Bump-Stock Gun Dynamics: a simple model

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Abstract

Bump-stock guns have been imfamously abused and thes been proposed for outlawing. A bump-stock gun a modification to a 'semi-automatic' gun to make it effectively fully automatic. This is done using a trivial device called a 'bump stock'. With this design the weapon slides forward and backwards, pushed forwards by the hand grasping the barrel and backwards by the recoil from firing. At the forward extreme, the trigger bumps up against a stationary finger on a hand on the stationary 'stock'. A simple model predicts an oscillation that, in the simplest case, is period 2 (there is a family of period-2 solutions). Trying to fire too fast, extinguishes the firing.

1 Introduction

The bump-stock style of automatic gun was made famous by its horrible use in a mass murder on October 1, 2017 in Nevada, and again by the disingenuous Feb 20, 2018 Trump suggestion in Florida, that it be outlawed. The idea of the bump-stock gun, and a simple analysis of it, is presented here. The idea is obviously not new; it's a thing you can buy. The analysis here is simple enough that many have probably done it, but we do not if or whrere it is written up.

In sequence, below, we explain guns, modern guns, semi-automatic guns and then the bump-stock. Then we give a simple model of the bump-stock and the solutions of the model equations. Finally, a few comments about things the analysis highlights.

1.1 What is a gun?

A gun is a small cannon. A sturdy hollow cylinder makes up the barrel, which is closed at one end. The open end points to the target. A propellent, like gunpowder, is packed at the closed end, followed by a heavy metal often-lead canon ball, slug or bullet. A small hole at the closed end allows ignition. In old-fashioned cannons this was a small string that would burn. In early guns, this was a flint. A small swinging hammer is spring loaded, 'cocked'. One finger on a trigger releases the hammer which then, propelled by the spring, swings towards the flint. When the flint is hit with the hammer, it sparks and ignites the gun powder. The gun powder explodes, or at least burns very quickly, propelling the slug (the bullet) down the barrel and out towards its target.

After a shot, one has to clean out the gun, then put in new gun powder and a new slug and possibly a new flint. And then cock the hammer to ready it for the next finger-press on the trigger.

1.2 What is a modern gun?

A modern gun uses a cartridges. One cartridge is a single disposable thing that contains the 'flint', or some ignition system, gunpowder and the bullet, all in a light metal shell. The informal word 'bullet' technically is the slug, but sometimes, in informal usage, refers to the cartridge ("I'm out of bullets'). At the base of the shell is a small slightly-moving part that, when the 'hammer' hits it, causes igition by a spark or be excitement of a pressure-sensitive 'primer', that ignites the gun powder and so on. That is, a modern bullet is itself a little one-time-use machine with 4 parts: shell, primer, powder and bullet.

After a shot, one takes out the empty shell, puts in new cartridge and cocks the hammer, readying it for the trigger release. On some guns, say a revolver holding 6 bullets, some of this is slightly automated. On a revolver the trigger finger pull takes some strength. A cylinder containing, say, 6 bullets is advanced so a new cartridge is lined up, and the hammer is cocked, possibly with a thumb action or possibly with

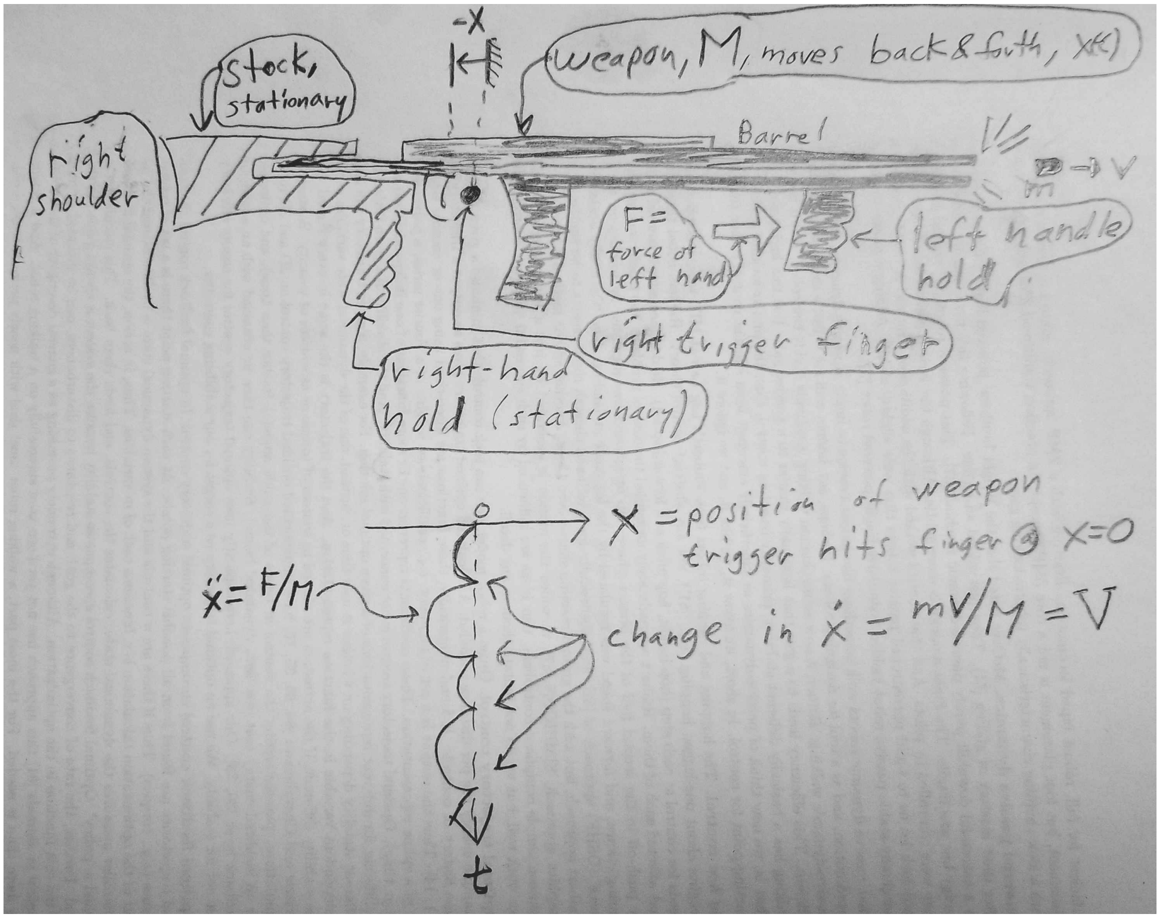

Figure 1: **Gun and its motion.** The gun is made up of the stationary 'stock' held against the shoulder and the weapon which moves back and forth with $x(t)$. The right hand holds the stock in place, the left hand tries to push the weapon away, the gun recoil pushes the gun back. The 'stock' the right hand and the right trigger finger are all stationary.

a mechanism from the trigger. Shots can be repeated with a strong finger pull, possibly supplemented with a hammer-cocking thumb action.

1.3 What is a semi-automatic gun?

In a modern 'semi-automatic' gun, the trigger finger only has to release the cocked hammer and do no more. This is only a small slight motion. The burning expanding propllent powers the bullet. This explosive power is also harnessed to eject the used shell, and the loading of a new bullet and the re-cocking of the firing hammer. This is a complicated fine machine design. So, with a semi-automatic gun, sequence of small trigger-finger pulls makes a sequence of gun firings, each involving a complex sequence of automatic actions (ejection, reloading, recocking). Once you have a semi-automatic gun you almost have a machine gun, all you need is a way to repeatedly pull the trigger quickly. One way to do that is with a 'bump stock'.

1.4 What is a 'bump stock'?

This is best explained with the short New York Times video. The gun has two big parts. The *stock* and the *weapon*. The 'weapon' part is all of the working parts of a semi-automatic gun, as described above: barrel, trigger, hammer and all the mechanism to eject and reload the shell and cock the hammer. The stock is just a piece of wood (or some plastic composite), for bracing the gun against the shooter's shoulder and for guiding the sliding weapon. The weapon can slide a few inches relative to the stock, along the direction of the barrel, which is the direction the gun shoots. There is no spring or mechanism, just two parts connected like a trombone slide that allows for relative sliding of the weapon relative to the stock.

The shooter holds the weapon with the left hand and the stock with the right. With his right hand he always holds the stock stationary against his right shoulder. The right stationary hand on the stock, has an extended stationary trigger finger. When the weapon slides forwards, when pushed by the left hand, the trigger (part of the weapon) bumps up against the stationary right trigger finger. The gun fires, recoils (every action has a reaction), and slides on the stock back towards the shooter. The shooter continues to push the weapon forwards with the left hand, causing the weapon to again slide forwards. The trigger again hits the stationary trigger finger, and the process repeats. Each shot is a forwards push until triggering, a consequent firing and backwards recoil, and then a reverse of direction of the weapon, by the continued forwards left-hand push, back towards another contact of the trigger up against the stationary trigger finger.
Minute 5:00 of this You Tube video also makes the bump-stock mechanism clear.

2 A simple model.

The weapon has mass M and the bullet mass $m \ll M$ so M may be approximated as constant through the process. The bullet exits the gun with a relative speed v causing a sudden change in the gun velocity by $V = mv/M \ll v$. All the while the left hand is pushing on the weapon with force F.
The simplest model. Assume the forwards force F of the left hand is a constant. Define the states $^-$ and $^+$ as just before and after the nth firing of the gun which is just after its nth recoil motion. The forwards motion of the gun x is measured relative to its position when the trigger is sufficiently deflected by the stationary finger. Thus we have these equations:

$$\text{The firing event (conservation of linear momentum):} \qquad \dot{x}_n^+ = \dot{x}_n^- - V \tag{1}$$

$$\text{Between firings } (F = ma): \qquad \ddot{x} = F/m \tag{2}$$

Between events, $x < 0$. The next event is triggered by x again reaching zero, from below. For any conservative left-hand force, for example a constant force or a spring-like force, the gun velocity $\dot{x}$ is the same magnitude just before one firing as it was just after the previous firing.

$$\dot{x}_{n+1}^- = -\dot{x}_n^+ \tag{3}$$

Putting these together we get the governing equation for the sequence of weapon speeds just before triggering:

$$\dot{x}_{n+1}^- = V - \dot{x}_n^- \tag{4}$$

After each firing, the gun speed is V minus the speed of the previous firing. In normal operation, the gun is going forwards before the shot ($\dot{x}_n^- > 0$) and backwards after a shot ($\dot{x}_n^+ < 0$). Associated with each gun speed $\dot{x}_n^-$ just before a shot, we have the duration since the previous shot. We also have the associated distance the gun moved back and up. These follow from the standard constant-acceleration formulas from freshman physics

$$\begin{aligned} &\text{Duration between firings:} && T_n = 2\dot{x}_n^- M/F \\ &\text{Net recoil distance (amplitude of motion)} && d_n = (\dot{x}_n^-)^2 M/(2F) \end{aligned} \tag{5}$$

The trigger does not reset unless the gun is sufficiently untriggered, that is, slides backwards enough relative to the stationar finger. There is a critical discance d_c, so we have the inequality constraint

$$d_n \geq d_c \qquad \text{for all } n. \tag{6}$$

Using equations 5 we get corresponding restrictions on the time between shots:

$$T_n \geq T_c = 2\sqrt{2 d_c M/F} \qquad \text{for all } n. \tag{7}$$

2.1 Solution of equations

These equations have the general solution

$$\dot{x}_n = \left[\begin{array}{ll} v_c < v_n < V & n = \text{even} \\ v_c < V - v_{n-1} < V & n = \text{odd.} \end{array} \right. \tag{8}$$

In each periodic motion the gun velocity oscillates between two values, with average value of $V/2$. We have associated durations between shots.

If we pick out the symmetric periodic motion then we have

$$\text{Gun speed} \qquad |\dot{x}^+| = |\dot{x}^-| = V/2 = mv/(2M), \tag{9}$$

$$\text{Time between firings:} \qquad T = 2VM/F = 2vm/F, \tag{10}$$

$$\text{Recoil distance:} \qquad d = V^2M/(2F) = v^2m^2/(2MF). \tag{11}$$

Recall that v is the bullet velocity as it leaves the gun (muzzle velocity). If we assume that there is a critical value d_c for the recoil distance, enough to reset the trigger, then we get a maximum firing-rate condition by using $d = d_c$, solving for F and finding the minimum firing time (the reciprocal of the maximum firing rate). Assuming symmetric solutions:

$$\text{Maximum force that avoids periodic misfirings:} \qquad F_c = m^2v^2/(2Md_c), \tag{12}$$

$$\text{Minimum firing time} \qquad T_c = 2Md_c/(mv) \tag{13}$$

3 Some observations

1. Superficially this is a second-order autonomous system. Thus it should not allow period-2 solutions. However, this system has impulses. 2nd order dynamical systems with impulses obey the usual dynamical systems features (*e.g.,* no chaos, no period-2 motions) so long as the impulses can be found as the limiting case of some $F(x, \dot{x})$. Normal bouncing with a coefficient of restitution, for example, fits in this class. The fixed impulse in this model, however, does not. Thus precluding the theorem which would have, if it applied, precluded the period-2 motions we find.
2. These equations are 'structurally unstable', like a 'center' in Dynamical Systems. This family presumably reduces to a single stable periodic motion, with every shot the same, for almost any model of F that includes some damping. On the other hand, the 'structural unstable' nature also implies that an appropriately chosen small perturbation to the model would lead to chaotic motions. That is, the simple model shows that the actual dynamics depends sensitively to details that are not in the model.
3. For any force F the solutions can be unsymmetric due to small deviations from the model. If the motion becomes sufficiently unsymmetrical, then the critical trigger reset will not occur. Thus a shooter can try to achieve a higher and higher shoot rate, applying a bigger and bigger force F, with increasing chance of the trigger not resetting and the firing sequence coming to a stop.

4 Parameters for the AR-15

We found some rough values for the physical properties of the AR-15, a popular semi-automatic assault rifle. These values vary significantly with the individual gun's hardware and configuration.

Bullet mass (e.g. .223 Remington):	$m \approx 3.6\,\text{gm} \approx 0.0036\,\text{kg}$
Loaded gun mass:	$M \approx 55\,\text{N}$
Trigger travel:	$d_c \approx 0.1\text{''} \approx 0.25\,\text{cm} \approx 0.0025\,\text{m}$
Muzzle velocity:	$v \approx 3250 ft/s \approx 1000\,\text{m/s}$

These numbers yield $T_c = 0.075s$, or a maximum symmetric firing rate of 13 rounds fired per second. The Las Vegas shooter fired a bit slower, 90 rounds in about 10 seconds (about 9 rounds per second). (NY Times). Perhaps

- the period-2 nature of the solution, mixed with the need to keep the distance of each slide $\geq d_c$, demands a mixture of a slower firing rate, and a coordination (damping in added to the force F), that keeps the firing rate below the maximum rate; or
- the Nevada shooter could have shot faster; or,
- one or another of our parameter values is off.

4.1 Model defects.

1. The force of the left hand is surely not constant. However, the sequence of gun velocities V_n^- only depends on the force being conservative. So, if the hand force F were instead spring-like, say $F = -kx$, the sequence of gun velocities would be the same. The sequence of times would be changed.
2. The above statement also applies if we include the force of the trigger, so long as the trigger is also conservative (*i.e.,* same force during releeas as during pressing).
3. A more realistic muscle-applied force the model would include a velocity dependence (for muscle dissipation). Similarly, if the trigger force is less during release than during pressing there is dissipation there. These dissipations, even if small, would presumably stability the period-2 solutions to a stable period-1 solution.
4. We have neglected the motion of the stock and the shoulder. No matter what model we would use for such, it would raise the order of the model to at least 4, increasing the complexity of the model. We guess that the gain in realism is not that high, but do not know that.

5 Punchline

Given a 'semi-automatic' gun, modification to make a machine gun is a small step. Almost not a step. A bump-stock is a single chunk of material (plastic or wood) with no moving parts of its own, easily made without the high-tolerances of the rest of the gun. You could also put something like a hand mixer against the trigger of a 'semi-automatic' gun for a similar effect. Given the existence of semi-automatic weapons, banning bump-stocks seems to have little gain, anyone can make one and there are other ways to do the same thing.

A semi-automatic weapon is a sophisticated machine that cannot be easily made, and thus should be easier to regulate.

As per the equations here, the firing rate of a bump-stock design could be restricted by demanding that the reset distance d_c on the gun, be large.

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