Speed, Efficiency, and Stability of Small-Slope 2-D Passive Dynamic Bipedal Walking

Mariano Garcia
Theoretical & Appl.Mech.
Cornell University
Ithaca, NY 14853

Anindya Chatterjee
Dept. Of Engr. Sci. & Mech.
Penn State University
University Park, PA 16802

Andy Ruina
Theoretical & Appl.Mech.
Cornell University
Ithaca, NY 14853

Abstract

This paper addresses some performance limits of the kneed and non-kneed passive-dynamic walking machines discovered by McGeer [10, 11]. Energetic *in*efficiency is measured by the slope γ needed to sustain gait, with $\gamma = 0$ being perfectly efficient. We show some necessary conditions on the walker mass distribution to achieve perfectly efficient walking. From our experience and study of a simpler model, only two gaits exist; the longer-step gait is stable at small enough slopes. Speed is regulated by energy dissipation. Dissipation can be dominated by a term proportional to speed^2 or a term proportional to speed^4 from normal foot collisions, depending on the gait, slope, and walker design. For special mass distributions of kneeless walkers, the long-step gaits are especially fast at small slopes. A period doubling route to chaos is numerically demonstrated for the kneed walker.

1 Introduction

This paper extends McGeer's work on passive-dynamic walking in the following ways that have not been described in previous publications: 1) near-zero slope walking is found for a class of kneeless and kneed 2-D walkers, 2) scaling laws are found for small slope walking for more than just the simplest walker, and 3) period doubling and chaos is found for kneed walking. The results may help those trying to build efficient robots, and to those trying to understand human walking. Energetic efficiency and speed maximization are obvious goals of both biological and artificial locomotion and transportation systems. Since animals and potentially-useful robot designs use legged walking motions it is interesting to consider the performance limits of such machines. In this paper, we address these questions in the context of two-dimensional passive-dynamic walking machines. We also describe some other newly found properties of these machines.

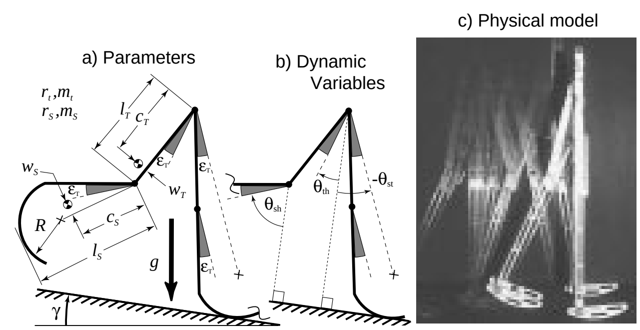

Figure 1: **(a)** Our description of McGeer's kneed walking model. Radii of gyration and masses of thigh and shank are denoted by r_t, m_t, r_s, and m_s, respectively. The circular-arc foot is centered at the $+$. ϵ_T is the angle between the stance thigh and the line connecting the hip to the foot center. A stop at each knee prevents hyperextension. **(b)** Dynamic variable values θ_{st}, θ_{th}, and θ_{sh} are measured from ground-normal to lines offset by ϵ_T from their respective segments. **(c)** A strobe photo of our physical model walking with dimensional parameters: $l_t = 0.35$m, $w_t = 0$m, $m_t = 2.345$kg, $r_t = 0.099$m, $c_t = 0.091$m, $l_s = 0.46$m, $w_s = 0.025$m, $m_s = 1.013$kg, $r_s = 0.197$m, $c_s = 0.17$m, $R = 0.2$m, $\gamma = 0.036$rad, $g = 9.81$m/s^2, $\epsilon_T = 0.097$rad.

2 Passive Dynamic Walking Machines

Passive-dynamic walking machines that walk on shallow slopes were first designed, simulated and built by Tad McGeer [10, 11]. These machines consist of hinged rigid bodies that make collisional and rolling contact with a sloped, rigid ground surface. They are powered by gravity and have no control. The 2-D kneed walking machine we study here, essentially

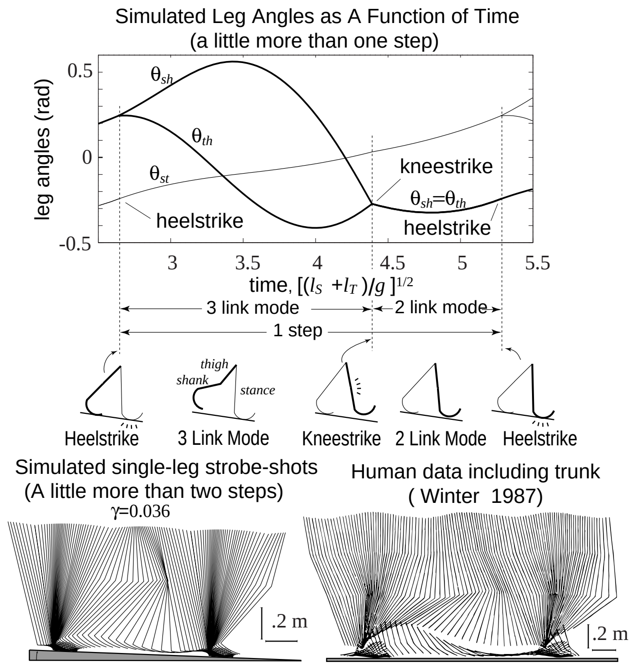

Figure 2: Simulated gait cycle (as per McGeer) of the walker in figure 1c. Angles of leg segments are shown from just before a heelstrike to just after the next heelstrike in a steady gait. The heavy line on the graph corresponds to the motion of the heavy-line leg on the small cartoon under the graph. At the start this is the stance leg, but it becomes the swing leg just after the first heelstrike, and again becomes the stance leg after the second heelstrike. The angular velocities of the joint segments have discontinuities at kneestrike and heelstrike, which appear as (barely visible) kinks in the curves. The strobe-like animation from the same simulation can be compared to measured human data (with a smaller scale and a longer stride).

a copy of McGeer's design, is shown schematically in figure 1. It consists of a *swing* leg (not in contact with the ground) and a *stance* leg (touching the ground), connected by a frictionless hinge at the hip. The non-kneed, or *kneeless* machines have the knee joint locked.

Figure 2 shows the simulated motion of such a machine.

The three features that make the McGeer-like models so intriguing for both robotics and the understanding of animal gait are these:

1. Existence of gait. A mechanism that resembles human legs in overall layout has an uncontrolled periodic motion that is rather anthropomorphic, as can be seen real models and simulations, or comparing simulated strobe data with human data (lower part of figure 2). Since passive walkers seem to be somehow close to human walkers, there is reasonable hope of learning something about human walking by studying these simpler passive models.

2. Efficiency. These machines can walk down shallow slopes. McGeer numerically found walking motions for slopes as low as about 0.005 radians and we will show here predictions of arbitrarily small slopes. Passive-dynamic based designs using other-than-gravity power schemes, e.g.,. toe-off, could have similarly high efficiencies. As argued clearly in, e.g.,[2], both evolutionary pressure and individual motivation push for high efficiency in animal locomotion.

3. Stability of gait. For certain parameter combinations, McGeer found *stable* limit cycle motion for both 2-D straight-legged and kneed walkers as [9] and [5] later repeated for some 2-D straight-legged walkers and we have repeated for kneed walkers (this work), and [3] found experimentally with a 3-D device. These stable motions indicate the possible role of passive-dynamics in *stabilizing* things which one might think need controlled stabilization.

It seems likely that the primary cost of locomotion is in the mechanical energy, and not the neural activity of control. It is thus natural to imagine that in evolution and learning of walking, a primary goal in perfecting walking motions would be energy-efficiency more than simplicity of control strategies. Such efficiency might even be achieved at the expense of passive stability, in contrast to item 3 above. Unstable limit cycle motions of mechanical systems can in principle be stabilized with 0+ energetic cost as has been addressed for a three-dimensional walking model by [4]. The tradeoffs, or lack thereof, between efficiency and stability for such systems are far from understood in these non-holonomic systems [12].

Method of analysis. The method used follows McGeer and is described in detail in, e.g., [5]. We find the Poincaré map for the change of the state of the walker in one step (starting just after heelstrike) by solving the Newton-Euler differential equations and collisional jump conditions numerically. Fixed points of this map are period 1 walking motions. We find both stable and unstable fixed points by numerical root finding. We evaluate the linearized stability of these gaits by numerically differentiating the Poincaré map. For general straight legged walkers the map of the fourth order system is three-dimensional (two for

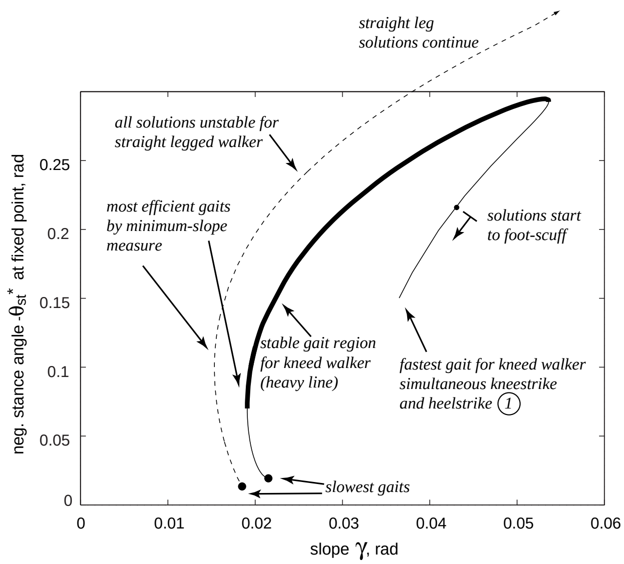

Figure 3: Calculated locus of solutions showing fixed points of stance angle as a function of slope for our physical kneed walking model (solid line) and for the same model but with the knees locked (dashed line). Each point on this graph is one periodic solution. The thick portion of the solid line denotes stable solutions for the kneed walker.

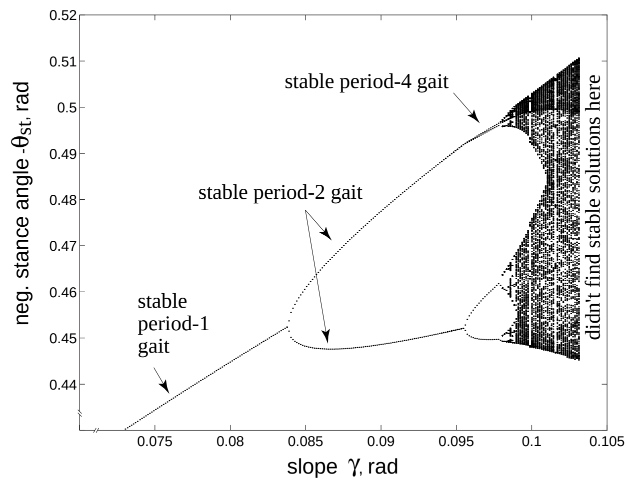

Figure 4: Period doubling route to chaos in *stable* kneed walking motions. The parameters are those of the zero-slope capable walker labeled (C) in figure 6.

some special cases). For the superficially 6th-order kneed walker the map is also only three-dimensional because the knee angle is not independent at heelstrike and because part of the swing-phase motion is only 4th order (Thus one of McGeer's numerically calculated eigenvalues near zero is actually zero). A plot of segment angles during a typical kneed gait cycle is shown in figure 2. For a straight-legged walker, the 3-link mode is absent, and $\theta_{sh} \equiv \theta_{th}$.

By assuming motions as described above, some of the periodic solutions we find might violate various physically-relevant inequality conditions [11]. We allow some of these violations since they could be circumvented by zero-energy-cost control action [6]. Allowed violations include scuffing of the swing foot (i.e.,. passing slightly underground when the two legs are parallel), unlocking torques at the stance knee, and hyperextension of the newly swinging leg.

3 Gaits of Generic Kneed and Straight-Leg Walkers

The number of gaits and their stability. Although the root finding involved in finding a gait cycle involves the solution of n equations in n unkowns (where n is the dimension of the return map) there is no *a priori* guarantee that any gait cycles will exist for a given passive-dynamic walking machine (i.e., a given set of masses, lengths, etc.), on a given slope γ. In practice, all searches with all designs have found either 0, 1, or 2 anthropomorphic solutions for given machine parameters and slope. Other non-anthropomorphic solutions may exist but are discounted. In these non-anthropomorphic solutions the leg swings forward and backward more than once, or the swing leg makes full revolutions.

Effect of varying slope. Figure 3 shows stance angle θ_{st} at the start of a gait cycle while slope is varied, for a given mass distribution, with and without knees. For both kneed and kneeless walkers there are slope γ regimes where there are either 0,1, or 2 solutions. Along the kneed curve, kneestrike occurs later and later in the step, until at one end of the locus of solutions (point 1), heelstrike and kneestrike occur simultaneously. At the other end of the locus of solutions the walker has just enough initial kinetic energy for the stance leg to make it past the vertical position. This is the slowest gait for these walkers. Neither of these walkers can walk at arbitrarily small slopes.

4 Chaos In A Kneed Walker

Like the simplest walker of [5], and the walkers of [13] and [9], kneed walkers can also exhibit period doubling and chaotic gait, as shown in figure 4.

5 Measures Of Performance

Since moving sideways in a gravitational field is workless (neglecting air friction), a rational dimensionless measure of work efficiency is somewhat problematic for locomotion on level ground. A natural measure of inefficiency, however, is the *specific cost of transport* η, (energy used)/(weight $\times$ distance travelled). It, as well as a few other reasonable measures of transport cost, reduces to the slope γ for small-slope passive-dynamic walking [6].

6 Walking At Near-Zero Slopes

Passive-dynamic walking at near-zero slopes has previously been demonstrated for the simplest walker [5]. Here we seek more general 2-D kneed and kneeless designs capable of zero-slope walking. Mathematical justifications for some of the arguments here can be found in [6].

Necessary Conditions on Mass-Distribution For Near-Zero-Slope Walking. Necessary conditions on the mass distribution for near-zero slope walkers are found as follows:

1. If walking motions do occur at very small slopes, these motions will be very slow [6]. The walker must be close to static equilibrium at all times. In the limit of zero slope, the walker configuration must approach a static equilibrium configuration. Thus the foot contact point must be where the foot-normal is directed towards the body center of mass.

2. At heelstrike both legs are straight and simultaneously touch the ground. As the slope (hence, step length) goes to zero, the spacing between the legs at this instant also goes to zero. In the limiting case, the foot contact point is seen to be that point on the foot which is farthest from the hip. Thus the normal to the foot contact point must pass through the hip.

3. From (1) and (2) the line from the hip through the body center of mass must intersect the foot curve normally at the nominal contact point at zero-slope walking. For circular feet this is equivalent to the collinearity of the center of mass of the whole body, the hip, and the foot center (see figure 5).

4. For the swing leg to be in static equilibrium in 3-link mode and to have zero knee-locking torque, the center of mass of the shank must lie directly under the knee, in the straight-leg configuration (see figure 5).

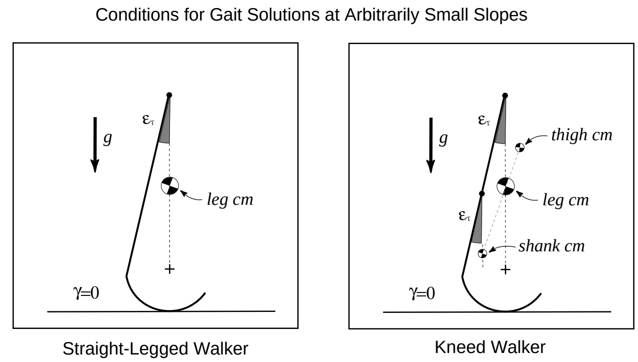

Figure 5: **Necessary conditions for near-zero slope walking.** The separations between the hips and the mass centers shown here are larger than those typical for our simulations. The necessary condition for elimination of first-order scuffing from knee flex is not shown.

These necessary conditions on the mass distribution do not guarantee that near-zero slope walking solutions exist. In simulations we have found these conditions lead to zero-slope walking designs if the body center of mass is close to the hip. These conditions are reminiscent of the "straight" joints suggested by Alexander [1] as a means of achieving efficiency.

The Simplest Walking Model. Alexander's *minimal biped* [1] has a point-mass at the hip but no mass in the legs. Hence the minimal biped can be (and needs to be) supplemented with further stride length conditions. To make the minimal biped leg swing most simply deterministic, the simplest walker adds vanishing point-mass feet. The simplest walker is the minimal biped with miniscule feet. The simplest walker is also the limiting case of the 2-D McGeer straight leg walkers with no knees, a finite point-mass at the hip, and a vanishing point-mass at the point feet. The results presented here are generalizations of the results found for the simplest walker by Garcia *et al.* in [5].

The simplicity of the "simplest" walker is that: the non-dimensional form of the simplest walker has as its only free parameter the slope γ; the stance leg dynamics are decoupled from the swing leg motions; the Poincaré map is only two-dimensional and; the motion was only studied in a regime where most of the governing equations were linear.

By a mixture of analytic and numerical means the simplest walker was found to have two gaits at all small-enough slopes. Of these, the long-step gait is stable at small slopes ($\gamma < 0.015$), while the short-step gait is unstable at all slopes. The simplest walker was found to have near-zero slope walking with speed proportional to the cube root of the slope. It was found to have a period doubling route to chaos. This period doubling and chaos were discovered independently and reproduced (respectively) by [7, 9] in studies of a less extreme point foot model. For both gait cycles of the simplest walker the stance angle θ_{st} or step-length is proportional to $\gamma^{1/3}$ at small slopes, while the step periods tend to (different) nonzero constants. This implies a walking power consumption proportional to the fourth power of speed for small speeds, for *both* gaits. That is, power $\propto$ (speed)^4. This power scaling can be derived from Alexander's [2, 1] minimal biped results by assuming speed is proportional to step length (as it is for both small-slope gaits of the simplest walker).

An interesting feature of the long-period gait is that, in the limit $\gamma \to 0$, has a time-reversal symmetry. The configuration with both legs vertical is passed. Defining $t = 0$ for this configuration, $\theta(-t) = -\theta(t)$ for both legs. Equivalently, a movie of this walker shown backwards looks like a movie of the walker walking forwards in the opposite direction. A consequence of this symmetry is that the long period gait has no component of foot velocity tangent to the surface at heelstrike.

This symmetry is approximately observed even for the somewhat generic kneed walker of figure 2 (after averaging the shank and thigh motions, the shape of the curves is nearly preserved by 180 degree rotation of the graph).

Not quite the simplest walker. For the simplest walker, with negligible mass, the only kinetic energy lost is that of the hip. When the feet have finite mass, however, they also lose energy at heelstrike. If the striking foot hits the ground with no tangential velocity, its loss still scales as step length to the fourth power. If, however, the foot collision has a grazing component, then the energy lost scales as the step-length squared.

The point-foot walker still has two solutions at small slopes, even with non-negligible foot mass. To first order in the slope, the long-step solution has time-reversal symmetry, no tangential foot collisions, and step length proportional to the cube root of the slope. The short-step solution has some tangential component in foot collisions. Because this involves a finite mass colliding at a speed proportional to step length, the $\gamma \to 0$ motion has step length proportional to slope. Thus for the long-step gait power $\propto$ (speed)^4.

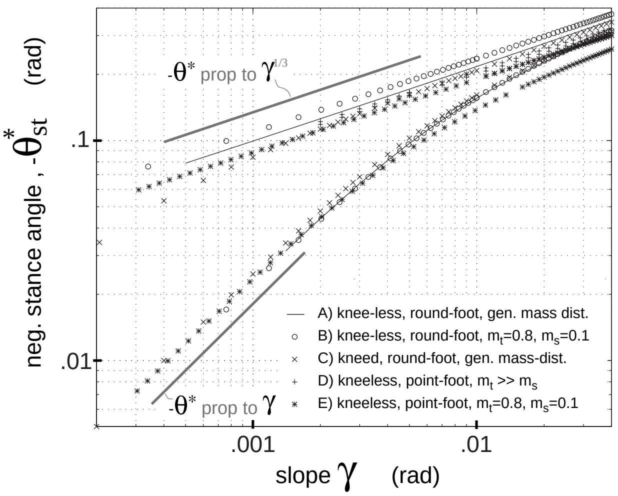

Figure 6: Low slope step-lengths of some zero-slope capable walkers vs slope γ. There are two gaits cycles at each slope, for each walker considered. Both step lengths for the simplest walker are proportional to $\gamma^{1/3}$ for small γ. The short-step gaits of the other walkers have step lengths proportional to γ for small γ. The long-step gaits for the other walkers have step lengths that are much longer than for the short-step gaits, approximately approaching $\gamma^{1/3}$ for larger γ. For a point-foot, kneeless walker with non-negligible foot mass, the step length of the long-step gait proportional to $\gamma^{1/3}$ for small γ.

For the short-step gait, power $\propto$ (speed)^2, at least at small speeds.

Scaling rule for more general straight-legged walkers. Near zero-slope the time period of steps for zero-slope capable walkers is asymptotically constant as $\gamma \to 0$. At steady walking the energy lost in collisions is balanced by the gravitational potential energy. The collision loss per step is proportional to the speed of the colliding foot contact point squared. The gravitational energy available is proportional to the product of step length and slope.

Taking account both the effects of the mass distribution on collisions and the kinematics of these walkers it is found that the dissipation per step is dominated by either $A{\theta^*}^2$ tangential foot collision term or a $B{\theta^*}^4$ normal foot collision term. This latter term is analogous to the point-mass collisional loss term for the point-mass-at-hub rimless wheel (or minimal biped) model of Alexander, but is derived for more general mass distributions of this 2 link mechanism [6]. Equating this loss with the gravitational energy available gives

$$A{\theta^*}^2 = \gamma\theta^* \quad \text{or} \quad B{\theta^*}^4 = \gamma\theta^*. \tag{1}$$

For zero-slope capable straight leg walkers, there is *no* tangential component of foot velocity at heelstrike for the long-period time-symmetric gait. Such walking motions follow the scaling rules found for the simplest walker [5]. Step length goes up with the cube root of slope. Power for walking increases with the fourth power of speed.

If the tangential foot velocity term is non-zero, the walkers have step length proportional to slope and the power for walking scales with the speed squared, at least as $\gamma \to 0$

Extension of scaling rule to kneed walkers. Kneed walkers dissipate kinetic energy in collisions at both heelstrike and at kneestrike. For heelstrike, the energy loss calculations described above still hold: the pre-collision velocities are determined from the straight-leg or 2 link configuration. The knee collision loss, for zero-slope walkers, scales with speed^2 and thus dominates at small enough slopes. However, the collisional loss of knee-strike is very small. In figure 6 it is only at the far left of the graph that the dominance of the knee-strike losses show as a switch from step length proportional to $\gamma^{1/3}$ to step length proportional to γ.

7 Conclusions

We have investigated the design of straight-legged and kneed passive-dynamic walkers that will walk at arbitrarily small slopes. At high speeds the power required for walking scales as v^4. At low speeds the scaling can be either with v^2 or v^4. On the one hand this shows how bad this kind of walking is when it is fast. On the other hand the v^4 scaling rule implies very small energy demand at low speeds. The essence of the scaling follows from the energy balance between collision losses and gravitational potential energy [5], rather like for the rimless wheel described in [10, 2, 1]. Perhaps one reason people in fact walk with higher frequencies at higher speeds, rather than to use a strictly pendulum-like fixed-period swinging motion, is to defeat the collisional losses which depend so strongly on step length.

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