Abstract:

This series of tests of the Faulhaber 2657CR012 motor were designed to give us a mathematical understanding of the motor’s mechanics and electronics. By machining a precise aluminum mounting frame, and creating a complex controlling application using National Instruments’ LabView 7.1, we were able to determine many of the motor’s properties. We fitted the widely recognized mathematical model of a DC motor (see equations 1.1-1.3) to our collected data, and we found: the motor constant $K$ is 0.0172 Nm/A and the motor’s effective resistance $R$ is 1.23 $\Omega$; The viscous friction $C_1$ is 0.012 Ns/rad and the constant friction $C_0$ is 0.021N. We also found that the common assumption of $PWM*V_{supplied}=V_{effective}$ is only fitting for very large approximations. Seeking a more universal relationship, we found $PWM = 0.0186\omega + 0.1276I$ is a good approximation for our particular equipment.
Introduction:

The Faulhaber 2657CR012 motor is being used by Cornell University’s Human Power and Locomotion Laboratory in their new bipedal robot. To achieve maximum efficiency of said robot, it is necessary to know the properties, both mechanical and electrical, of all its components. In order to gain such an understanding of this motor, we used a special testing rig and controlling program, both derived from past experiments.

Past experiments:

Physical setup:

- (Refer to Fig. 1) We used a powerful Thermax DC motor supplied with a constant 12V connected end-to-end to the output of the Faulhaber 30/1S 14:1 planetary gearbox on our Faulhaber motor. We varied the current supplied to the Thermax motor to provide different levels of resistance to motion. A digital load cell was attached to a cable wound around the bigger motor, which was allowed to rotate freely, measuring the torque output of our Faulhaber motor at different velocities and PWM values. A Faulhaber HEDS5500 D06 optical encoder measured the motor velocity, and a ST Micro VNHT2 H-bridge provided the motor with varying PWM values supplied by our controlling program. In addition, a current sensor was attached in series between the H-bridge and the power supply.

![Figure 1](image)

Controller Setup

- Using National Instruments’ LabView 7.1, we created a controlling program for our setup. This program ran a 2-second loop of functions over and over again. It used a “sequence structure” inside the main loop which made sure that the PWM was raised by 0.02 and that the program waited 500 mili-seconds before any measurements were made to allow the motor to stabilize after sudden acceleration. In addition, the sequence structure included a nested timed loop after the wait period which ran 100 10-millisecond iterations, during each of which it would: measure motor velocity; measure the force on the load cell, converting it into the effective torque supplied by the Faulhaber motor; and finally measure the current going through our current sensor.
Once the sequence structure was completed, the data points were averaged by a “mean” function provided in LabView which added them together and divided them by the number of data points. This proved to be a problematic process when dealing with the velocity data because the first iteration of the loop calculated the velocity over zero time, yielding extreme spikes in the data. We solved this problem by “dropping” the velocity measurement of the first iteration of the nested timed loop. As done in our previous experiments, the data were compiled into a text file specified at the beginning of the program.

**Data Analysis:**

The analysis of the collected data was not overly sophisticated, consisting of a chart of torque vs. angular velocity at constant PWM values (Graph 1), and simple calculations using a mathematical model of a DC motor found in Marks’ Standard Handbook for Mechanical Engineers\(^1\) (Eq. 1.1-1.3) to find physical properties of the motor.

\[ T_m = I_a K \] \hspace{1cm} (1.1)

\[ V I_a = I_a^2 R + T_m \omega \] \hspace{1cm} (1.2)

\[ V = P V_{\text{battery}} \] \hspace{1cm} (1.3)

**Note:** K is the motor constant, V is the terminal voltage, \( V_{\text{battery}} \) is the voltage supplied to the H-bridge, \( I_a \) is the armature current, and R is the armature resistance; \( T_m \) is the output torque, \( \omega \) is the motor velocity in rad/sec, and P is the supplied PWM value.

Current experiments:

Physical setup:

- (Refer to Fig. 2) This year we are utilizing a Maxon F-2260.885 DC motor with a matching 2962.70 planetary gearbox to provide the resisting torque: the resisting motor change was made to provide a wider range of resistances, as well as making the testing setup easily adaptable to test future motors. This motor rests on a platform which is free to pivot about one of its edges, the opposite edge being supported by a load cell (Fig. 3): when torque is applied to any portion of the platform, the load cell has to provide a tension to keep the system from moving. The torque applied can then be calculated from this tension. The resisting motor is connected in series to the Faulhaber motor by means of a double-universal joint, which ensures that minute shaft misalignments do not cause pressure to be applied on the platform, so only torque is transmitted. The current sensor is being used again, but now it measures current between the motor and h-bridge, giving us a clearer picture of the motor’s properties.

![Figure 2](image1.png)

![Figure 3](image2.png)

- We also utilized a voltage sensor, not shown in the diagram, connected in parallel to the Faulhaber motor. In addition, we decided to utilize the optical encoder attached to the resisting motor (Maxon HP HEDS 6010-3417) to measure angular position and velocity: this will allow us to use the same controlling program in the future when we need to test new motors, as none of the data-gathering hardware will need to change.
Controller Setup

- Instead of developing a new controlling program from scratch, we decided to simply build on and improve our previous LabView application. Key changes are as follows:
  - Added a manual PWM feature, which allows the user to manually input the desired PWM of the system.
  - Added an increasing vs. decreasing PWM switch, which allows the user to choose whether the program should run the desired tests from low to high PWM values, or from high to low.
  - Changed the nested loop to run 20 10-millisecond iterations to accommodate more data inputs.
  - Added a data input for voltage measurements to the nested loop.
  - Changed all data acquisition applications to gather 500 samples at 23000 Hz and average, rather than one sample on demand. The 23000 Hz sample frequency is off-phase with the H-bridge’s PWM wave, which has a frequency of 20000 Hz, allowing us to gather data at different points of the PWM signal.
  - All other functions remained unchanged for this controlling application.

Testing Procedure

- Negative resistance
  - We ran a series of tests in which we controlled the resisting motor’s voltage to apply a torque opposite to the one being applied by the Faulhaber motor. Keeping this voltage constant, we used the LabView application to sweep up a set of PWM values (0.01 to 0.85 at intervals of 0.02), and then back down (0.85 to 0.01 at intervals of 0.02). This procedure was conducted at seven different resisting voltages: 0.5, 1, 1.5, 2, 3, 4 and 5 volts.

- Positive Resistance
  - Next we ran the same experiments, except now the resisting motor was set to turn in the same direction as the Faulhaber motor; we used the same resisting voltages. This procedure proved troublesome because once the current being generated by the resisting motor matched the current that was being put out by its power supply, it would start feeding current to the supply. Because we were unsure whether the power supply could handle the back-feed, we connected a set of resistors in parallel to the circuit which provided enough current draw to make up for this new current source.

Data Analysis

- For this set of experiments, most of the data analysis was conducted through Mathwors’ MATLAB by graduate student Pranav Bhounsule. His assistance was critical to our understanding of the experiments and the Faulhaber motor.
- The data analysis also relied heavily on MATLAB subroutines Polyfitn.m and polyvaln.m, found on Mathworks’ website. These programs allow us to fit 3-dimensional polynomials \( f(x,y) \) to our sets of data. For our fits, we needed 5 variables: \( V \), the terminal voltage, \( I_a \), the

\[ \text{http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=10065&objectType=File} \]
armature current, $T_m$, the output torque, $\omega$, the motor velocity, and $P$, the supplied PWM value. In our case, $P$ is a control variable, and $T_m$ is obtained from equations of motion: we have 3 unknown variables ($V,I,\omega$) and we need three equations to relate them.

- Recalling eq. 1.1 and 1.2, we see that substituting of 1.1 into 1.2 and dividing by $I_a$ we get $V = IR + Gk\omega$, where $G$ is the gear ratio on the Faulhaber motor. A least squares fit using Polyfitn.m gave:

$$V = 0.2420\omega + 1.2261I \quad --- (I)$$

The gear ratio was 14, and thus $K=0.0172 \text{ Nm/A}$ and $R=1.23 \Omega$

- Assuming a mechanical model for the motor of

$$T = GkI + C_1\omega + C_0 \frac{\omega}{|\omega|}$$

where $C_1$ is viscous friction and $C_0$ is static friction, a least-squares fit of our data yields

$$T = 0.2228I + 0.0174\omega + 0.0286 \quad \text{Fit for negative speed}$$

$$T = 0.2465I + 0.007\omega - 0.0126 \quad \text{Fit for positive speed}$$
Averaging the two,

\[ T = 0.234I + 0.012\omega + 0.021\frac{\omega}{|\omega|} \quad (II) \]

Again, we find that with \( G = 14 \), \( K = 0.017 \text{ Nm/A} \), \( C_1 = 0.012 \text{ Ns/rad} \) and \( C_0 = 0.021 \text{ N} \).

- Finally, note that PWM is a measure of the voltage applied to the motor; hence we are trying to fit PWM to speed and current. This type of “black box” fit is necessary because the application of our model, the Cornell Ranger, utilizes PWM values to as a control variable. However, we can find no physical significance to the constants thus obtained.

\[ P = 0.0186\omega + 0.1276I \quad (III) \]

**Conclusions:**

During our series of experiments, we were presented with data which, at first, seemed to contradict the accepted DC motor model: frictional torques, from initial calculations, were coming out to be close to zero and even negative; theoretical torques, calculated from gathered electrical data, seemed to be less than the actual output torques we measured. This was before we implemented the voltage measurement: we realized that the common assumption of \( PV_{bat} = V_{act} \) is not accurate at the resolutions at which we were collecting our data. As is apparent from solution (III), PWM also depends on current and the angular velocity. From solutions (I) and (II), we can see that the motor model does seem to fit pretty closely: after attempting to fit higher order polynomials, we noticed that the root-mean-square of the plots did not decrease significantly, so we are confident in the linear fit approximation.