4.5.9: Beam ABCD is simply supported at A and C and has reactions at each end (see figure). The span length is L and each overhang has length L/6. A uniform load of intensity q acts along the entire length of the beam.

Draw the shear force and bending moment diagrams for this beam.

Solution

Span length: distance between B and C = L.

Length of each overhang = L/6.

FBD for beam ABCD:

\[ F_D = \frac{qL}{2} \]

By symmetry, \( R_B = R_C = \frac{qL}{2} \).

In general details:

\[ 2 \frac{qL}{2} \cdot 0 = q \left( L + \frac{L}{3} + \frac{L}{3} \right) + R_B + R_C \]

or

\[ R_B + R_C = \frac{qL}{6} \]

To find other equations to solve for reactions, do moment balance about B.

\[ M_B = 0 \]

\[ + \frac{q}{2} \left( L \cdot \frac{L}{2} \right) + R_C \cdot L = q \left( \frac{L}{2} + L \right) \left( \frac{L}{2} + L \right) \]

\[ = 0 \]

\[ q \left( \frac{L}{2} - \frac{L}{2} \right) \]

\[ = 0 \]

\[ q \left( \frac{L}{2} + L \right) \]

\[ = 0 \]

\[ M = q \left( \frac{L}{2} + L \right) \]

\[ = \frac{qL}{2} \]

\[ M = -\frac{qL}{2} \]

\[ M = -\frac{qL}{2} \]

M = 0 will have 2 roots.

Cut C-C can continue the same way by taking all the forces and moments.

Or by starting over from the other side.

Using Eqs. 1, 3, 6, draw shear force diagram.

Note: Diagram not to scale.
Method 2:

- Use the equations: \( \frac{dV}{dx} = -q \quad \frac{dM}{dx} = V \)
  - Integrate, find out \( V \) and \( M \) in terms of \( x \).
  - \( M \) in terms of \( x \) and \( V \) in terms of \( x \).

In this problem \( q = q_x \) (a constant).

\[
\frac{dV}{dx} = -q, \quad 0 \leq x \leq \frac{L}{3}
\]

Integrate, \( x \) measured from left-hand side.

\[ V = V(0) - \int q \, dx = V(0) - q \frac{x}{3} \]

To calculate \( V(0) \), use \( V = 0 \) at \( x = 0 \), so \( V(0) = 0 \).

So \( V = -q \frac{x}{3} \), \( 0 \leq x \leq \frac{L}{3} \). \( -1 \).

Similarly,

\[
\frac{dM}{dx} = V \Rightarrow M = \int V \, dx + M_0
\]

So \( V = -q \frac{x}{3} = M + M_0 \) + \( \frac{5}{6} \left( q \frac{x^2}{2} \right) \)

\( \text{or} \quad M = M_0 + q \frac{x^2}{2} \) \( -2 \).

To calculate \( M_0 \) use \( M(0) = 0 \).

So \( M_0 = 0 \), \( 0 \leq x \leq \frac{L}{3} \). \( -2 \).

See that the equation we got here is the same as when we took the cut AB calculated \( V \) in \( M \).

Now take \( \frac{L}{3} < x < \frac{2L}{3} \). [Have to take \( x \) in the region where there is no discontinuity in load or moment (no concentrated load or moment].
4.5-10 (cont'd)

Solution

FBD for the cantilever beam

\[ V_b = -\frac{q_0 L^2}{2} \]

\[ M_B = \frac{q_0 L^3}{6} \]

Now to draw shear force & bending moment diagram take a cut \((n-n)\) & look at \(V\) & \(M\).

Method 1

FBD for Section A

\[ F_X = 0 \Rightarrow \frac{V}{2L} = 0 \]

\[ V = \frac{q_0 x^2}{2L} \]

\[ M = 0 \Rightarrow M + \left(\frac{q_0 x^2}{2L}\right)(\frac{L}{3}) = 0 \]

\[ M = \frac{q_0 x^2 L}{6} \]

Using (1) & (2), draw shear force & bending moment diagram. [Diagram not to scale]

Method 2

Use

\[ \frac{dV}{dn} = -g x \frac{dM}{dn} = +V \]

Here \(g\) is a function of \(x\).

\[ g = \frac{q_0 x}{L} \]

\[ \frac{dV}{dn} = \frac{-q_0 x^3}{L} \]

\[ V = V(0) - \int_0^x \frac{q_0 x^2}{L} dx \]

\[ V(0) = V(0) - \frac{q_0 x^3}{2L} \]

\[ V(0) = 0 \Rightarrow V(0) = 0 \text{ at } x = 0 \]

So

\[ V = \frac{-q_0 x^3}{2L} \]

\[ M = M(x) + \int_0^x V dx \]

\[ M = M(0) - \int_0^x \frac{q_0 x^2}{L} dx \]

\[ M = \frac{-q_0 x^3 + M(0)}{6} \]

\[ M(0) = 0 \Rightarrow M = 0 \text{ at } x = 0 \]

\[ M = \frac{-q_0 x^3}{6L} \]

Use (1) & (2) to plot bending moment & shear force diagrams.
5.4.2 A copper wire having diameter \( d = 3 \text{ mm} \) in bent into a circle and held with the ends just touching (see figure). If the maximum permissible stress in the copper is \( \sigma = 200 \text{ MPa} \), what is the maximum length \( L \) of wire that can be used?

Solution: \( d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m} \),

\( \sigma = 200 \text{ MPa} = 2 \times 10^5 \text{ N/m}^2 \)

\( R = \text{radius of curvature} = \frac{L}{2\pi} \)

\( \frac{R}{d} = \frac{L}{2\pi d} \)

\( \sigma = \frac{E\pi d^4}{8R L} \)

\( \frac{\pi d^4}{8R} = \frac{2 \times 10^5}{2 \times 10^5} = \frac{1}{2} \)

\( \frac{d^4}{L^2} = \frac{2 \times 10^5}{2 \times 10^5} = \frac{1}{2} \)

\( d = 2.36 \text{ mm} \)

5.5.2

A simply supported wood beam AB (see figure) is subjected to a uniform load of intensity \( q = 6.4 \text{ kN/m} \) (see figure). Calculate the maximum bending stress \( \sigma_{\text{max}} \) due to the load \( q \) if the beam has a rectangular cross section with width \( b = 150 \text{ mm} \) and height \( h = 300 \text{ mm} \).

\( M = \frac{qL^2}{8} \)

\( \sigma_{\text{max}} = \frac{My}{I} = \frac{qL^2}{bh^2} \)

\( I = \frac{bh^3}{12} \)

\( \sigma_{\text{max}} = \frac{qL^2}{12} \)
A freight-car axle AB is loaded as shown in the figure, with the force P representing the car load (transmitted through the axle box) and the force R representing the rail load (transmitted through the wheels). The diameter of the axle is d = 90 mm, the wheel gauge is L = 1.45 m, and the distance between the forces P and R is h = 200 mm. Calculate the maximum bending stress \( \sigma_{\text{max}} \) in the axle if \( P = 46.5 \text{ kN} \).

Solution: \( R = P = 46.5 \text{ kN} \).

The bending moment diagram looks like this:
5.5-8 The horizontal beam ABC of an oil-well pump has the cross section shown in the figure. If the vertical pumping force acting at end C is 35 kN, and if the distance from the line of action of that force to point B is 4.5 m, what is the maximum bending stress in the beam due to the pumping force?

(a) Find $M_{max}$

In this problem, we are not given the angle $AB$, but this will not bother us, let's look at Fig. 5.5-8 of $ABC$.

$M_{max} = \frac{P \times L}{2} = \frac{35 \times 4.5}{2} = 83.25 \text{ kNm}$

Now we can find the bending moment distribution in $ABC$.

$M_{A} = \frac{P \times L}{2}$

So the bending moment distribution are simply straight lines.

$M_{max} = P L / 2$, occurs at $x = L / 2$, and $I_{s}$ is given in this problem.

5.5-9 (Cont'd)

(b) Find $I$ for an $I$-beam

Method I

\[ I = I_{1} + I_{2} + I_{s} \]

where

\[ I_{1} = \int_{A_{1}}^{A_{2}} y^{2} \, dy = \frac{k_{1} h_{1}^{3}}{12} \]

\[ I_{2} = \int_{A_{2}}^{A_{3}} (y - h_{2})^{2} \, dy = \frac{k_{2} (h_{1} - h_{2})^{3}}{12} \]

\[ I_{s} = \frac{b_{s} t_{s}^{3}}{12} \]

Substitute numbers into $I$.

\[ I = \frac{k_{1} h_{1}^{3}}{12} - \frac{k_{2} (h_{1} - h_{2})^{3}}{12} + \frac{b_{s} t_{s}^{3}}{12} \]

\[ I = \left( \frac{500 \text{ mm}}{2} \right) \left( 500 \text{ mm} - 50 \text{ mm} \right)^{3} \]

\[ I = 6.31 \times 10^{9} \text{ mm}^{4} \]

5.5-12 A small disc of height $h = 24$ m is composed of vertical wood beams $AB$ of thickness $t = 100$ mm, as shown in the figure. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress $\sigma_{max}$ in the beam, assuming that the weight density of wood is $y = 9.81 \text{ kN/m}^{3}$.

5.5-13 (Cont'd continued)
Find \( R_0 \) or \( R_x \) (reaction forces):

\[
\begin{align*}
\text{FBD:} & \quad & & \text{FBD:} \\
& & & \text{where } R = \frac{P}{2} \left( \frac{1}{2} \right) h = \frac{1}{4} (bh) \\
& & & = \frac{1}{2} b h^2 \\
\Sigma M_y = 0 &= R \left( \frac{1}{2} \right) - R_0 (h) \\
& \Rightarrow R_0 = 2 b h^2 / 6 \\
\end{align*}
\]

Find \( M_{max} \):

\[
\begin{align*}
\text{FBD:} & \quad & & \text{FBD:} \\
& & & \text{where } R_0 = \frac{R_{xu}}{h} = 2x \\
\text{this FBD is equivalent to the following one:} & \quad & & \text{this FBD is equivalent to the following one:} \\
&R_0 x = \frac{1}{4} (bh) \\
\Sigma M_x = 0 &= M_x + R_u (\frac{1}{2}) - R_0 x \\
& \Rightarrow M_x = R_0 x - R_0 \frac{x}{2} = \frac{1}{2} b h \left( \frac{3}{4} - \frac{x}{h} \right) \\
\text{To find } M_{max}, \text{ we set } \frac{dM}{dx} = 0. \\
\frac{dM}{dx} = 0 &= \frac{1}{2} b h \left[ \frac{3}{4} - \frac{x}{h} \right] \Rightarrow x = h / 6 \\
\text{Therefore, } M_{max} &= M(x = h / 6) = \frac{1}{2} b h \left[ \frac{3}{4} - \frac{h / 6}{h} \right] = \frac{1}{4} b h^2 (\text{Continued})
\end{align*}
\]