Hip Spring for the Cornell Ranger

Senior Design Report
M&AE 429
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Biorobotics and Locomotion Laboratory
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1. Introduction

The Cornell Ranger is a powered 4-leg walking robot. The two inner legs are connected to each other, as well as the two outer legs, and it has no knees so it resembles a person walking on crutches. The degrees of freedom are the hip, connected inner ankle, and outer ankles. Steering is controlled remotely by slightly altering outer ankle movements. Because of the constrained motion of the legs, we call this robot a "4-leg biped."

Application of this research is very promising. Initially the dynamics of a human walking may seem simple, but in fact there is much more too it than meets the eye. The details of the way the body coordinates and controls motion is beyond our scope of knowledge. It's no coincidence the Ranger robot resembles a human in its bipedal walking. If a mechanical model can be built that simulates the coordination of this motion, we will greatly develop our understanding of human locomotion.

The Ranger robot has made great achievements in walking efficiency. It has already walked 1 km on much less power than comparably sized robots require. This is due to the passive dynamics of the robot, which is its ability to utilize the momentum of swinging limbs to carry it into the next step. However, the robot has difficulty on irregular or uneven surfaces. My project was focused on increasing the robustness of the Ranger by adding a restoring spring to the hip so that it can handle practical surfaces without falling over or getting tripped up.

2. Background

The basic design of the robot is illustrated in Figure 1. The distance of each leg is roughly 1 meter, so the robot stands about waist height. Mass of the robot is approximately 6 kg with the majority distributed on the outer legs and near the hip axis of rotation. Inertia of the inner and outer legs is roughly equivalent. Batteries provide power to the Ranger, which requires about 40 W to operate. The robot currently carries about 80 W-hr of power, and could double that by adding a second battery. Due to the constrained motion of the legs, we usually refer to them as the inner and outer leg.

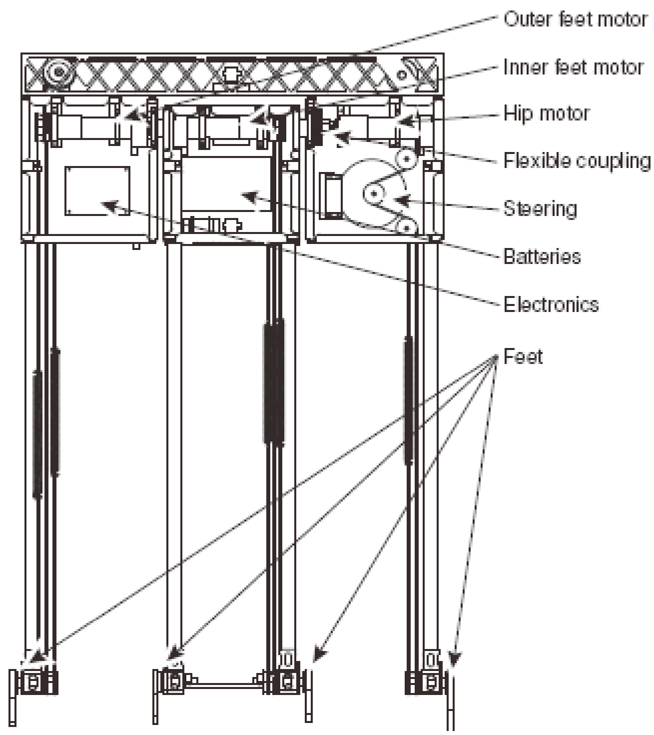

Figure 1: Basic design of the robot

The Ranger uses a Java computer program to operate, with the basic control loop running once every millisecond. Feedback such as when the heel strikes the ground each step is used to prompt the next movement. An array of telemetry, such as various torques, currents, and angles, can be read remotely for quick tuning. Onboard motors can provide a hip torque and ankle actuation via a spring, cable, and pulley system. The Ranger has various sensors to calculate its current state.

On December 3, 2006, the Ranger walked approximately 1 km without falling, being touched, or battery recharge on the indoor track of Barton Hall, Cornell University. A slack safety cable was used to prevent damage should it fall. Failure occurred when the robot encountered an uneven surface (the cover to a sand pit) and was not able to swing its legs into the next stride as planned.

3. The Hip Spring

The design requirement of the hip spring was to produce 6 N-m of restoring torque when the angle between the legs was .55 radians. We did not at this time have a strong preference for the torque value at intermediate angles between the legs; though we knew the higher it was, the faster the legs would swing back together in between steps. In theory this quick snap of the legs back together will prevent tripping up. Figure 2 shows the basic idea of restoring springs. Each spring is held at either end by a hook.

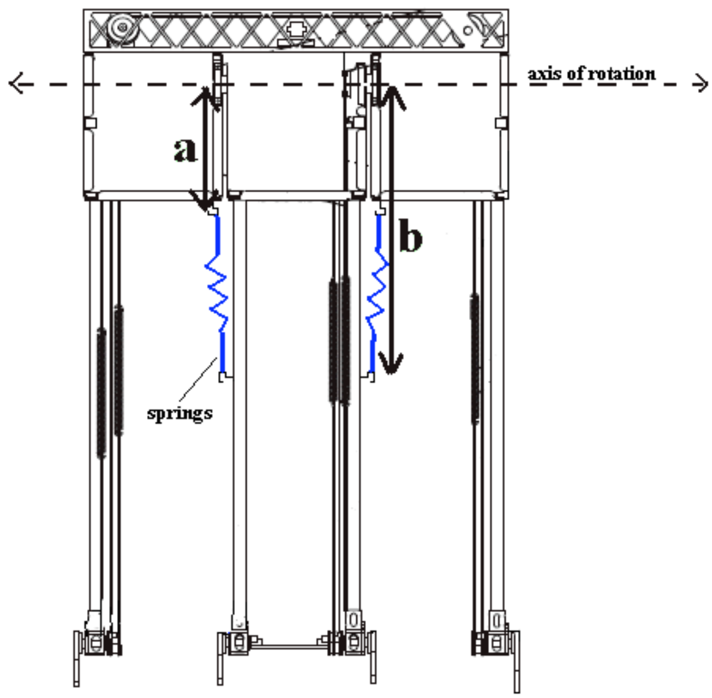

Figure 2: Restoring springs connecting the inner and outer legs

To minimize reaction forces and for simplification of calculation, we decided to position the springs as vertically as possible. By placing the upper attachment hook adjacent to the inner edge of the outer leg boxes, the springs are very near vertical. In our analytical calculations, we assume the springs are vertical and estimate the error associated with this assumption is negligible.

With vertical springs, we can create a simplified 2-dimensional model to predict spring behavior for a range of angles between the legs. We define this angle as θ and generally consider only the absolute value of θ because the spring produces a restoring effect regardless of which leg is in front of the robot. Considering the plane perpendicular to the hip axis of rotation, we define the geometry as in Figure 3.

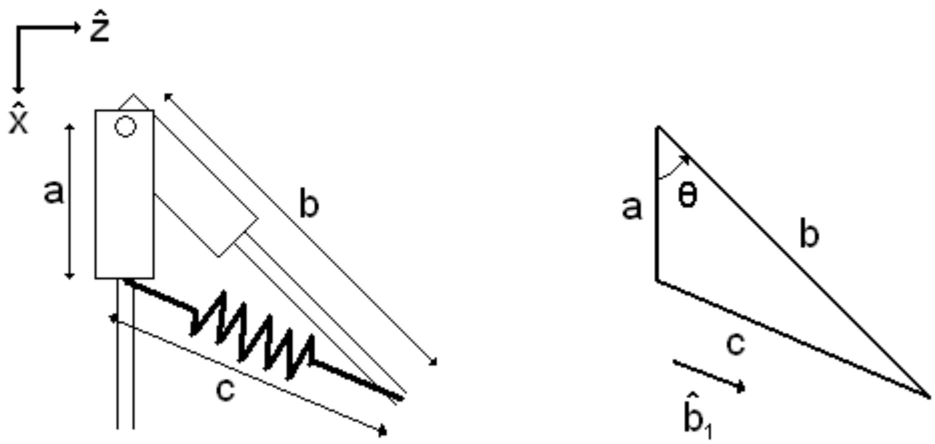

Figure 3: Geometry of the hip spring, shown in the x-z plane

The absolute value of torque produced about the y-axis will be the restoring torque of the hip spring. Tension will be produced along the triangle edge *c*, so we need to find the length of this edge as a function of θ. The law of cosines states that

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta} \tag{1}$$

In a linear spring, the force is a function of the spring constant *k*, the stretched length *c*, and the spring equilibrium point (zero-force displacement) c_{eq} and is given by

$$\vec{F} = -k(c - c_{eq})\hat{b}_1 \tag{2}$$

Dotting this force with the unit vector in the z-direction, we have

$$F_z = \vec{F} \bullet \hat{z} = -k(c - c_{eq})\frac{b\sin\theta}{c} \tag{3}$$

The restoring torque produced about the hip axis (y-axis) will be equal to twice (because the design involves two springs) the product of the z-component magnitude of the spring force and the distance *a*.

$$T_s = 2a\left|F_z\right| = \frac{2kab(c - c_{eq})}{c}\sin\theta \tag{4}$$

$$T_s = \frac{2kab\left(\sqrt{a^2 + b^2 - 2ab\cos\theta} - c_{eq}\right)}{\sqrt{a^2 + b^2 - 2ab\cos\theta}}\sin\theta \tag{5}$$

We estimated that the gearbox for the hip motor cannot handle torques greater than 6 N-m without risk of breaking. Thus we planned the hip spring to produce a maximum of 6 N-m, which would occur at the maximum angle between the legs (~.55 rad). Running our Matlab program *rangerspring*, we can calculate this torque for a variety of leg angles and plot our results. A spring with a *k* value of 2.83 lbf/in and an equilibrium point equal to its relaxed length could produce the following results. The code for *rangerspring* is provided in Appendix A.

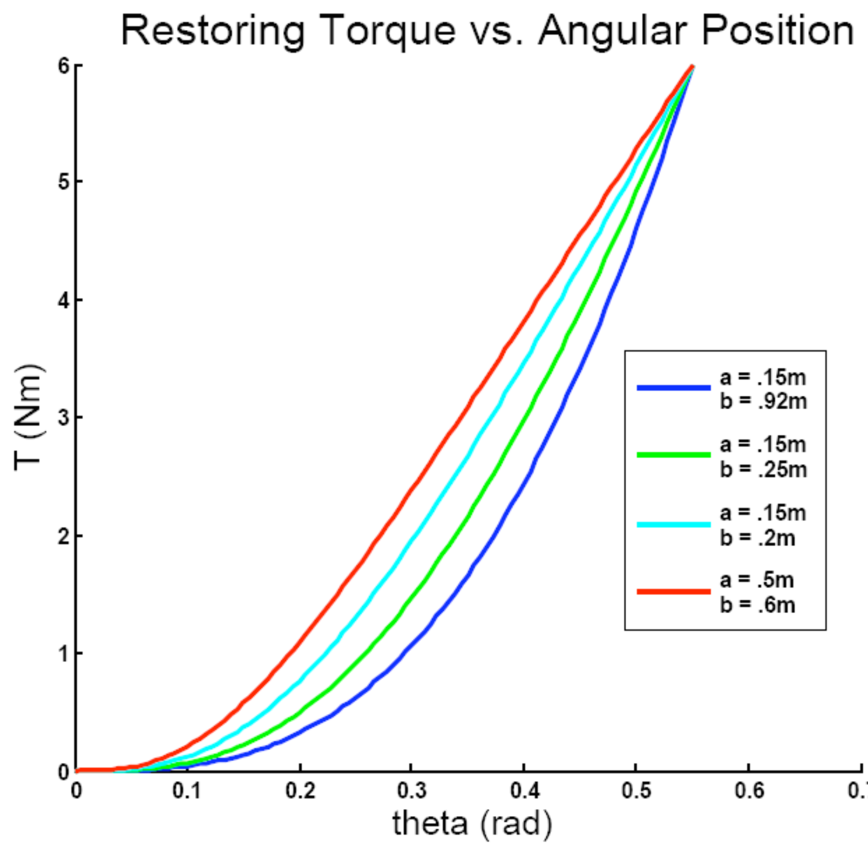

Figure 4: Torque results for varying θ and geometry parameters

Note the zero slope near $\theta = 0$ rad. This result would only be possible if the spring were installed to be at its rest position at $\theta = 0$ rad and the equilibrium point was the same as the spring's relaxed length. We discovered that nearly all available extension springs had pretension, that is, the equilibrium point was less than the relaxed length. Despite this, we decided that springs producing a more linear pattern (such as the red curve in Figure 4) would probably be preferable because they will cause the legs to swing faster into their next stride.

The function *rangerspring* also outputs a summary of information such as the parameters a, b, c_{eq}, c_{max} (the maximum length the spring will be stretched), k, and F, the minimum tension force in each spring for quick comparisons. If the user desires to solve for a different parameter with the torque as an input, they can make a small number of modifications to code. This approach was used to get a ballpark figure for spring constants by inputting the maximum torque (6 N-m) and the maximum angular position (.55 rad). See the appendix of this report for the code of *rangerspring*.

Using predictions from *rangerspring*, sets of two different springs were ordered. Additionally data was taken for two 4 inch springs combined in series (Spring C) and one 4 inch and one 6 inch spring combined in series (Spring D). Table 1 summarizes the properties of these springs, most of which were experimentally determined in the laboratory.

Table 1: Spring properties

	Spring A	**Spring B**	**Spring C**	**Spring D**
Relaxed Length	4 in	6 in	8 in	10 in
Equilibrium Point	3.91 in	4.69 in	7.82 in	9.10 in
Spring Constant	2.76 lbf/in	2.82 lbf/in	1.38 lbf/in	1.41 lbf/in
Mass	16.5 gm	45 gm	33 gm	61.5 gm
Material	Zinc-plated steel music wire	Type 302 stainless steel	Zinc-plated steel music wire	-
Outside Diameter	3/8 in	½ in	3/8 in	-
Wire Diameter	.048 in	.063 in	.048 in	-
Max. Load	13.82 lbf	14.61 lbf	13.82 lbf	13.82 lbf
Deflection at Load	4 in	4 in	8 in	8 in
Pretension	.23 lbf	3.3 lbf	.23 lbf	3.3 lbf

The equilibrium points in Table 1 were calculated using extrapolated experimental data. A force sensor was used to stretch the springs to a variety of displacements, and these points were recorded and plotted. It is best to not use the factory given pretension values, as they conflicted with experimental results. Figure 6 illustrates spring data taken in the laboratory.

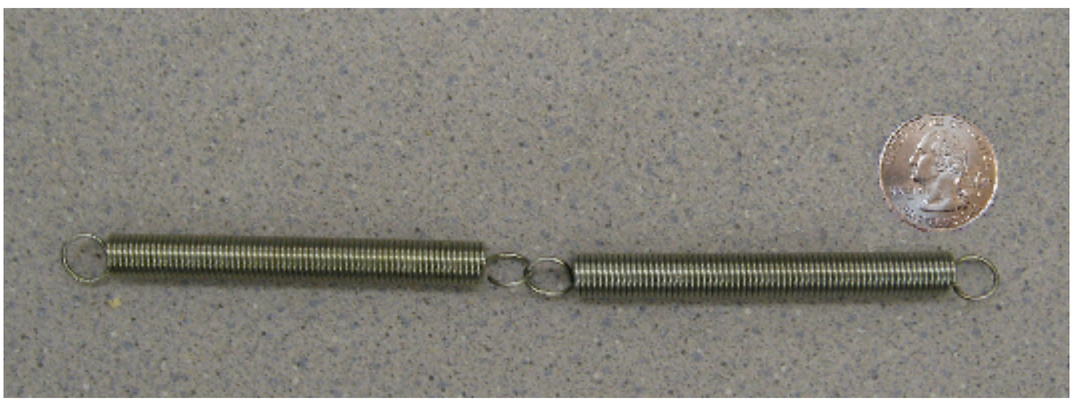

Figure 5: Spring C (two 4 inch springs in series)

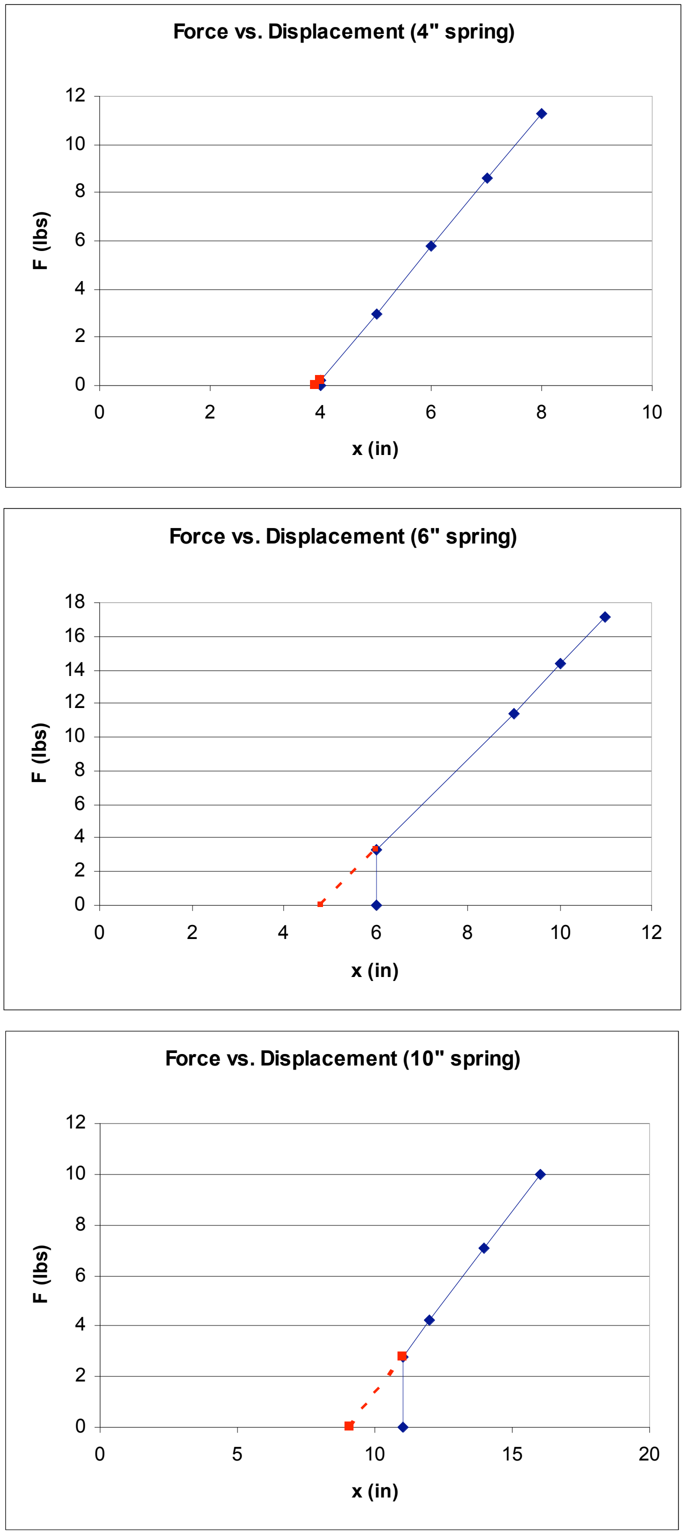

Figure 6: Experimental spring force measurements

Springs A and B were soon realized to have too large of spring constants when their pretension had to be overcome, so the combined configuration was pursued. The parameter *a* was to be fixed at .155 meters by installing a hook directly into the leg boxes, as shown in Figure 2. We drilled a small hole in the box and an installed an eyebolt, with excess threads trimmed away and filed, with washers and self-locking nuts. This needs to be sufficiently secure so it doesn't twist loose during the periodic loading of a long duration walk. Parameter *b* was decided to be adjustable, as the leg mounting method needed to not damage the composite legs.

The 10 in spring was then inputted into *rangerspring*, and the parameter *b* was varied until the desired maximum torque was achieved. Figure 7 shows the torque pattern, which is very close to linear in θ.

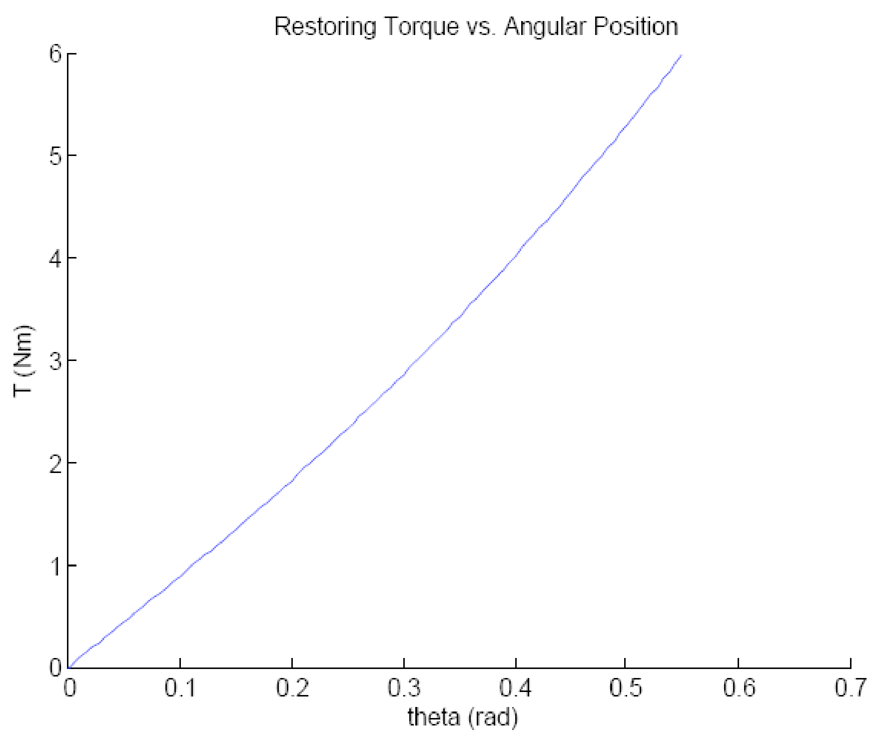

Figure 7: Torque vs. θ for the 10 in spring

Our analysis thus far has neglected the effect of gravity on the total hip restoring torque. To obtain a better model, we introduced gravity by assuming the robot is rigid and applying a single force through the center of mass. The gravity torque is also restoring, so this means we must reduce the hip spring torque to match our earlier requirement of 6 N-m at θ = .55 rad. Figure 8 shows the basic setup used to evaluate the gravity torque.

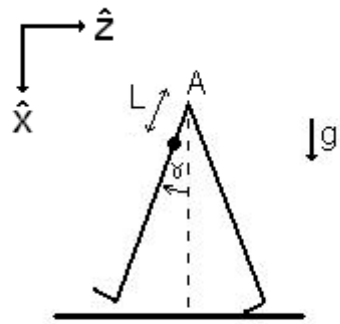

Figure 8: Gravity torque evaluated as a point mass approximation

Summing moments about point A and dotting with a unit vector in the y direction, we have

$$T_g = \left|\vec{M}_A \bullet \hat{y}\right| = mgL\sin\gamma \qquad (6)$$

Roughly speaking, γ will be equal to half of the leg separation angle θ. A working value for the point mass m is the average leg mass, that is, the total mass of the Ranger divided by 2.

$$m = \frac{total\_mass\_of\_Ranger}{number\_of\_independent\_legs} = \frac{6kg}{2} = 3kg \qquad (7)$$

L has been experimentally measured by other laboratory members as near .16 kg for the outer leg. The addition of gravity did not change the curve of the torque as a function of angle noticeably. Rather, the value of b had to be reduced to maintain a maximum of 6 N-m of torque, meaning the springs would be less stretched for any given θ.

At this point it was time to determine a method of attaching the bottom end of the spring the inner leg. It had to be adjustable, not damage the Ranger's legs, secure, and easy to remove. For adjustability a clamping method was preferred. Electrical tape can be wrapped around the Ranger's legs where the clamping points are to protect the legs and allow a tighter fit. The clamp needed to screws or bolts that could be fastened repeatedly. Figure 9 shows a design concept.

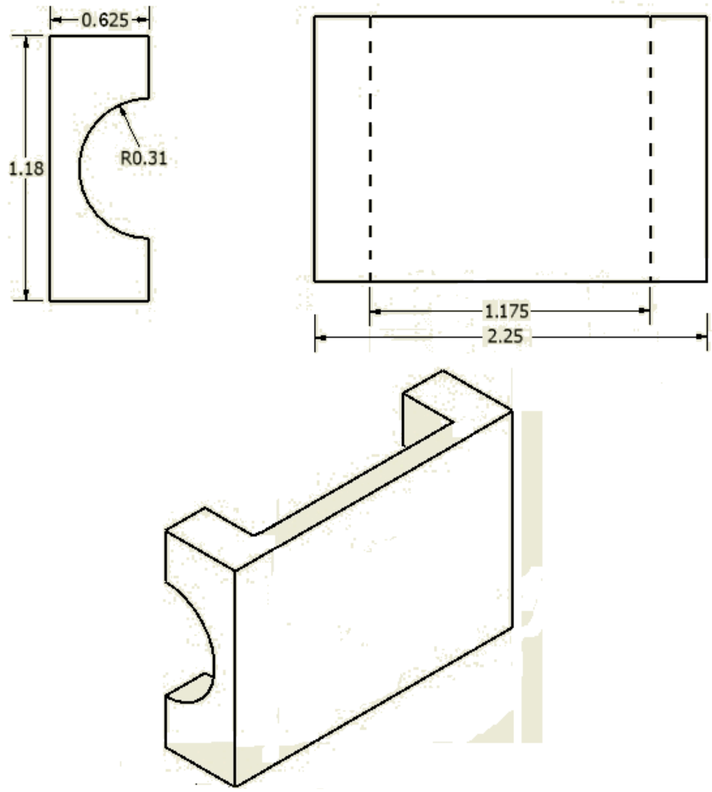

Figure 9: Design concept for inner leg brackets

This piece shown in Figure 9 was machined using a mill and serves as the main body of the bracket. Additional pieces were needed to screw in to form a full clamp. Holes were drilled and threaded into the main bracket body. The same type of eye bolt that serves as a hook for the upper end of the spring was installed into this bracket as depicted in Figure 10.

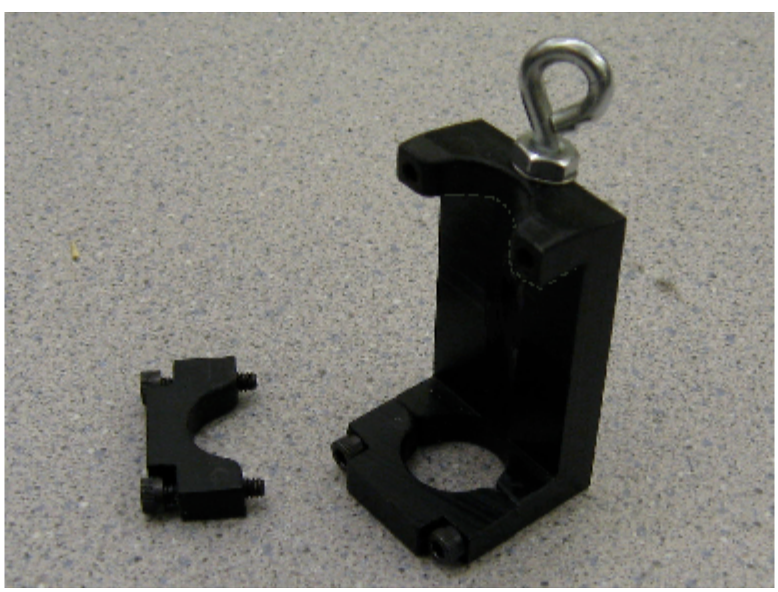

Figure 10: The inner leg bracket as machined

Both of the “C” shaped pieces in Figure 10 screw into the main bracket body with 2 small bolts each. The eye bolt shown is screwed into a hole that was drilled and threaded at approximately a 12° angle off the vertical. This was machined by using a small clamp in the machine shop that allowed an angular adjustment. The clamp itself was then clamped into the milling station and the hole was drilled. In preparation for assembly, the leg contact edges (around the inner circle) were filed down to ensure a smooth contact surface and no sharp edges.

Several issues were discovered during assembly. Due to the tolerance of the hole bored, five wraps of smooth, tightly pulled electrical tape are needed to ensure a tight and secure fit. The clamping down of the bracket tended to pinch the electrical tape where the bracket components met. This destabilized the mount so the bracket had to be filed extensively at this meeting point. With this improvement the small bolts can be torqued down to only slightly tighter than hand tight and still maintain adequate grip. These effects can be seen in Figure 10. Also a tensioning spring on the left inner leg used for ankle actuation contacted the bracket when it was installed on the upper half of the leg. This led to a later decision to install the brackets low on the leg (extending *b*). Finally it is important to keep each of the “C” shaped pieces attached to the same side of the main bracket and with the same orientation. The hole was bored with these pieces in place, so for proper alignment they need to be matched to their corresponding side. Figure 11 shows photographs of the bracket properly installed onto the leg.

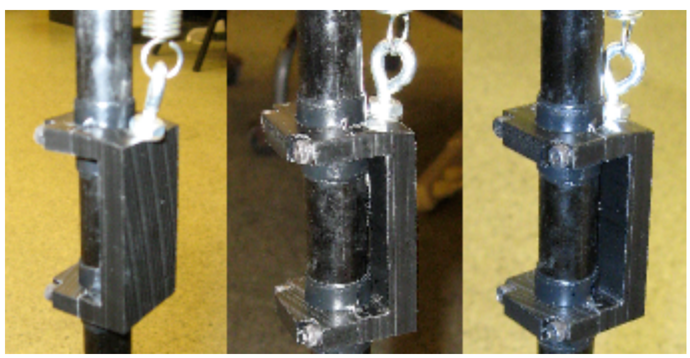

Figure 11: A bracket installed with electrical tape for grip and protection

With the hip spring installed, it was time for an experimental torque test to confirm theoretical predictions. The Ranger was laid on its side (to avoid gravity effects) and its inner leg was pulled perpendicularly at a known location with a force sensor. The angle between the legs was measured and data was taken at several angles. Table 2 and Figure 12 show the results of this test.

Table 2: Experimental torque test results

θ (rad)	T (N-m)
.185	1.25
.38	3.68
.523	5.71
.55	5.90

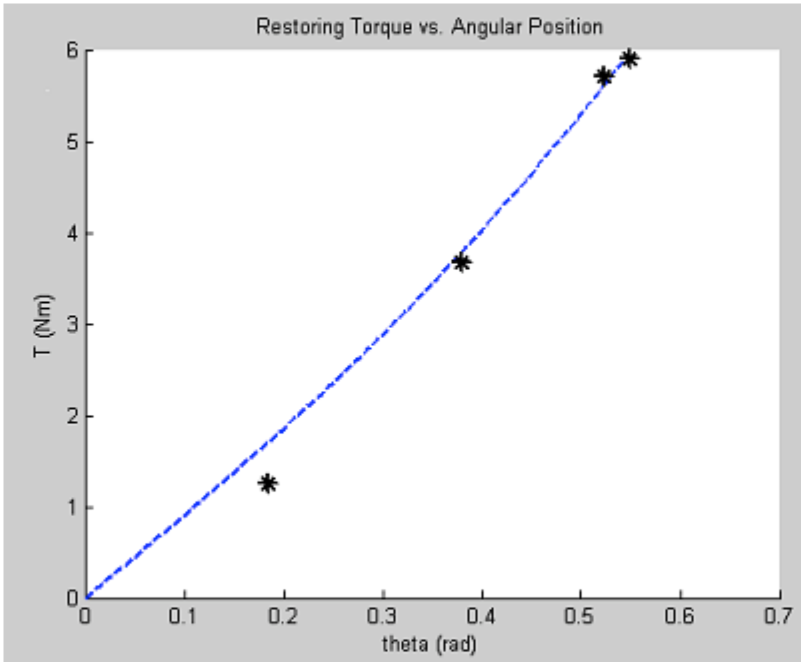

Figure 12: Experimental data (black stars) and theoretical predictions (blue curve)

4. Dynamic Simulation

The next step was to perform a dynamic simulation to estimate the time history of the angle between the Ranger's legs with the hip springs. This helped us determine how the hip spring will actually affect the Ranger in motion. The model for this simulation is the double pendulum. For simplification the "stance leg," the leg that is in contact with the ground, is assumed to be affected negligibly by the motion of the "swing leg," the leg that is swinging through the air. Figure 13 shows the geometry we are working with.

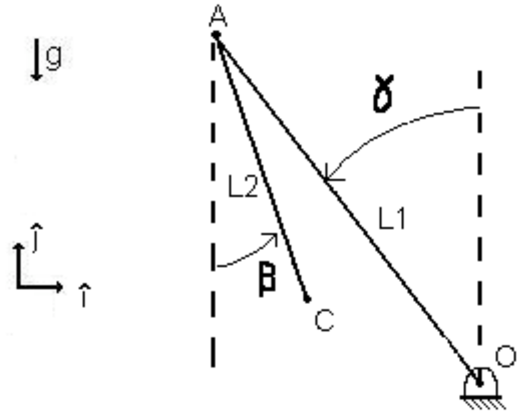

Figure 13: Double pendulum setup for the dynamic simulation*

*Note that in the following equations and Matlab code, θ is used rather than γ.
Summing moments about point A, we have

$$\vec{M}_A = \dot{\vec{H}}_{/A} \tag{8}$$

$$T\hat{k} = \vec{r}_{C/A} \times m\vec{a}_c \tag{9}$$

The acceleration of point C can be expressed in terms of the absolute acceleration of point A and the relative acceleration of point C with respect to A.

$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A} \tag{10}$$

$$\vec{a}_C = -L_1\dot{\theta}^2\hat{e}_{RA} + L_1\ddot{\theta}\hat{e}_{\theta A} - L_2\dot{\beta}^2\hat{e}_{RC} + L_2\ddot{\beta}\hat{e}_{\beta C} \tag{11}$$

Substituting equation (11) into equation (9) and transforming into the i-j coordinate system, we can arrive at an expression for $\ddot{\beta}$. The details of this expression are shown in the code of Appendix C. We use ode45 in Matlab to numerically integrate this expression over a time period to obtain time histories for the angles of interest.

The stance leg time history was attained by assuming it takes exactly 1 second to start at .275 radians (half of the maximum angle between the legs) off the vertical, flip over the top, and return to .275 radians on the other side. A simple pendulum equation was integrated and a cubic curve fitting scheme was applied to extract θ and its derivatives. Figure 14 shows the results of the dynamic simulation calculated from the code in Appendix B.

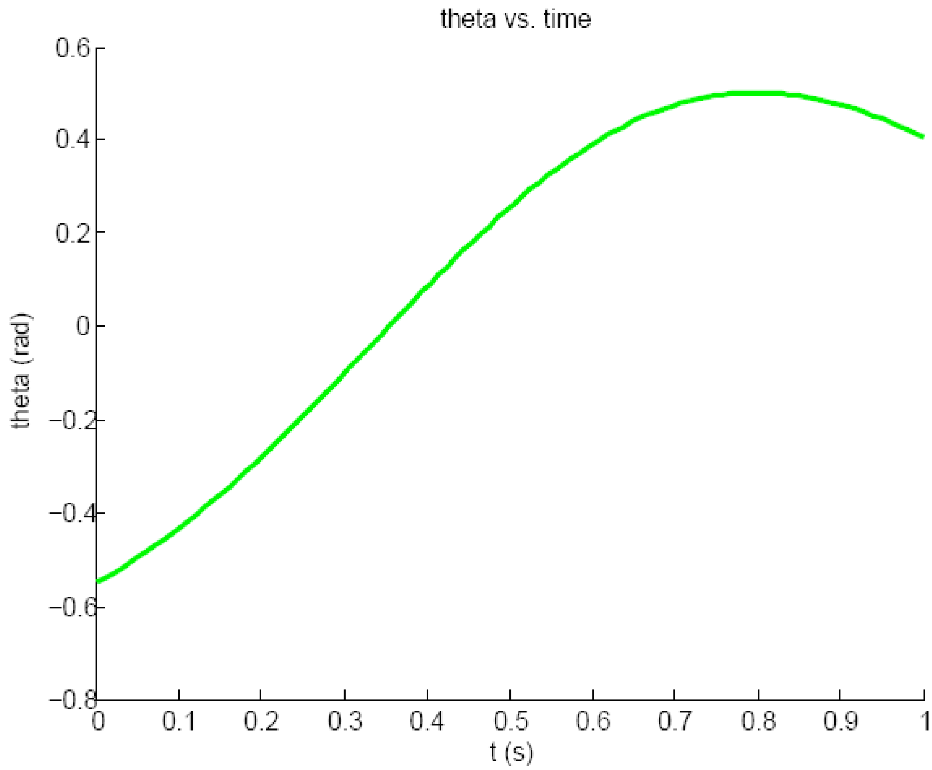

Figure 14: Angle between the legs as a function of time

As one can see from the graph, it takes about .8 seconds for the legs to swing through one stride. This is significantly faster than the leg swing time of the Ranger before the hip spring was added. We hope that this quicker leg swing will increase the robustness of the Ranger by preventing the robot from tripping over irregular and uneven surfaces.

5. Future Recommendations

The hip spring will inevitably be adjusted as the Ranger project proceeds. Motivations for adjusting the spring include redistribution of mass, use of different springs, or a different desired torque function. My recommendations for further work on the hip spring are:

- A better gravity calculator for *rangerspring*. The current method makes many assumptions, including equal mass and same centroid location for both the inner and out legs. Also, *rangerspring* assumes gravity only acts on the swing leg and that the angle off the vertical of the swing leg is always equal to half of the angle between the legs.
- Improvements in the dynamic simulation. The current model assumes the stance leg is unaffected by motion in the swing leg. A full double pendulum analysis will provide more accurate results.
- Installation of the brackets in the same orientation. Each piece of the brackets was machined to fix exactly as it was installed at the time of this report. Interchanging or rotating "C" shaped pieces will result in a misalignment of the hole and thus a poor fit onto the leg.
- Weight reduction in the inner leg brackets. The lower section of the brackets (away from the eye bolt) has excess material. Most of the stress in the brackets is concentrated around eye bolts and the areas near the leg clamps. Material can be removed from the lower portion of the brackets
- Further testing with the Ranger. The Ranger successfully walked with the hip spring installed, and the leg swing is noticeably quicker. However further tuning is required to get a better sense of the Ranger's potential performance with the hip spring.
- Energy efficiency analysis. As the Ranger project progresses, it will be important to know how each component affects the walking efficiency. The hip spring's effect on this is not well understood and will require testing.

Appendix A

```
%James Doehring
%Senior Project
%Fall 2007

function rangerspring

clc
format compact

%**********variables definitions**********
%a is the length from spring attachment to rotation axis on outer leg in
    %meters
%b is the length from spring attachment to rotation axis on inner leg in
    %meters
%c is the total length of the spring in meters as a function of theta (see
    %Figure 3 in report)
%ceq is the net equilibrium point (zero-force displacement) of the spring in
    %meters, which is usually different from the relaxed length of springs
    %due to pretension
%cmax is the maximum total length of the spring in meters (a function of
    %thetamax)
%cmin is the minimum total length of the spring in meters (occurs at theta = 0)
%cz is the z-component of the total length of the spring in meters(a function
    %of theta)
%theta is the angle between the legs in radians
%thetamax is the maximum angle between the legs in radians (occurs at
    %thetamax)
%k is the spring constant in N/m
%Fz is the z-component of the total spring force (both springs) in N
%F is the tension in one spring in N at theta = 0
%m is the mass that gravity acts on the produce a restoring torque in kg
%L is the distance from the hip axis to the centroid of the section of the
    %Ranger that gravity acts on to produce a restoring torque, in meters
%T is the total restoring torque in N-m


%NOTES:  (1) The current design has the variable 'a' fixed at .155 m
%        (2) When measuring 'b', consider the end of the spring itself,
%            rather than the top of the hook to define this distance
%        (3) Adding a cable to the spring changes all the c values,
%            including ceq.  In this case the proper ceq is the ceq of the
%            spring itself plus the additional length aquired as result of
%            adding the cable
%        (4) If springs are used in series, the effective spring constant
%            and other spring properties need to be used


%adjustables
b = .725; %[m], adjustable by changing height of inner leg bracket
ceq = 0.5236; %[m], spring property
k = 241.7; %[N/m], spring property
thetamax = .55;  %[rad], max angle between legs

%fixed parameters
```

```
a = .155; %[m], fixed
cmin = b-a; %[m]
cmax = sqrt(a^2 + b^2 - 2*a*b*cos(thetamax)); %[m], law of cosines
F = k*(cmin-ceq); %[N]

%gravity torque values
m = 6; %[kg]
L=.16; %[m]

T = []; %initializing the torque vector

%loop runs to calculate torque at varying theta, see report for calculation
    %details
for i = 1:100 %100 data points
    theta = (i-1)*(thetamax/99); %[rad], the current theta
    c = sqrt(a^2 + b^2 - 2*a*b*cos(theta));  %[m], law of cosines
    cz = b*sin(theta); %[m]
    Fz = 2*k*(c-ceq)*(cz/c);  %[N], z-component only
    Torque = Fz*a + L*m*9.81*sin(theta/2); %total torque, including gravity
    T = [T Torque]; %concatenation
end
    Fz*a
    L*m*9.81*sin(theta/2)
%plotting Torque vs. angle
theta = linspace(0,thetamax); %creating a theta vector

hold on
plot(theta, T, 'b-') %solid blue curve

xlabel('theta (rad)')
ylabel('T (Nm)')
title('Restoring Torque vs. Angular Position')

%print results
disp('metric')
fprintf('a = %.3fm\n',a);
fprintf('b = %.3fm\n',b);
fprintf('ceq = %.3fm\n',ceq);
fprintf('cmin = %.3fm\n',cmin);
fprintf('cmax = %.3fm\n',cmax);
fprintf('k = %.3f N/m\n',k);
fprintf('F = %.3f N\n',F);

disp('  ')

disp('Imperial')
fprintf('a = %.3fin\n',a*39.3700787);
fprintf('b = %.3fin\n',b*39.3700787);
fprintf('ceq = %.3fin\n',ceq*39.3700787);
fprintf('cmin = %.3fin\n',cmin*39.3700787);
fprintf('cmax = %.3fin\n',cmax*39.3700787);
fprintf('k = %.3f lbf/in\n',k*.005709917);
fprintf('F = %.3f lbf',F*.224808943);
disp('  ')
```

Appendix B

```
%James Doehring
%Senior Project
%Fall 2007

%function dynsim

%Dynamic simulation of hip spring
%all units are SI (meters, kilograms, seconds)
%see report for variable explanations and details

clc
% Initial conditions and time span
tspan= linspace(0,1,100); %Integrate for 1 second

z0 = [-.275 1.005 .275 0]'; % initial theta, thetadot, beta, betadot

%solve ODE
[t z] = ode45(@rhs, tspan, z0);

% Unpack the variables
theta = z(:,1); % theta is first column of z
thetadot = z(:,2); % thetadot is second column of z
beta = z(:,3); % beta is third column of z
betadot = z(:,4); % betadot is fourth column of z

angle = theta - beta; %this is the actual angle between the legs

% Plot the results
figure(1)
hold on
plot(t, theta, 'b');
plot(t, thetadot, 'r');
plot(t, angle, 'g');
plot(t, beta, 'c');

title('theta vs. time')
xlabel('t (s)')
ylabel('theta (rad)')

%%%%%%Torque section
%a, b, ceq, k as defined in rangerspring.m
a = .155;
b = .46;
ceq = .23114;
k = 253.92;

T = []; %hip spring torque
Tg = []; %gravity torque
for i = 1:100
    anglei = angle(i); %current angle
    c = sqrt(a^2 + b^2 - 2*a*b*cos(anglei));  %full length of spring
    cz = b*sin(anglei);
    F = 2*k*(c-ceq)*(cz/c);  %z-component only
    Torque = F*a; %hip spring component
```

```
torq_g = -3.78*9.81*.15*sin(beta(i)); %gravity component
    Tg = [Tg torq_g];
    T = [T Torque];
end

%plotting
Tor_tot= T+Tg;
figure(2)
hold on
plot(t, T, 'k--')
plot(t, Tg, 'b--')
plot(t, Tor_tot, 'r-')

title('Torque vs. time')
xlabel('t (s)')
ylabel('T (N-m)')
```

Appendix C

```
%James Doehring
%Senior Project
%Fall 2007

%function rhs

function zdot = rhs(t,z)
% all variables in SI (meters, kg, seconds)

% unpack z into readable variables
theta = z(1); thetadot = z(2);
beta = z(3); betadot = z(4);

%variables
g = 9.8; %acceleration of gravity
I = .4086; %moment of inertia
a = .155;
b = .46;
ceq = .23114;  %equilibrium point of spring
k = 253.92; %spring constant
l1 = 1.1; %distance from hip axis to ground
l2 = .87; %distance from ground to center of mass of swing leg
mouter = 3.78; %mass of outer leg

%curve fit polynomial coefficients
  p1 = 0.81032;
  p2 = -1.2163;
  p3 = 0.9521;
  p4 = -0.27338;

theta = p1*t^3+p2*t^2 + p3*t + p4; %[rad]
thetadot = 3*p1*t^2 + 2*p2*t + p3; %[rad/s]
thetadoubledot = 6*p1*t + 2*p2; %[rad/s^2]

c = sqrt(a^2 + b^2 - 2*a*b*cos(theta-beta));  %full length of spring
cz = b*sin(theta-beta);
F = 2*k*(c-ceq)*(cz/c);  %z-component only

Torque = F*a;  %torque (F*a)

 %finding betadoubledot
 X = sin(beta)*(-l1*thetadot^2*cos(theta)-
l1*thetadoubledot*sin(thetadoubledot)+l2*betadot^2*cos(beta)) + ...
     cos(beta)*(l1*thetadot*sin(theta)-l1*thetadoubledot*cos(theta)-
l2*betadot^2*sin(beta));

 betadoubledot = (1/l2)*(Torque/(mouter*l2) -g*sin(beta) - X);

z1dot = thetadot; z2dot = thetadoubledot; % rename variables
z3dot = betadot; z4dot = betadoubledot;
zdot = [z1dot z2dot z3dot z4dot]'; %outputs

end
```

Sources

J. G. Daniël Karssen. *Design and construction of the Cornell Ranger, a world record distance walking robot*. January 26, 2007.

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