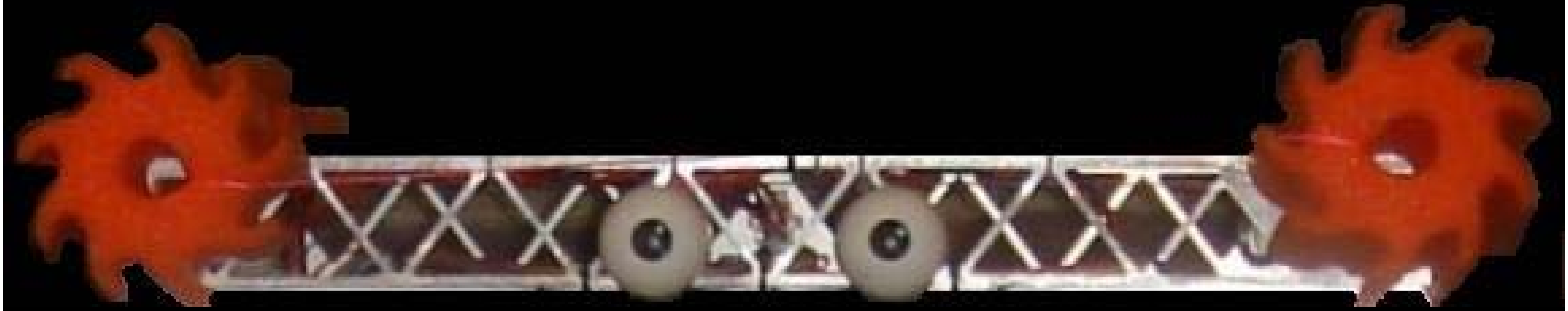
Cornell University

Physics of Walking Robots



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Biorobotics and Locomotion Lab

Dept. of Theoretical and Applied Mechanics

Biorobotics and Locomotion Lab:

- Robot, animal, and human walking and running
- Muscle performance
- Bicycle balance and stability
- Human power
- Rowing and other sports
- Mechanics of prosthetic devices

Inspiration for our robot research

Passive Dynamic Walkers

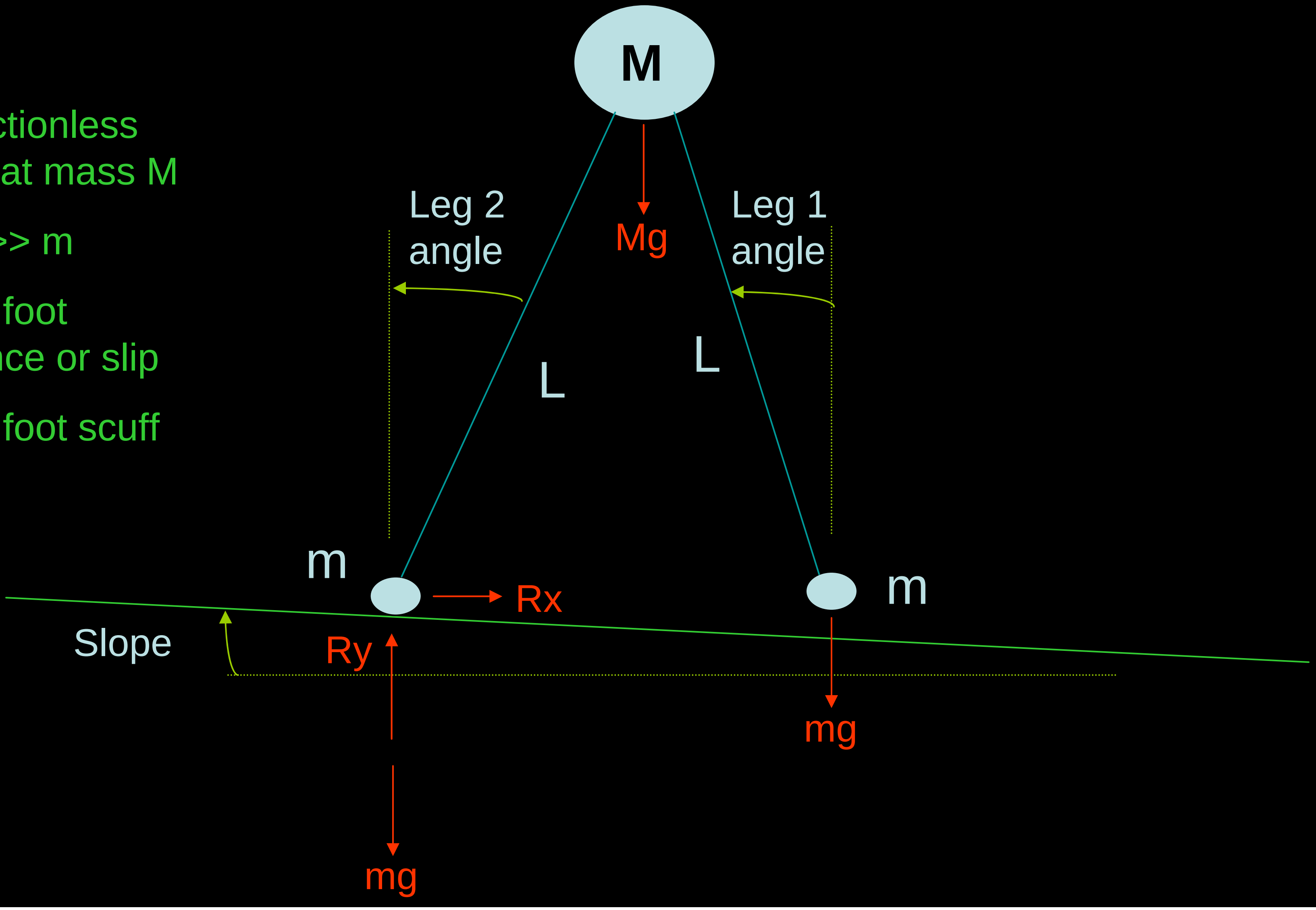
Research tools

- Analytical methods
- Numerical simulations
- Study walking in humans, animals
- Build robots

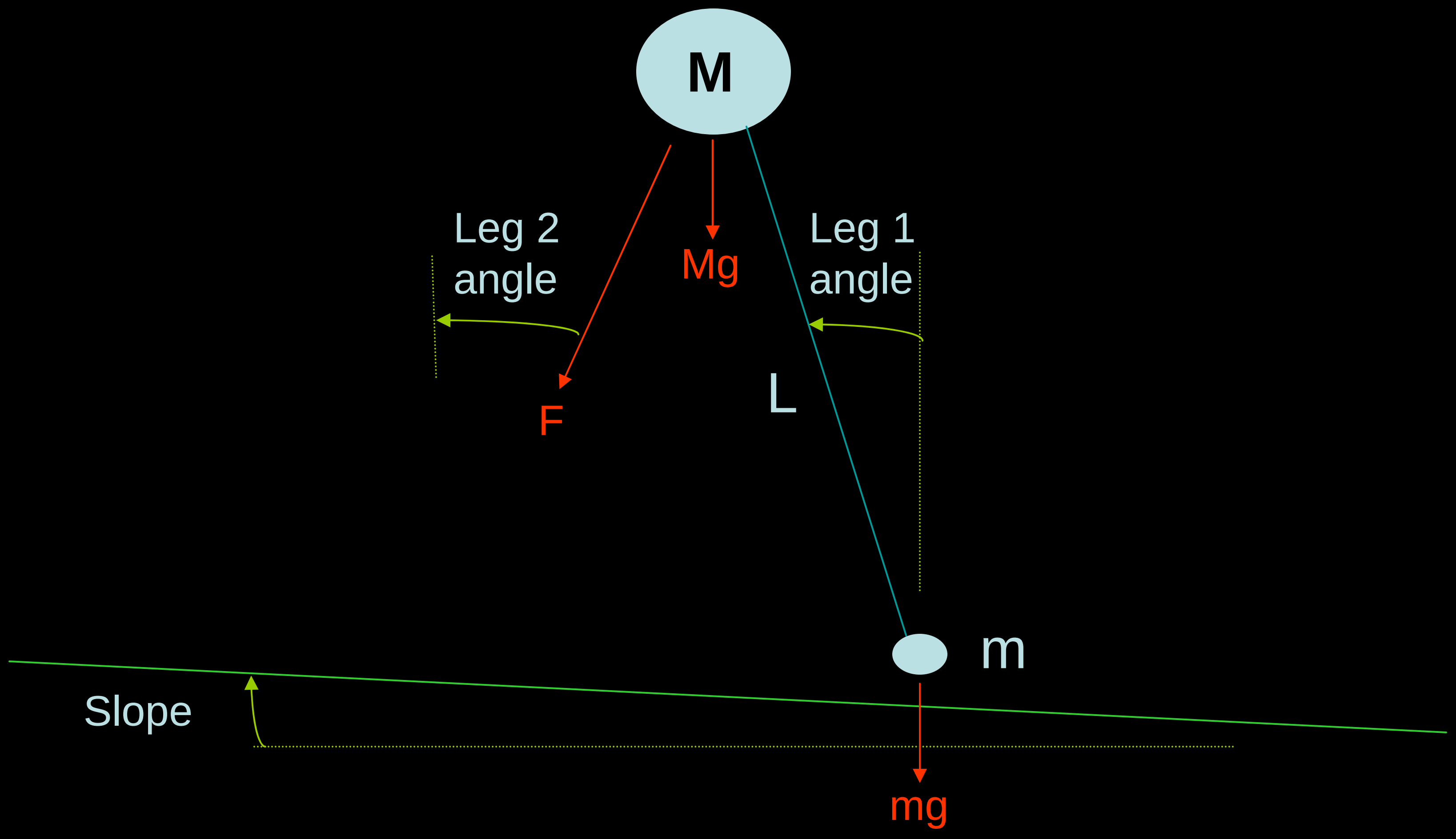
A simple walking model

Assume:

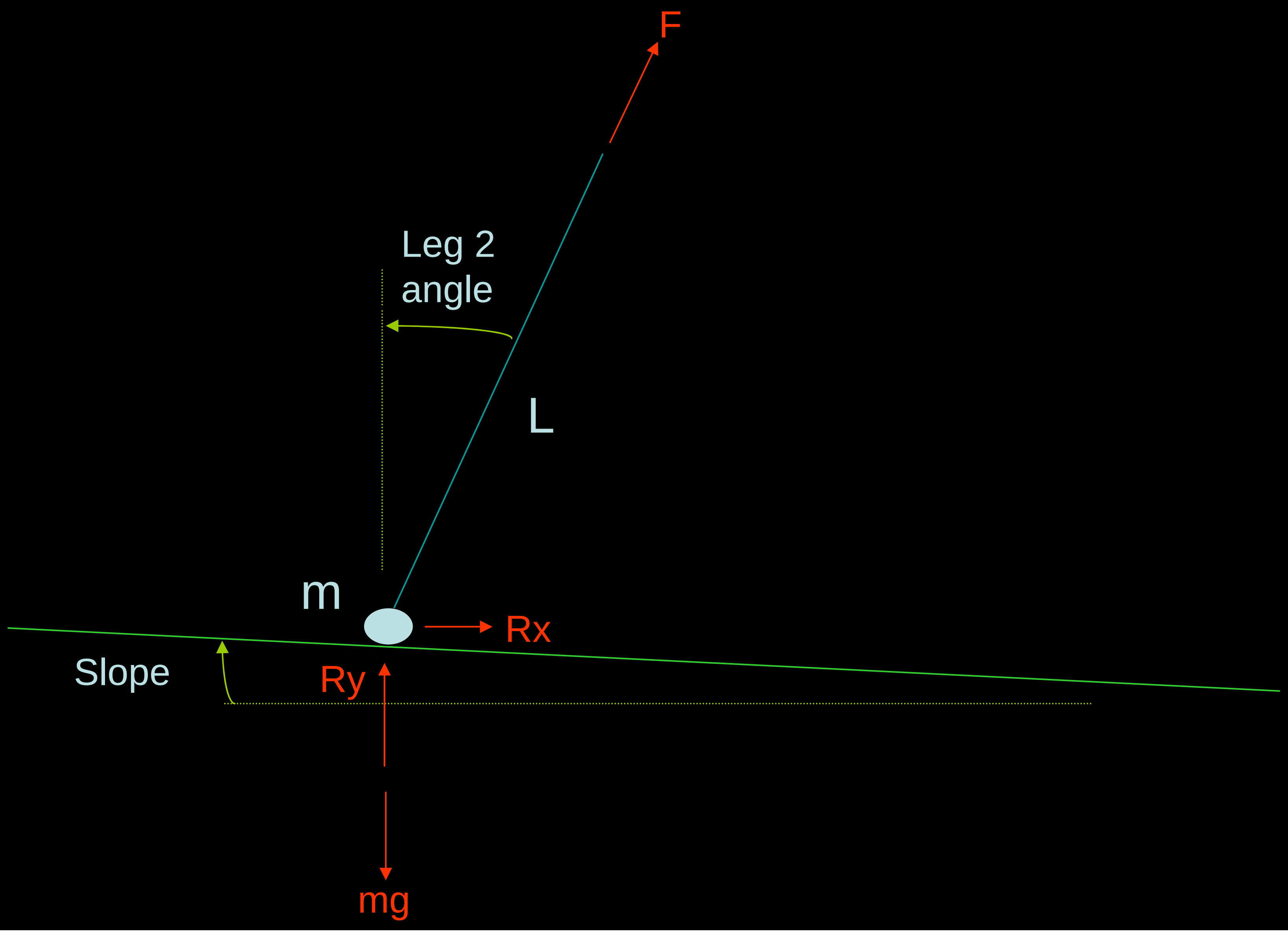
- 2D
- Frictionless joint at mass M
- M >> m
- No foot bounce or slip
- No foot scuff



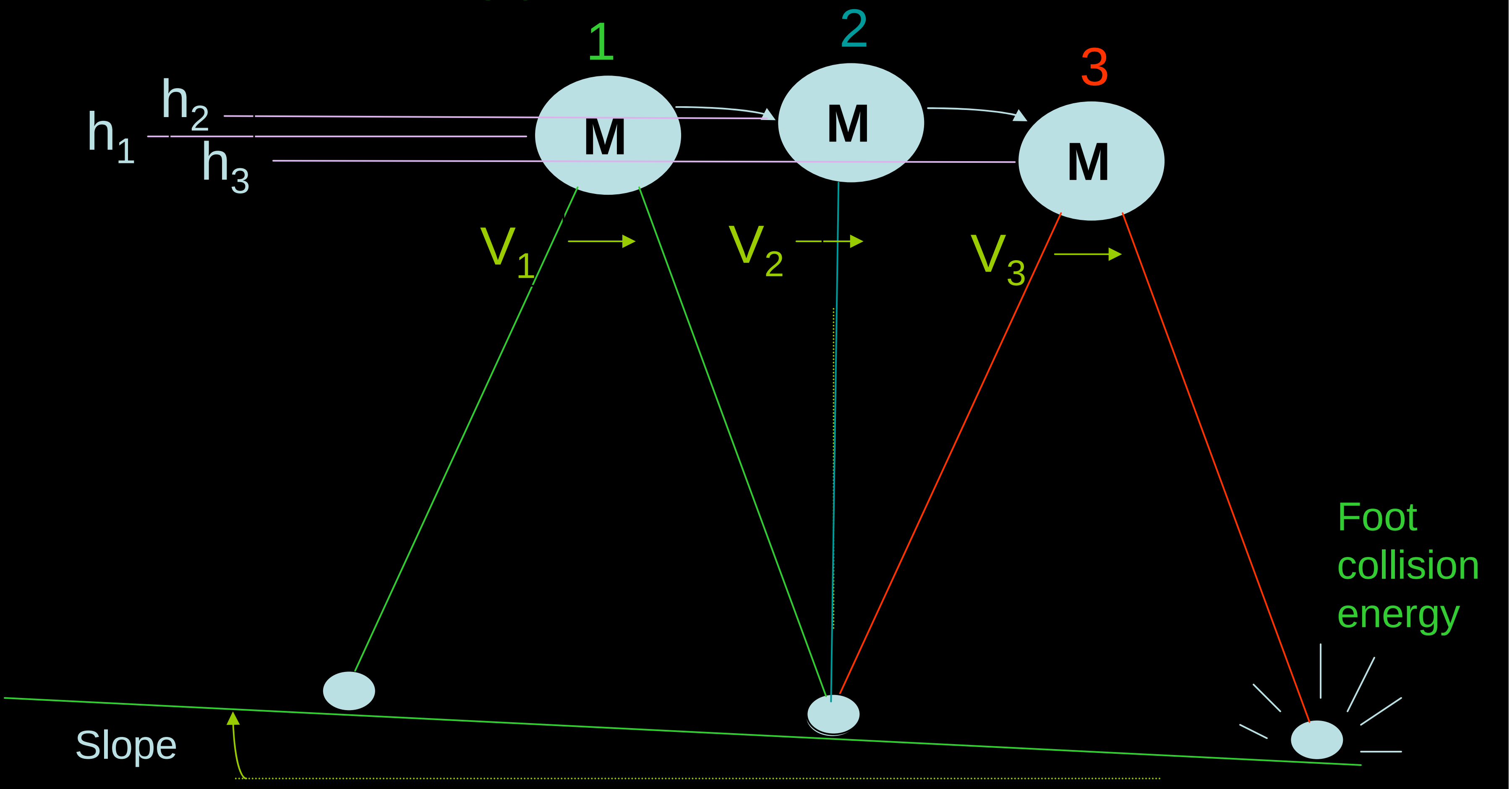
Front leg FBD



Rear leg FBD



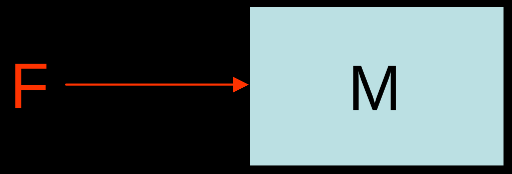
Energy of the walker



Energy change per step = Final – Initial

$= (Mgh_3 + MV_3^2/2 - \text{collision}) - (Mgh_1 + MV_1^2/2)$

Differential equations of motion



$F = Ma \xrightarrow{\qquad\qquad} a = F/M$

Choose state variables:

$Q_1 = x$ and

$Q_2 = v$

a = change in velocity v/change in time

v = change in position x/change in time

Let Q_1' mean "change in Q_1/change in time"
(rate of change of Q_1)

Then our differential equations of motion become:

$$Q_1' = Q_2$$

$$Q_2' = F/M$$

Numerical simulation

Equations of motion:

$Q_1' = Q_2$

$Q_2' = F/M$

Initial conditions of the state variables:

$Q_1(0) = x_0$ $Q_2(0) = v_0$

What the computer does:

Time step	Q_1	Q_2
0	$Q_1(0)$	$Q_2(0)$
1	$Q_1(0) + Q2(0)$	$Q_2(0) + F/M$
2	$Q_1(1) + Q2(1)$	$Q_2(1) + F/M$
…		
N	$Q_1(N-1) + Q2(N-1)$	$Q_2(N-1) + F/M$

Walker equations of motion

```
dYdt2 = [th1dot; th2dot; ...
    -(-2*m*sin(th1)*sin(th1)^2*L*cos(gma)*cos(th2)*th1dot^2 ...
    +2*m*sin(th2)^2*sin(th1)*L*cos(gma)*th1dot^2*cos(th1) ...
    -m*sin(th2)^2*sin(th1)*g-m*cos(th1)*L*cos(gma)*th1dot^2*sin(th1) ...
    +m*cos(th2)*L*cos(gma)*sin(th2)*th1dot^2-m*cos(th2)*cos(th1)*g*sin(th2) ...
    -
m*L*cos(gma)*th2dot^2*sin(th2)*cos(th1)+m*L*cos(gma)*th2dot^2*cos(th2)*sin(t
h1) ...

+M*g*sin(th1)+m*g*sin(th1))/L/cos(gma)/(2*m*sin(th2)*sin(th1)*cos(th2)*cos(th1)
...
    +2*m*sin(th2)^2*sin(th1)^2-m*sin(th1)^2-m*sin(th2)^2-M); ...
    (cos(th2)*m*L*cos(gma)*th1dot^2*sin(th1)-
cos(th1)*m*L*cos(gma)*sin(th2)*th1dot^2 ...
    +cos(th2)*m*L*cos(gma)*th2dot^2*sin(th2) ...
    -2*sin(th2)*sin(th1)^2*m*L*cos(gma)*th2dot^2*cos(th2) ...
    -sin(th1)*m*L*cos(gma)*th2dot^2*cos(th1) ...
    +2*sin(th2)^2*sin(th1)*m*L*cos(gma)*th2dot^2*cos(th1) ...
    -cos(th2)*cos(th1)*M*g*sin(th1)+m*g*sin(th2) ...
    -sin(th2)*sin(th1)^2*m*g-cos(th2)*cos(th1)*m*g*sin(th1) ...
    +L*cos(gma)*cos(th2)*th1dot^2*sin(th1)*M ...
    -L*cos(gma)*sin(th2)*th1dot^2*cos(th1)*M ...
    -
sin(th2)*sin(th1)^2*M*g+g*sin(th2)*M)/L/cos(gma)/(2*m*sin(th2)*sin(th1)*cos(th2)
*cos(th1) ...
    +2*m*sin(th2)^2*sin(th1)^2-m*sin(th1)^2-m*sin(th2)^2-M)];
```

Ranger robot at the track

Details:

- 9 kilometers
- 45 laps
- 27,724 steps
- 5 hours 12 min.
- 1.75 km/hour
- 25 watts
- 126 watt-hours
- 8.5 kg
- 0.6 cost of transport (energy per unit weight per unit distance)

Ranger design goals:

Goals:

- Record walk distance
- Basic robot for research

Requirements:

- Low energy use
- Reliability
- Robustness
- Simple to build
- Evolvability

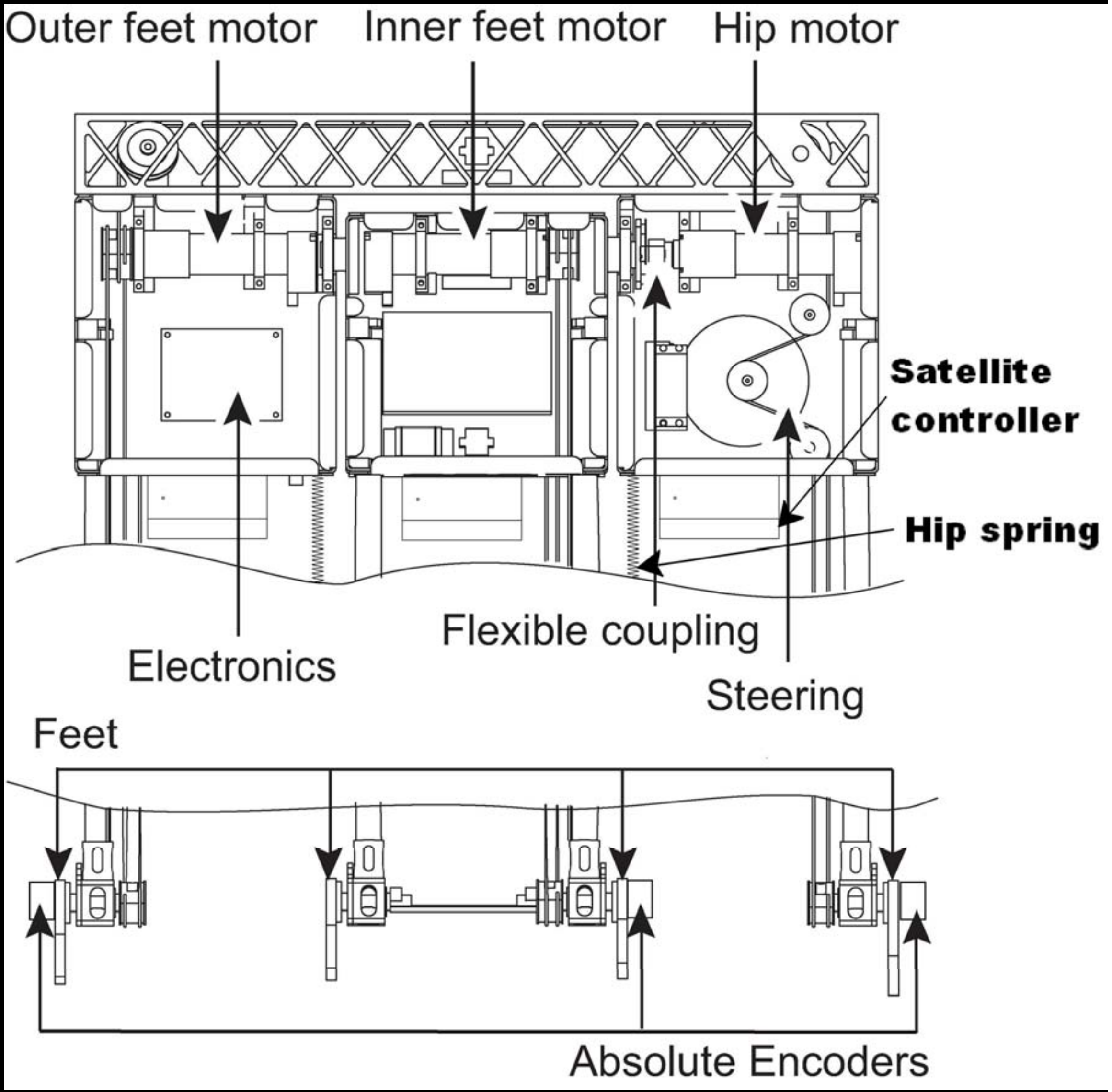
Ranger design: Overview

- 2-D (or 4-leg) biped
- Three degrees of freedom
- Plus a little steering
- Three motor encoders
- Four RLS AM8192 encoders
- MicroStrain InertiaLink IMU
- Machined aluminum top truss
- Sheet metal boxes for rigidity
- Epoxy-bonded chassis
- Four microcontrollers

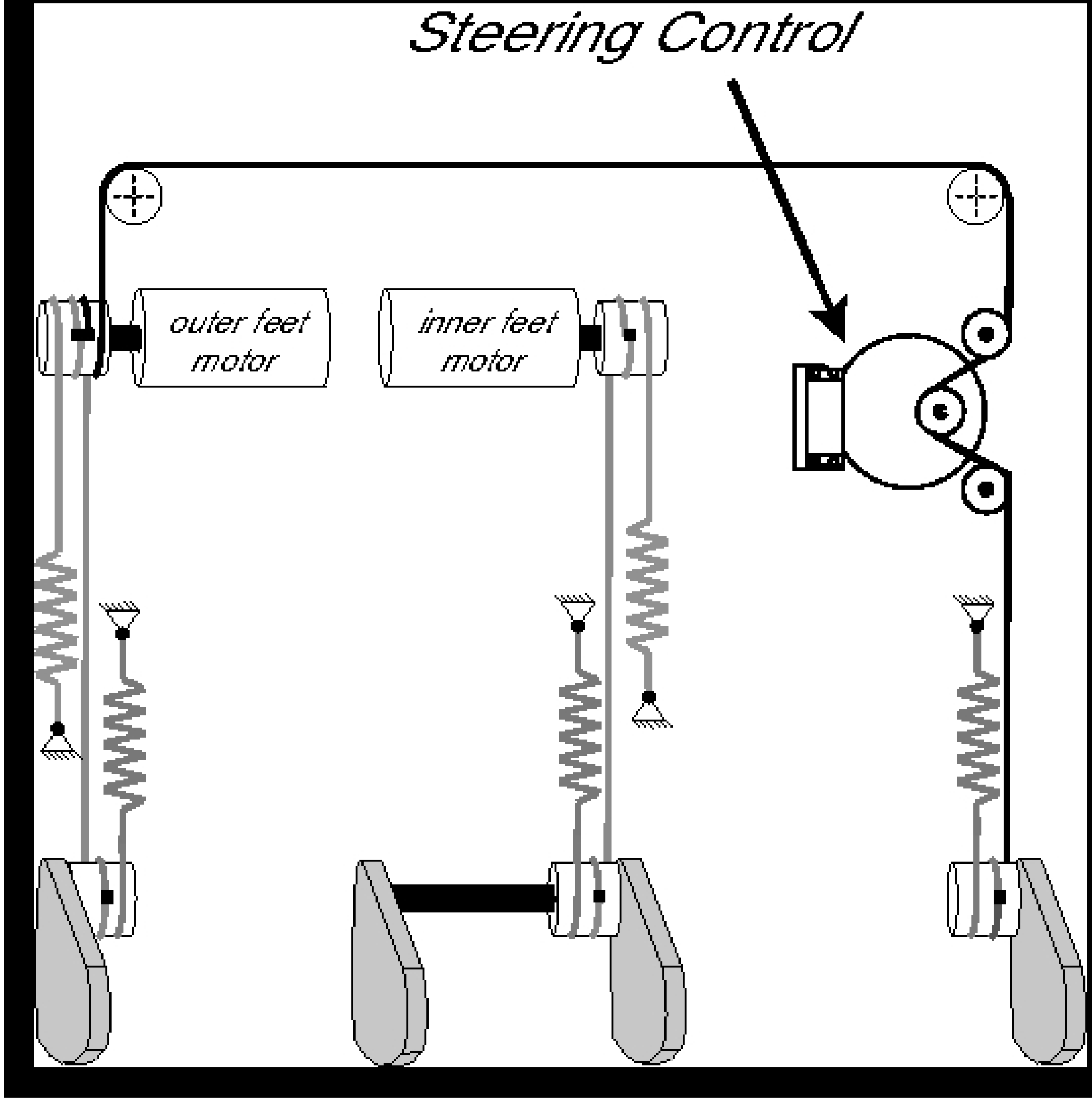
  1 – 56F8347

  3 – 56F803

  CAN network

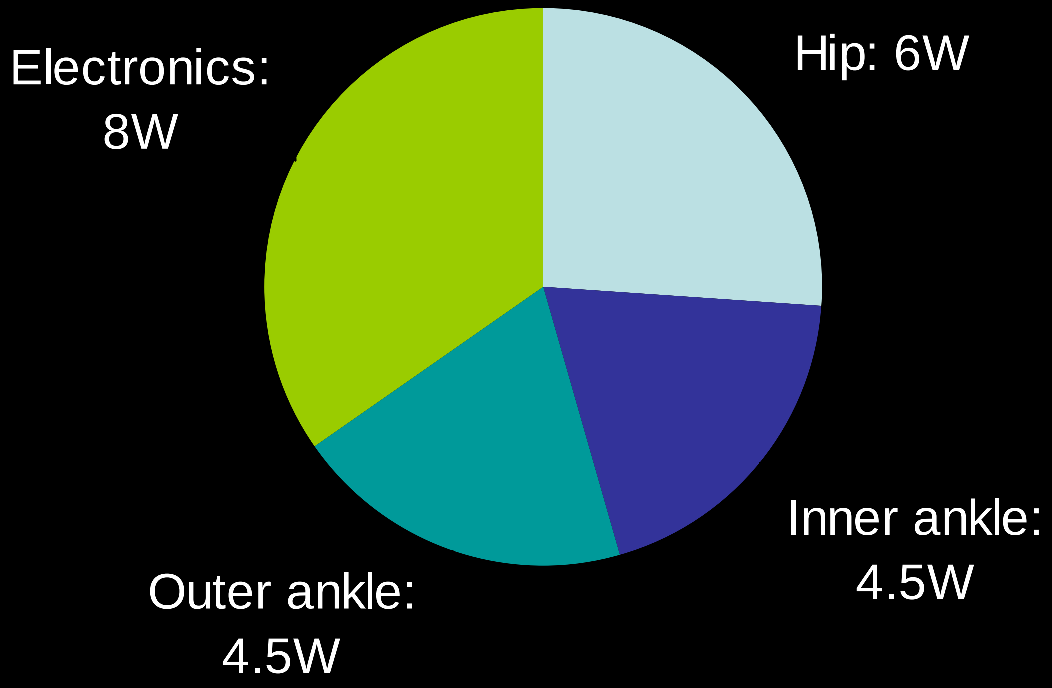


Steering a 4-leg biped?



With the mechanism shown at left, Ranger's left outer foot can rotate up or down relative to its right foot. The lower foot hits first during heelstrike, so the robot tends to rotate around that foot.

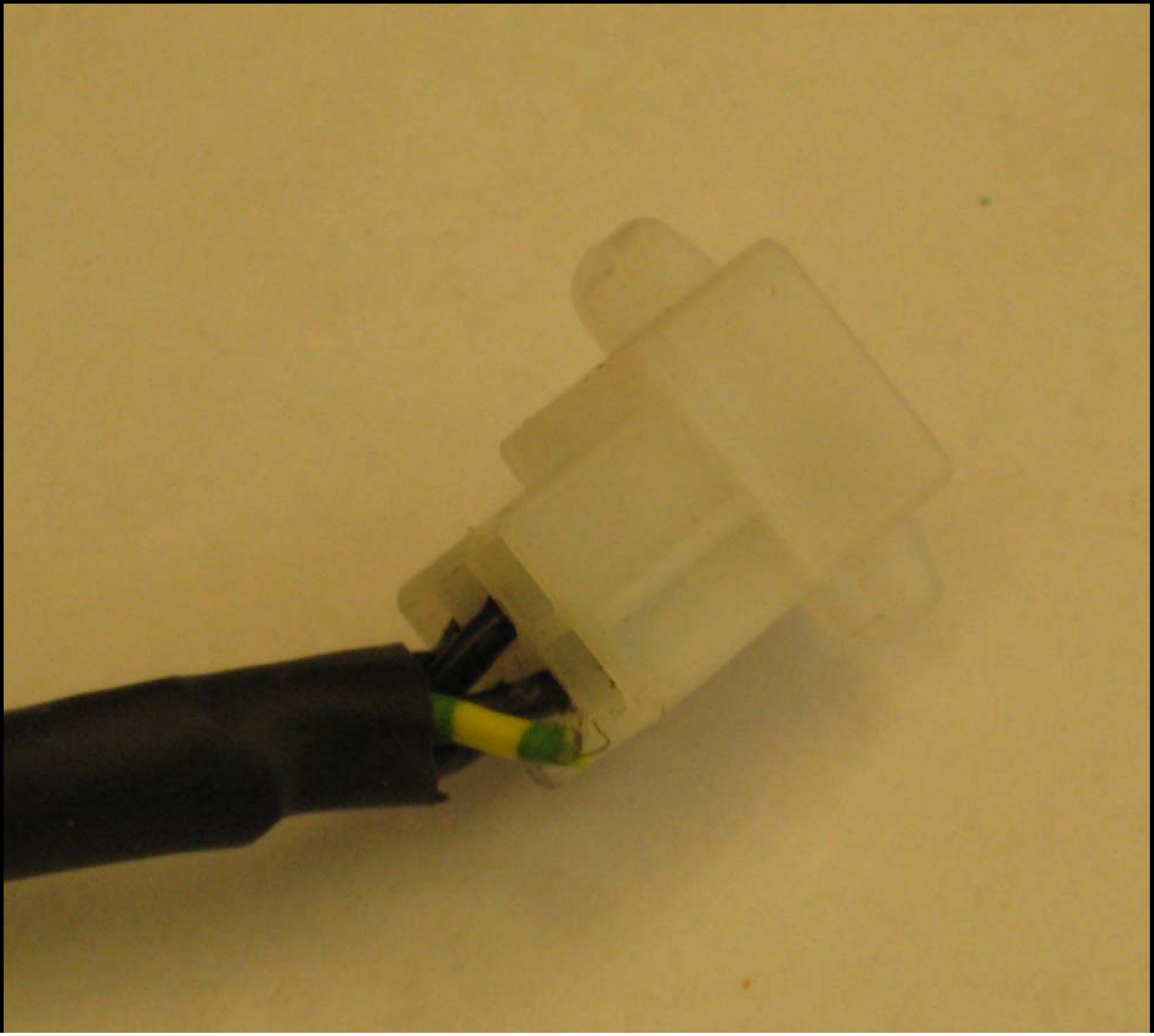
Power usage by Ranger motors and electronics – straight walk



Outer ankle motor power is 7.5W during turns;
the other power values are unchanged.

Reliability

Why it fell:



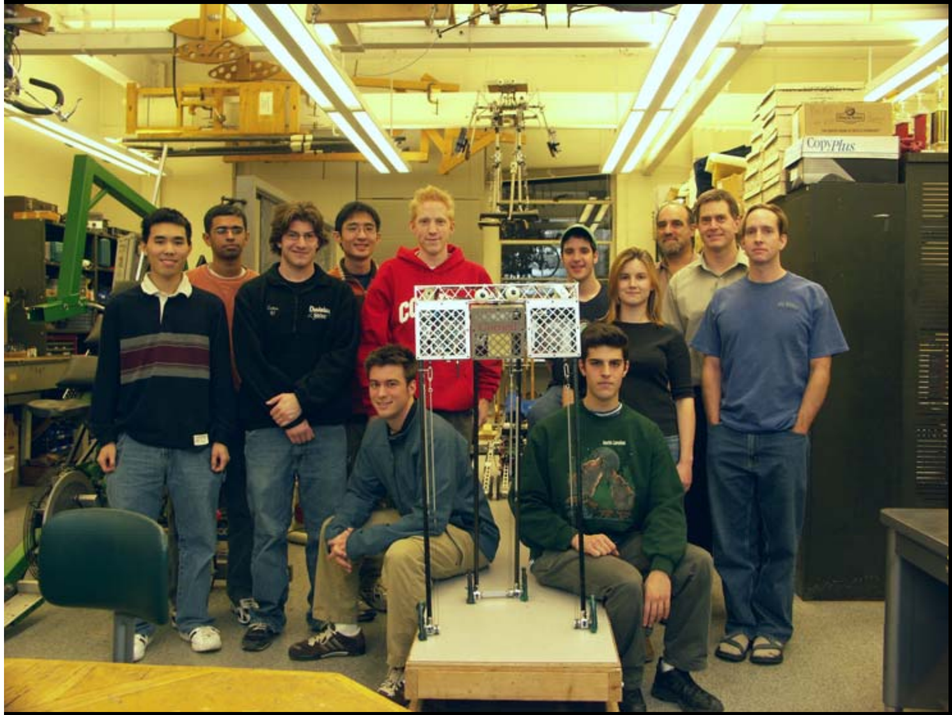
Robustness

What we want – but don't really have yet:

Swing-leg retraction

Acknowledgements

Ranger team 2006



Ranger team 2008

Andrew Mui, Pranav Bhounsule, Alex Gates, Sam Lee, Matt Haberland, Daniel Karssen, Andrew Spielberg, Carlos Arango, Megan Berry, Andy Ruina, Jason Cortell, Gregg Stiesberg, Stephane Constantin, Leticia Camargo, John Buzzi, Bram Hendriksen, Greg Falco. Not shown: Yingyi Tan, Alexis Collins, James Doehring, Ben Oswald, Ko Ihara, Max Wasserman, Rohit Hippalgaonkar, Reubens Lee, Andrew LeClaire,

Special thanks to Delft visitors Daniel Karssen and Bram Hendriksen, for their huge efforts (and excellent drawings)!