 + \frac{1}{12} P x^3 + C_1 x + C_2 \]

- Plug in B.C.S at \( A(x=0): y = 0 \Rightarrow C_2 = 0 \)

\[ \theta = 0 \Rightarrow C_1 = 0 \]

\[ B(x=L): \theta = 0 \Rightarrow M_0 x + \frac{1}{4} P L^2 + C_1 = 0 \]
Cont. 5.

\[ M_x = -\frac{1}{4} P L \]

\[ \Rightarrow \quad y = \frac{1}{EL} \left( -\frac{1}{8} P L x^2 + \frac{1}{12} P x^3 \right) \]

\[ y = \frac{P x^2}{24 EI} \left( 3L - 2x \right) \]

when \( x = L \):

\[ y = -\frac{PL^3}{24EI} \quad \text{for} \quad x = L \]

Comparing the \( y \) at \( x = L \) for case a and (b):

\[ y^{(b)} = \frac{1}{8} y^{(a)} \]

which means the deflection of the cantilever case is 8 times the clamped on.

Now we slove by shortcut:

for case (a), we already got:

\[ y = -\frac{P x^2}{6 EI} \left( 3L - x \right) \]

The deformation of case (b) is:

\[ A \quad B \quad C \quad D \]

This means \( \theta = 0 \) at \( L \), and the curve for \( AB \) are symmetric to \( BC \)

( Beam \( AC \) is symmetric about \( B \) )

Also, curve for \( BC \) is symmetric about \( D \), where \( D \) is the center of \( BC \)

The curve for \( BD \) is equal to the curve caused by case C.

Case (c)

It's \( \frac{P}{2} \) because when you cut at \( D \) and draw the FBD of the right part

\[ \frac{P}{2} \]

Compare case (a) and case (c), they're all cantilever beam except the length of (c) is half of case (a) and the load in case (c) is half of \( P \).

Using the eqn for cantilever beam (a)

\[ y_D = -\frac{P L^3}{3EI} \quad \text{here} \quad P = \frac{P}{2} \quad L = \frac{L}{2} \]

\[ = -\left( \frac{P}{2} \right) \left( \frac{L}{2} \right)^3 \]

\[ = -\frac{1}{2} \left( \frac{P L^3}{24EI} \right) \]

but \( y_c \) is only half of the deflection of the center \( B \) (look at the deformation curve)

\[ \Rightarrow \quad y_B = 2y_D = -\frac{P L^3}{24EI} = \frac{1}{8} y_B \]