5.6-16. A beam having a cross section in the form of a channel (see figure) is subjected to a bending moment acting about the x-axis. Calculate the thickness of the channel in order that the bending stresses at the top and bottom of the beams will be in the ratio 7:3, respectively.

![Diagram of the channel beam](image)

1. First, we can use the condition \( \frac{\tau_t}{\tau_b} = \frac{7}{3} \) to find the position of neutral axis.
   \[ \tau = \frac{M}{I_y} \Rightarrow \frac{\tau_t}{\tau_b} = \frac{h_b}{h_a} = \frac{7}{3} \]
   Total height \( h = h_a + h_b = 75 \text{ mm} \)
   \[ h_a = 52.5 \text{ mm}, \quad h_b = 22.5 \text{ mm} \]

2. Now, the area \( A_i \) (\( i = 1, 2, 3 \)) at the position of the neutral axis depend on the thickness \( t \), but the moment of the whole section is zero, i.e.,
   \[ \frac{1}{2} \rho_1 Y_i A_i = \rho_2 Y_b = 0 \]
   where \( \rho_1 \) is the centroid of the area \( A_i \). (\( i = 1, 2, 3 \)) (Continued)
5.6-20 Water pressure acts against an inclined panel ABC that serves as a barrier (see figure). The panel is pivoted at point B, which is height $h$ above the base, and press against the base at A when the water level is not too high (note that the panel will rotate about the pin at B if the depth $d$ of the water exceeds a certain maximum depth $d_{max}$. The panel has thickness $t$ and is inclined at an angle $\alpha$ to the horizontal. The allowable bending stress in the panel is $\sigma_{allow}$.

Derive the following formula for the minimum allowable thickness of the panel:

$$t_{min} = \sqrt{\frac{E t_{allow}}{\sigma_{allow} h^2}}$$

(Note: To aid in deriving the formula, observe that the maximum stress in the panel occurs when the depth of the water reaches the maximum depth $d_{max}$. Also, consider only the effects of bending in the panel, disregard the weight of the panel itself, and let $y$ be the weight density of water.)

**Solution:**

Let $d_{max}$ be the maximum depth of water.

(There is no reaction at end $A$ when the water depth equals $d_{max}$)

$b$ = width of panel perpendicular to plane of figure

$q_{o} = \text{Max. intensity of distributed load on panel} = Y d_{max} b$

Resultant of the triangular load acts through the centroid of triangle. This must also act through B.

$$FBD of \text{part BC of panel:}$$

$$\text{Maximum bending moment occurs at section B.}$$

$$M_{max} = M_{b} = \frac{1}{2} \left( \frac{2q_{o}}{3} \right) \left( \frac{2h}{\sin \alpha} \right) \times \frac{1}{2} \left( \frac{2h}{\sin \alpha} \right)$$

$$= \frac{4q_{o}h^{2}}{9\sin^{2} \alpha}$$

$$q_{o} = Y d_{max} b = Y (3h) (b)$$

$$M_{max} = \frac{4Yhb^{3}}{3\sin^{2} \alpha} \rightarrow (1)$$

Also, $M_{max} = (\text{allow}) \left( \frac{bt^{2}}{6} \right) \rightarrow (2)$

From $(1, 2)$, we get (by equating $M_{max}$)

$$\text{allow} \left( \frac{bt^{2}}{6} \right) = \frac{4Yhb^{3}}{3\sin^{2} \alpha}$$

$$\Rightarrow t_{min} = \frac{\sqrt{8Yh^{3}}}{\text{allow} \sin^{2} \alpha}$$

Hence, the result:
9.3-11 Derive the equation of the deflection curve for a cantilever beam AB supporting a load \( P \) at the free end (see figure). Also, determine the deflection \( \delta \) and angle of rotation \( \theta \) at the free end. (Note: Use the second-order differential equation of the deflection curve.)

PROF. 9.3-11

Take a section at a distance \( x \):

\[ M = \frac{V(L-x)}{L} \]

We know that

\[ EI u'' = M = -P(L-x) \]

So

\[ u'' = -\frac{P}{EI} \left( L-x^2 - \frac{x^3}{3} \right) \]

Integration

\[ u' = -\frac{P}{EI} \left( x^2 - \frac{x^3}{2} \right) + C_1 x + C_2 \]

One more integration

\[ u = -\frac{P}{EI} \left( \frac{Lx^2}{2} - \frac{x^3}{6} \right) + C_1 x^2 + C_2 x \]

\[ u(0) = 0 \Rightarrow C_2 = 0 \]

\[ u'(0) = 0 \Rightarrow C_1 = 0 \]

\[ \delta = u(L) = -\frac{PL^3}{6EI} \]

\[ \theta = u'(L) = -\frac{PL^2}{2EI} \]

9.3-13 A cantilever beam AB supporting a uniformly distributed load of maximum intensity \( q_0 \) is shown in the figure. Derive the equation of the deflection curve and then obtain formula for the deflection \( \delta \) and angle of rotation \( \theta \) at the free end. (Note: Use the second-order differential equation of the deflection curve.)

PROF. 9.3-13

Take right part of the FBD with section at a distance \( x \):

\[ M = \frac{V(L-x)}{L} \]

\[ Z \delta = 0 \Rightarrow M + \frac{1}{2} (L-x) \int L(L-x) \delta \theta \]

\[ M = -\frac{3P}{2L} (L-x)^3 \]

Integrating \( \theta \)

\[ \theta = \frac{q_0}{6EI} \left( L-x \right)^6 + C_1 \]

At \( x = 0 \)

\[ \theta(0) = 0 \Rightarrow C_1 = 0 \]

\[ C_2 = \frac{q_0}{48EI} \left( -\frac{3L^6}{8} + \frac{L^5}{10} \right) \]

Integrating \( \theta \)

\[ \theta = \frac{q_0}{24EI} \left[ -\frac{(L-x)^5}{5} - \frac{(L-x)^4}{4} \right] + C_2 \]

At \( x = 0 \)

\[ \theta(0) = 0 \Rightarrow C_2 = 0 \]

\[ \delta = \frac{q_0}{120EI} \left[ \frac{L^6}{5} - \frac{L^5}{4} \right] \]

At \( x = L \)

\[ \theta = -\frac{q_0}{30EI} \left( L^5 - 5L^4 \right) \]

\[ \delta = \frac{q_0}{120EI} \left[ \frac{L^3}{6} - \frac{L^2}{4} \right] \]

\[ \theta(0) = 0 \Rightarrow C_2 = 0 \]

\[ \delta = \frac{q_0 L^3}{120EI} \]

\[ \theta = \frac{q_0 L}{30EI} \]

\[ \theta(0) = 0 \Rightarrow C_2 = 0 \]

\[ \delta = \frac{q_0 L^3}{120EI} \]

\( \theta \) is negative \( \Rightarrow \) Clockwise rotation.
9.3-4 A gold-alloy microbeam attached to a silicon wafer behaves like a cantilever beam subjected to a uniform load (see figure). The beam has length \( L = 75 \mu m \) and rectangular cross section of width \( b = 15 \mu m \) and thickness \( t = 0.87 \mu m \). The total load on the beam is 44 \( N \).

If the deflection at the end of the beam is 1.3 \( \mu m \), what is the modulus of elasticity \( E \) of the gold alloy? (Note: We use the formula of Example 9.3.)

Consider a section at a distance \( x \) from the fixed support (Fig. 3.9-4),

\[
M = -\frac{w}{2}(L-x)^2
\]

\[
EI\frac{d^2w}{dx^2} = M = -\frac{w}{2}(L-x)^2
\]

\[
\frac{d^2w}{dx^2} = -\frac{w}{2EI}(L-x)^2 - 0
\]

Integrating (1),

\[
w = \frac{w}{6EI}(-\frac{1}{3}(L-x)^3 + C_1)
\]

At \( x = 0 \), \( w(0) = 0 \)

\[
C_1 = 0
\]

\[
w = \frac{w}{6EI}(-\frac{1}{3}(L-x)^3)
\]

At \( x = L \),

\[
w(L) = \frac{1}{2EI}(-\frac{1}{4}L^4 - L^4)
\]

\[
v(L) = -\frac{qL^3}{6EI}
\]

\[
E = \frac{d^2w}{dx^2} = \frac{1}{8}\delta_b
\]

\[
I = \frac{bL^3}{12} = \frac{(15\mu m)(0.87\mu m)}{12}
\]

\[
E = \frac{44N}{8\delta_b} = 8 \times 82 \mu m
\]

\[
E = 80.32 \mu m
\]
Structure and geometry description:
Torsion bar AB is welded to a rigid support at its left end at A and is supported by a bearing at B. Rigid beam DBC is welded to, and rotates with, the right end of the bar AB. The load F is applied to point C on DBC. The width of this beam can be neglected when considering the length of AB. Tension rod DE hangs from a pin joint at D and pulls up on a pin joint at E. Cantilever beam HE is welded to the wall at H and has its end pulled up by the pin at E.

The load F tries to rotate the beam DBC clockwise (as viewed from the right). This motion is resisted by the torsional stiffness of rod AB and would also be resisted by the bending stiffness of HE but for the compliance of tension rod DE which diminishes this resistance.

Assume linear elastic behavior throughout. The structure is stress-free when there is no load (F = 0).

Given:
\[ L_{AB} = L_{HE} = L_{DB} = L_{BC} = L_{DE} = 0.5 \text{ m} \]
\[ r_{AB} = 2 \text{ cm}, \ G_{AB} = 80 \text{ GPa}, \]
\[ d = 2 \text{ mm}, \ E_{DE} = 200 \text{ GPa}, \]
\[ b = 2 \text{ cm}, \ h = .5 \text{ cm}, \ E_{HE} = 200 \text{ GPa} \]
\[ F = 1000 \text{ N}. \]

a) What is the deflection of point C?
b) What is the maximum shear stress on any surface in bar DE?
c) What is the maximum tension stress in bar HE?
d) What is the maximum tension stress on any surface in bar AB?
Synthesis problem

Synthesis HW. Solution

1. Rigid Member DBC

\[ \delta_D = \frac{M_{AB} L}{E I} \]

2. Deflection of point D in member CBD will be same as the sum of deflection of E at F due to bending and extension of member DE due to tension.

\[ \delta_D = \frac{M_{AB} L}{E I} \]

So

\[ \delta_D = \delta_E + \delta_{DE} \]

Geometry

\[ \delta_E = c c' = D D' \]

Rotation of DBC due to twist \( \phi_{AB} \)

also

\[ \delta_D = \delta_E + \delta_{DE} \]

from (b) and (ii)

\[ \phi_{AB} L = \delta_E + \delta_{DE} \]  

Strength of material result.

A) Twist of AB

\[ \phi_{AB} = \frac{M_{AB} I_{AB}}{G_{AB} J_{AB}} \]

\[ J_{AB} = \frac{\pi b^4}{3} \]

\[ I_{AB} = \]
b) Stetch of DE
\[ \delta_{DE} = \frac{T_{DE}}{E_{DE} A_{DE}} \]  
\[ A_{DE} = \frac{d^2}{3} - l_{DE} \times L \]

\[ l_{DE} = \frac{d}{2}, \frac{3}{12} \times L_{DE} \times L \]

\[ \delta_E = \frac{T_{DE}}{3 E_{HE} I_{HE}} \]  
\[ I_{HE} = \frac{6d^4}{12}, \frac{L_{HE} \times L^3}{2} \]  
(from problem 4.3/ii)

FBDs and Mechanics

Consider FBD of DBC

\[ 2 M_B = 0 \Rightarrow M_{AB} + T_{DE} L - F \cdot L = 0 \]

From eq. (1)

\[ \delta_{AB} \cdot L = \delta_E + \delta_{DE} \]

Use eqs (3) and (4)

\[ \frac{M_{AB} L}{G_{AB} I_{DE}} = \frac{T_{DE} L}{E_{DE} d^2} + \frac{T_{DE} L^3}{3 E_{HE} \left( \frac{d}{12} \right)^2} \]

and from eq (6)

\[ T_{DE} = (FL - M_{AB}) \frac{1}{L} \]

\[ \Rightarrow \frac{M_{AB} L}{G_{AB} I_{DE}} = \left( \frac{1}{E_{DE} d^2} + \frac{1}{3 E_{HE} \left( \frac{d}{12} \right)^2} \right) \times \left( FL - M_{AB} \right) \frac{1}{L} \]
b) Deflection of pt C

To: \Delta = \Delta_D

From eq(1)

\[ \delta_C = \delta_D = \frac{DL^2}{8EI} \]

As: \[ L_D = L_E = 5 \text{ m} \]

\[ \delta_C = \frac{DL^2}{8EI} \]

\[ = \frac{496.9 \text{ N} \cdot \text{m}^2}{80 \text{ kPa} \cdot \text{m}^3} \]

\[ = 6.18 \text{ mm} \]

b) Max. Tension stress in beam HE

\[ \sigma_{max} = \frac{M_{max}}{I} \]

\[ M_{max} = T_{DE} \cdot L \]

\[ \sigma_{max} = \frac{T_{DE} \cdot L}{I \cdot 2} \]

\[ = \frac{(618 \text{ N} \cdot \text{m}) \cdot (0.5 \text{ m})}{(2 \text{ cm})(0.5 \text{ cm})^3} \cdot \frac{2}{12} \]

\[ = 37.068 \text{ MPa} \]

d) Max. shear stress on any surface

In DE, DE is under pure tension and the max. shear stress will be at an angle 45° from the longitudinal axis and its value will be

\[ \tau_{max} = \frac{\sigma_{x}}{2} \]

\[ = \frac{T_{DE}}{A_{DE}} \]

\[ = \frac{618 \text{ N}}{2 \times 10^{-3} \text{ m}^2} \]

\[ = 772 \text{ kPa} \]

d) Max. shear stress in beam AB

\[ \sigma_{max} = \frac{2M_{AB}}{J} \]

\[ = \frac{2 \cdot 416.91}{\pi \left(2 \times 10^{-3} \text{ m}^3\right)^3} \]

\[ = 39.53 \text{ MPa} \]