HW 10 solution provided by Chongqing Bao.

1.8-3 A nylon bar having diameter $d = 0.75$ in. is placed inside a steel tube having inner diameter $d_i = 2.75$ in. (see figure). The nylon cylinder is then compressed by an axial force $P$. At what value of the force $P$ will the stress between the nylon bar and the steel tube be closed? (For nylon, assume $E = 450$ ksi and $v = 0.4$.)

**PROB. 1.8-3**

\[ P = \frac{F \cdot A}{E \cdot \varepsilon} = E \cdot V \cdot \varepsilon \]

where $\varepsilon$ is the axial strain. Thus

\[ \varepsilon = \frac{\varepsilon'}{\sigma'} = \frac{(d_i - d)}{d_i} \]

Thus, in magnitude,

\[ P = E \cdot \varepsilon \cdot \left( \frac{d_i}{d} \right)^2 \]

\[ = 450 \text{ ksi} \cdot \frac{0.75}{d} \text{ in.}^2 \cdot \frac{0.01}{\text{ksi} \cdot 0.75 \text{ in.}} \]

\[ = 243 \text{ kip. i.e. } 342 \times 10^3 \text{ lb} \]

Note: $\sigma' = \frac{P}{A} = \frac{243 \times 10^3 \text{ lb}}{\pi \left( \frac{0.75}{4} \right)^2} = 41 \text{ ksi}$. And $d$ for nylon is 0.13 in. By of nylon is not available. Here, we have to assume it is still in elastic range for $\sigma' = E \cdot \varepsilon'$ to be valid.

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1.8-4 The connection shown in the figure consists of five steel plates, each 0.1 in. thick, joined by a single 4-mm-diameter bolt. The total load transferred between the plates is 6000 N, distributed among the plates as shown.

(a) Calculate the largest shear stress in the bolt, disregarding friction between the plates.

(b) Calculate the largest bending stress acting against the bolt.

**PROB. 1.8-4**

\[ \text{FBD of bolt: (this is the key part.)} \]

\[ 1800 N \]

\[ 3000 N \]

\[ 3000 N \]

\[ 1800 N \]

\[ 1800 N \]

\[ 3000 N \]

\[ 3000 N \]

\[ \text{Section A-A, shear force } |V| = 1800 N \]

\[ \text{Section B-B, shear force } |V| = 1200 N \]

Thus, $|V|_{\text{max}} = 1800 N$

\[ \text{Maximum shear stress in bolt} \]

\[ \tau_{\text{max}} = \frac{V}{\pi \left( \frac{d}{2} \right)^2} = \frac{1800 \text{ N}}{\pi \left( \frac{4}{2} \right)^2} \]

\[ = 35.3 \text{ MPa} \]

\[ \text{Maximum bearing stress} \]

The maximum bearing force $F_b_{\text{max}}$ on the bolt is 3000 N.

\[ F_b = \frac{F_b_{\text{max}}}{d} = \frac{3000 \text{ N}}{6 \text{ mm} \cdot 5 \text{ mm}} \]

\[ = 100 \text{ MPa} \]

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1.8-11 A spherical fiberglass buoy used in an underwater experiment is anchored in shallow water by a chain (see part (a) of the figure). Because the buoy is positioned just below the surface of the water, it is not expected to collapse from the water pressure. The chain is attached to the buoy by a shackle and pin (see part (b) of the figure). The diameter of the pin is 0.5 in. and the thickness of the shackle is 0.25 in. The buoy has a diameter of 60 in. and weighs 1800 lb on land (not including the weight of the chain).

(a) Determine the average shear stress $\tau_{\text{ave}}$ in the pin.

(b) Determine the average bearing stress $\sigma_{\text{ave}}$ between the pin and the shackle.

**PROB. 1.8-11**

\[ \text{FBD of the buoy:} \]

\[ T = \text{buoyant force of water pressure (sea water)} \]

\[ = \frac{8.3 \text{ kip} \cdot \text{ft} \cdot \text{in} \cdot \text{lb} \cdot \text{in}}{12 \text{ in} \cdot \text{lb}} \]

\[ = 614 \text{ lb} \cdot \text{ft} \cdot \text{in} \cdot \text{lb} \cdot \text{in} \]

\[ = 4190 \text{ lb} \cdot \text{ft} \cdot \text{in} \cdot \text{lb} \cdot \text{in} \]

\[ \text{Equilibrium: } T = F_b - W = 4190 - 1800 = 2390 \text{ lb} \cdot \text{ft} \cdot \text{in} \cdot \text{lb} \cdot \text{in} \]

\[ \text{Average shear stress in pin (under double shear)} \]

\[ \tau_{\text{ave}} = \frac{T}{A} = \frac{2390 \text{ lb} \cdot \text{ft} \cdot \text{in} \cdot \text{lb} \cdot \text{in}}{1.05 \text{ in} \cdot \text{lb} \cdot \text{in}} \]

\[ = 608 \text{ psi} \]

(Continued)
Bearing stress between the pin and shackle:

The bearing area is
\[ A = 2.5 \text{ in} \times 0.25 \text{ in} = 0.3125 \text{ in}^2 \]

\[ \sigma_b = \frac{F_b}{A} = \frac{2340 \text{ lb}}{0.3125 \text{ in}^2} = 7480 \text{ psi} \]

Thus, the ultimate stress governs.

Allowable compressive force:
\[ P_{\text{allow}} = \frac{F_b}{A} = 7480 \text{ psi} \]

\[ \sigma_{u} = 310 \text{ Mpa} \]

Calculating the allowable compressive force \( P_{\text{allow}} \), if the factors of safety with respect to the yield stress and the ultimate stress are 3 and 5, respectively.

\[ A_{\text{yield}} = \pi \left( \frac{d}{2} \right)^2 - \pi \left( \frac{d}{2} - \frac{t}{2} \right)^2 \]

\[ A_{\text{yield}} = 176.7 \text{ mm}^2 \]

\[ \sigma_{y} = 270 \text{ Mpa} \quad \text{with F.S. = 4} \]

\[ \sigma_{u} = 310 \text{ Mpa} \quad \text{with F.S. = 5} \]

\( P_{\text{allow}} = \frac{F_b}{A} = \frac{270 \text{ Mpa}}{4} = 67.5 \text{ Mpa} \)

\( P_{\text{allow}} = 67.5 \text{ Mpa} \)

The next thing is to find \( \sin \theta \):

\[ \begin{align*}
\text{suppose } |AC| &= x, \quad |BC| = y \\
\frac{x + y}{L} &= \frac{L}{L} \\
\frac{x \cos \theta + y \sin \theta}{L} &= \frac{L}{L} \\
\sin \theta &= \frac{\sqrt{L^2 - x^2}}{L} \\
\cos \theta &= \frac{\sqrt{L^2 - y^2}}{L}
\end{align*} \]

\[ P = 2T \sin \theta = 2T \sin \left( \sqrt{L^2 - x^2} \right) \]

With breaking strength \( S = 200 \text{ N} \) and F.S. = 4, \( T_{\text{allow}} = \frac{S}{T_{\text{allow}}} = \frac{200 \text{ N}}{4} \)

\[ P = T_{\text{allow}} \sqrt{1 - \left( \frac{L}{L} \right)^2} \]

\[ P = \frac{200}{3} \sqrt{1 - \left( \frac{L}{135} \right)^2} \]

\[ P = 80 \text{ N} \]
1.8-10 A bar of rectangular cross section is subjected to an axial load $P$ (see figure). The bar has width $b = 60$ mm and thickness $t = 10$ mm. A hole of diameter $d$ is drilled through the bar to provide for a pin support. The allowable tensile stress on the net cross section of the bar is 140 MPa, and the allowable shear stress in the pin is 80 MPa.

(a) Determine the pin diameter $d_p$ for which the load $P$ will be a maximum. (b) Determine the corresponding value $P_{max}$ of the load.

**Allowable load based on tension in bar**

$$P_t = 0.05 \times A_{net}$$
$$= 140 \text{ MPa} \times (b - d) \times t$$
$$= 140 \text{ MPa} \times (60 \text{ mm} - d) \times 10 \text{ mm}$$
$$= [1400 \times (60 - d)] \text{ N}$$

$d$ in mm

**Allowable load based on shear in pin**

$$P_{shear} = 2 \times T_{allow} \times (\frac{d}{2})^2$$
$$= 2 \times 80 \text{ MPa} \times \frac{\pi}{4} \times \left(\frac{d}{2}\right)^2$$
$$= 40 \pi d^2 \text{ N}$$

$d$ in mm

Graph of eqn 1 and 2 (please see next page) and we can find where the two lines cross each other. And at that point, we will have $max(P)$.

(Continued)