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A. Project Summary

Passive, Nonlinear-Dynamic Study of Walking: Simulation, Analysis, and Experiment.

Andy Ruina, Theoretical and Applied Mechanics, Cornell University

Human walking might be approximated as a mechanical process governed by Newton's laws of motion and not controlled. Tad McGeer first demonstrated, and Ruina's lab has confirmed, that a two dimensional legged mechanism with four moving parts can exhibit stable, human-like walking on a range of shallow slopes with *no* actuation and *no* control (energy lost in friction and collisions is recovered from gravity). More recently, Ruina's lab has found a simple walking mechanism that also balances from side to side. That is, there is much that might be understood about walking by considering it as a natural motion of a simple uncontrolled and unpowered dynamical system, or a *passive-dynamic* system.

Ruina's work is intended to address a range of questions about the role of mechanics in animal movement. What are the limits to the stability and efficiency of these passive-dynamic walkers? To what extent can the properties of passive-dynamic machines mimic and thus give insights into human walking? How much of human coordination is governed by the brain and how much is governed by mechanics? To what extent are the physical aspects of biological design dominated by stability and/or efficiency considerations? Do more degrees of freedom limit the possibilities of self-stability?

Insight into the answers of these questions will come from mechanics-based theoretical and physical models. Ruina proposes to extend his lab's locomotion research using numerical simulations guided by non-linear dynamical systems approaches, and by building and experimenting with physical passive legged mechanisms. He plans to develop and build increasingly complex walking mechanisms that have efficient walking motions. For example, he plans to find a theoretical model that explains his labs 3-D walking and balancing toy, he plans to find theoretical 2-D passive walking models that have upper body parts, and he plans to investigate theoretical and physical models that have (possibly non-linear) springs and dampers. The efficiency and stability of various theoretical models, or lack thereof, provides guidance for understanding where control is really needed. If some reasonable approximation to the human body can be made to walk passively and efficiently in 3-D, then this might be a guide for how to design efficient prosthetic devices or improve abnormal gait.

Ruina also proposes to investigate simple actuation of increasingly complex theoretical models by adding simple power systems to passive gait cycles. McGeer and Ruina's lab have shown by means of simulation that some simple actuation schemes work as well as gravity for a simple 2-D theoretical model. A simple powered physical model may also be built. What are the similarities and differences between various simple actuation schemes and power from gravity? What can we learn about muscle activation patterns by these comparisons? This minimal-actuation approach could lead to more efficient applications of Functional Neuromuscular Stimulation (FNS), since simplicity, stability, and limited muscle usage are all critical factors in obtaining clinical effectiveness.

As seen from a control perspective, the proposed work largely involves investigation of control parameters which are physical properties rather than the traditional active-control parameters (such as feedback gains, neural net parameters, genetic algorithm reward schemes, etc.). While the second extreme – that of adjusting control algorithms and optimization criteria – is being explored by others, the other extreme – that of adjusting mechanical parameters in uncontrolled theoretical and phsyical models – remains relatively unexplored.

In summary, Ruina is testing the hypothesis that human walking is largely an uncontrolled mechanical process by designing, building, and studying uncontrolled or minimally controlled walking devices and seeing how well they mimic human motion.

C. Project Description

Passive, Nonlinear-Dynamic Study of Walking: Simulation, Analysis, and Experiment.[1]

Andy Ruina, Theoretical and Applied Mechanics, Cornell University

Introduction

Coordinated motion, locomotion, and walking in particular, are central aspects of human behavior. So furthering our understanding of them has a wide range of applications. Because human motion is controlled by the nervous system and powered by muscles the role of nerves and muscles is of central interest. One way to understand the role of nerves and muscles is to learn how much can be done without them. Human walking, for example, might be modeled for some purposes as an uncontrolled mechanical process. The role of the nerves and muscles in walking might be more to gently guide than imposingly control. This advantages of this *passive dynamic* approach to control are gaining acceptance [43].

The approach here was originally pioneered by McGeer (1989-1993) [35, 37, 40, 39, 38, 33]. McGeer demonstrated that a somewhat anthropomorphic, two-dimensional, four-link mechanism is capable of stable, human-like gait down a shallow slope with no activation (besides gravity) and no control. McGeer's passive-dynamic theory of bipedal locomotion describes gait as a natural repetitive motion of a dynamical system or, in the language of nonlinear dynamics, a *limit cycle*. In preparation for this funding proposal I and my graduate students Camp, Chatterjee, Coleman, and Garcia (who helped prepare this proposal), and some undergraduates, have duplicated and extended McGeer's work using computer simulation, non-linear dynamics techniques, and physical experiments. Our simulated stick figure in figure 3f on page 8 (or a video of the unpowered robot) show the similarity of McGeer-like passive mechanisms to human gait. This human likeness suggests that a good way to learn about human walking may be to learn about passive-dyanamic walking.

I propose here to continue our work on passive-dynamic, or nearly passive-dynamic, models of human locomotion. The results could be useful in the theory of gait synthesis, in diagnosing gait disorders, in prosthetic design, and in robotics. Specific applied problems that could gain from this research, for example, are functional neuromuscular stimulation (FNS), where minimizing muscle usage is a key strategy, and prosthetic design, where actuators with complex controls are expensive and difficult to maintain.

Muscles, nerves and gravity

Because the nervous system controls, and the muscles power walking, most gait simulations incorporate varying amounts and types of joint-angle or model-muscle control in an effort to mimic human gait (e.g. [45, 53, 27]). Some theoretical gait-synthesis models use sophisticated control strategies and generator patterns, such as the neural networks of [52]. Non-linear dynamics approaches, similar in some ways to what I propose here, have also been used [28, 8]. An attempt at a more realistic muscle and kinematics model is represented by the 3-D partial-step theoretical model of Yamaguchi [57]. Muscle forces are used by humans for more than just power. The torques preventing knee unlocking, for example, may represent necessary control.

A drawback of some previous theoretical muscle-control gait models, is that they only study a part of a step (e.g., [46, 58, 31]). However, in order to understand the efficiency and stability of gait, the entire gait cycle (i.e. a whole step) must be taken into account (as also argued by [52, 29, 30]).

[1] This project description is an improved version of a description submitted to the NSF in March 1996.

Because animal nerve systems are so capable, because the energetic cost of thinking is so low, and because minimizing food use is advantagous, I believe that an energy-based optimization approach is likely to well describe much of how people move, e.g. [7, 2, 4]. Other possible optimization criteria include peak muscle force, minimum-jerk, minimum-stress, etc.[15, 44, 25].

Electromyographic signals from muscles (EMG) show a low level of muscular activity in human and gorilla legs during walking [6]. The minimal muscle activity in leg swing motivated (what seems to be the first) passive-dynamic (or *ballistic*) partial-gait simulations [42].

There *is* a need for full-gait-cycle optimization simulations of gait using complex muscle-activation descriptions. The results of such optimizations, like the results of animal evolution, will probably show limited use of muscles in walking. Given the uncertainties and complexities of many-degree-of-freedom optimization studies, and the likely prediction of small muscle usage, there is hope for insight from simpler approaches which do not include these muscles.

Although some muscular power is needed for walking, it might be neglected in some analysis like engine power can be neglected for much of the study of airplane flight [35]. A small simple energy source, gravity, is then used as a proxy for the small but essential muscle use of humans. It is hoped, as must be checked, that most results will be insensitive to the choice of the energy source. However, the use of gravity as an energy source (as opposed to a simple muscle approximation) eliminates some arbitrariness, and simplifies simulation and physical experimental verification.

The control aspects of muscle use involve small energetic cost, at least in principle. The role of low-energy control actions may be better understood by finding the limits of passive strategies.

Passive-dynamic walking

The emphasis of this proposal is research on pure passive-dynamic models, built (theoretically or physically) from passive elements (rigid bodies, springs, dashpots, hinges, frictional and rolling contact) with power coming only from gravity. These uncontrolled models can have one, two, or three of these remarkable properties.

1. **Existance of gait.** With no control they have periodic motions that look like walking.
2. **Efficient gait.** The passive walkers can have remarkably high efficiency, approaching perfect efficiency (at least in theory).
3. **Stable gait.** For some parameter combinations the gait limit cycles are stable. After small perturbations steady gait is reached again.

Thus, I believe that deeper study of passive-dynamic models will provide clues about the design of the human body and the brain's underlying strategies for motion synthesis.

Our passive-dynamic research to date.

Here I summarize our progress in passive-dynamic locomotion research. [10, 13, 11, 14, 12, 20, 19, 18]. (Some reprints, preprints, reports and video clips can be downloaded from:
http://tam.cornell.edu/programs/humanpower/humanpower.html).

Wheels, etc.

Two intimately related ways to support a translating weight over approximately level ground are with wheels and with legs. McGeer [35] studied two wheel-like devices: the *synthetic wheel*, a non-physical device which we have not investigated, and the spoked but *rimless* wheel which we have studied in some detail.

Rimless Wheel in 2 and 3 Dimensions

A rimless wheel pivots and collides with the ground on rigid spokes instead of rolling. It shares with walking the feature that translation occurs by intermittent non-slipping contact. When a spoke collides with the ground, the trailing spoke instantaneously loses contact so that, except at the moment of collision, only one spoke is contact with the ground. We assume the spoke collisions are instantaneous, have no-slip, and are perfectly inelastic. The only non-contact force is gravity. Unlike an ideal rigid dissipation-free round wheel, the rimless wheel cannot roll steadily on level ground because it loses energy at each collision.

Results and insights from the theoretical rimless wheel models

My student Coleman completed a non-linear analysis of the rimless wheel constrained to 2 dimensions [11], extending McGeer's linearized analysis. We also analyzed a rimless wheel free to move in 3 dimensions [13].

The speed of a 2-D rimless wheel is regulated by dissipation from collisions [33]. The gravitational energy available per step is independent of speed and proportional to step length, whereas the kinetic energy lost per step in collisions increases with the square of the speed and also (approximately) the square of the spoke spacing (see also [1, 4]). Balance of these energies determines the speed of the wheel. Our 2-D rimless wheel analysis did not produce any suprises, but it is the simplest example that yields, at least in part, the scaling rules we have discovered which apply to the more complex theoretical walking models.

Unlike the 2-D rimless wheel, the 3-D rimless wheel is not constrained from falling down sideways. Because rolling coins, wheels, disks, etc. don't fall over, the stability of the rimless wheel might not seem surprising. However, rolling flat disks are only neutrally stable against lean perturbations (perturbations never decay), whereas the 3D rimless wheel can be asymptotically stable (small perturbations decay). Our discovery that intermittent contact augments side to side stability in rolling raises the possibility that a similar passive processes could contribute to human side-to-side balance.

The simplest walker in 2 dimensions

The simplest walking mechanism with swinging legs that can fall down, and thus has an interesting balance, is the simplified point-foot straight-leg 2D walker of figure 1a [19]. It is a double pendulum with a big point mass at the 'hip' and much smaller point masses at the 'feet'. It is a simpler version of the theoretical model being studied independently by Goswami and others in France [21, 54, 22, 23] (who have independently found and/or reproduced some of the results discussed below). The simplest walker is a deterministic generalization of Alexander's non-deterministic theoretical "minimal biped," [1].

For the simplified point-foot walker the stance leg is an inverted pendulum, while the swing leg is a pendulum whose hinge point moves. At heelstrike, angular momentum balance determines the jumps in joint-angle rates. In our simulations we allow a no-impact swing through at the otherwise inevitable foot scuffing of all 2D straight legged walkers. We have conducted only theoretical studies of the simplest walker.

Some Results from the simplified 2-D point-foot walker

This drastic simplification of walking has suprising properties that carry over to the kneed walking theoretical models and perhaps to human gait [1, 19, 18].

After nondimensionalizing the governing equations, this walking model has *no* free parameters other than the ground slope γ. No motions or results depend on parameter fits.

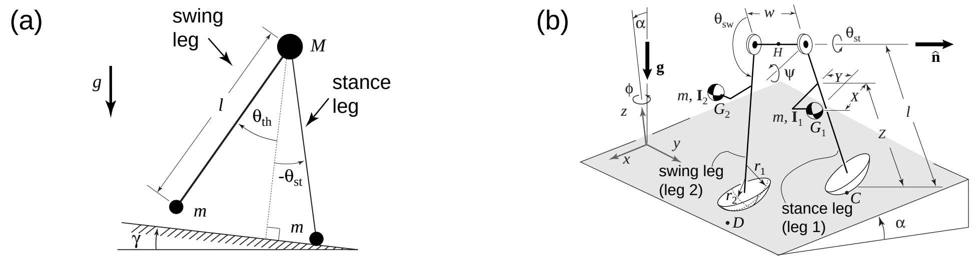

Figure 1: **(a) The 2-D point-foot theoretical walking model** from [19]. Hip mass dominates foot mass. **(b) A general 3-D knee-less theoretical model**: from [14]. This theoretical model may be sufficient to explain the working physical tinkertoy walker.

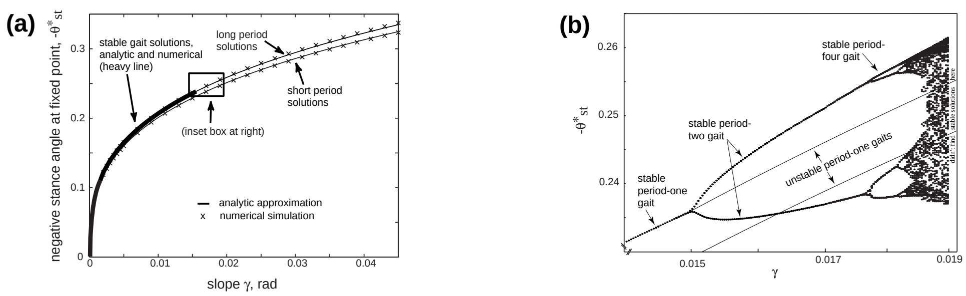

Figure 2: (a) Point-foot stance angle at fixed point as a function of slope, both numerical and analytic predictions are shown, from [19]. At zero slope, the gait is perfectly efficient and infinitessimally slow. The short period semi-analytic solution is $-\theta^*{}_{st} \approx 0.943976\gamma^{1/3} - 0.244101\gamma$. The long period semi-analytic solution is $-\theta^*{}_{st} \approx 0.970956\gamma^{1/3} - 0.234372\gamma$. (b) Period doubling of stable walking motions, inset from previous figure. Unstable period-one cycles are shown for reference. (dotted lines represent stable cycles while solid lines represent unstable ones.) No *stable* walking was found at slopes above ≈ 0.019.

We found two gaits (period one limit cycles) at every small slope. One of these gaits is unstable and one is stable at shallow slopes ($\gamma < 0.015$). For both of these gaits the stance angle, the angular velocities of the legs, and average transit velocity, scale as the cube root of slope $\gamma^{1/3}$. Swing period τ is constant to first order in γ.

Figure 2a shows stance angles for the short and long period-one gaits, plotted as a function of γ. The region of stable period-one gait bifurcates into a stable period-two gait as the period-one motion becomes unstable. As γ is varied from 0.017 to 0.019, we observed the standard period-doubling route to chaos (e.g. [51]), as shown expanded in figure 2b. The period doubling (limping) and possibility of chaotic walking (stumbling) exhibited by this simplest of theoretical models might suggest a relevance to the variety of human gait styles.

Efficiency of the 2D point-foot walker

The standard measure of transport cost or transport inefficiency is energy used per unit distance traveled per unit weight carried (where a value of zero is perfectly efficient). This measure is the slope γ for passive downhill machines on small slopes. If the walker could walk steadily on level ground ($\gamma = 0$), it would be perfectly efficient. It turns out that stable walking motions persist down to arbitrarily small slopes for this theoretical model.

After redimensionalizing the equations, the small-slope scaling rule governing gravitational power usage is [19],

$$(\text{Power}) = C \cdot m \cdot g^{-1/2} \cdot \ell^{-3/2} \cdot v^4 \tag{1}$$

where C is, for example, $pi^3/8$ for the short period gait. For a 50 kg, 1m legged person walking at one meter per second this predicts a somewhat high 60 watts.

We have begun to see why other theoretical models, such as that shown in figure 3, violate (1) [18]. The strong speed dependence in (1) hints at a possible basic reason for gait transition as mentioned in [1] for his related scaling rule. However, 1 also reveals the extremely low energy cost of low speed walking.

Straight legged walker in 3-D

McGeer [40] and Fowble and Kuo [17] were unable to find stable walking motions for a 3-D passive walker. Although the asymptotic stability of the rimless wheel (above) inspires some hope, it has gyroscopic terms to help with stabilization that theoretical and physical walking models cannot access. However, work on skateboards by [26] as well as our previous work with bicycles [24] and boats [9] shows that passive balance stability does not necessarily depend on gyroscopic terms.

Thus informed, we have begun investigation of a point-foot walker in three dimensions. The special mass distribution of the simplest 2D point-foot walker has singular equations of motion in 3-D, so we have used a more general mass distribution. For certain mass distributions that are planar or have planar symmetry, the 3-D walker is known from 2-D analysis to have 2-D walking solutions that are stable against in-plane perturbations.

Although our numerical attempts to find stable 3-D walking have failed thus far, they led to some insight into stabilizing techniques, which in turn led to a simple succesful physical 3-D passive-dynamic walker ([14]). This is the only known (to us) three dimensional passive-dynamic walker that can stably walk, but that cannot stand still in any configuration. Prior to this device, stable three-dimensional passive walking machines with more than one link have yet to be found in theory, simulation, or physical experiment, excepting statically-stable toys with low mass-centers and/or broad feet.

Although the mass distribution in this physical model is not anthropomorphic, its success hints at a possible role for passive dynamics in side-to-side balance as well as fore-aft balance.

Passive dynamic walking with knees

Our 2-D kneed walking theoretical and physical models [18], based closely on McGeer's models, are shown in figure 3a-c on page 8. Figure 3d shows one of our dynamic simulations using the theoretical model for just over one step.

The physical 2D kneed walker of figure 3c exhibits stable limit cycle motions which strikingly resemble human gait. These gait cycles change in nature as the slope angle γ varies. We have traced out the theoretical solution-locus diagram shown in figure 3e. This plot shows stance angle at a fixed point plotted as a function of slope. McGeer plotted the stable part of this curve for a similar walker, but did not study unstable fixed points (although he mentions their existence). For comparison, 3e also shows the locus of solutions for the corresponding kneeless walker.

On the solution locus in figure 3e, the ratio of time until kneestrike to total step time grows monotonically until kneestrike and heelstrike are simultaneous, and the curve ends at the point of fastest gait. The most anthropomorphic looking gaits (by subjective judgement) are found near here. The other the other end of the locus curve corresponds to the smallest step possible. Other quantifiers of gait (e.g., step period, initial conditions, velocity, efficiency, etc.) also vary along this curve.

As inspired by the straight-legged walker results, we found how to theoretically predict perfectly efficient, walking at slope $\gamma = 0+$, kneed and straight-leg walkers [18]. Simulations verify that such mass distributions do lead to perfectly efficient walking. The efficient mass distribution requires colinearity of the nominal contact point, the center of mass, and the hip-joint with the

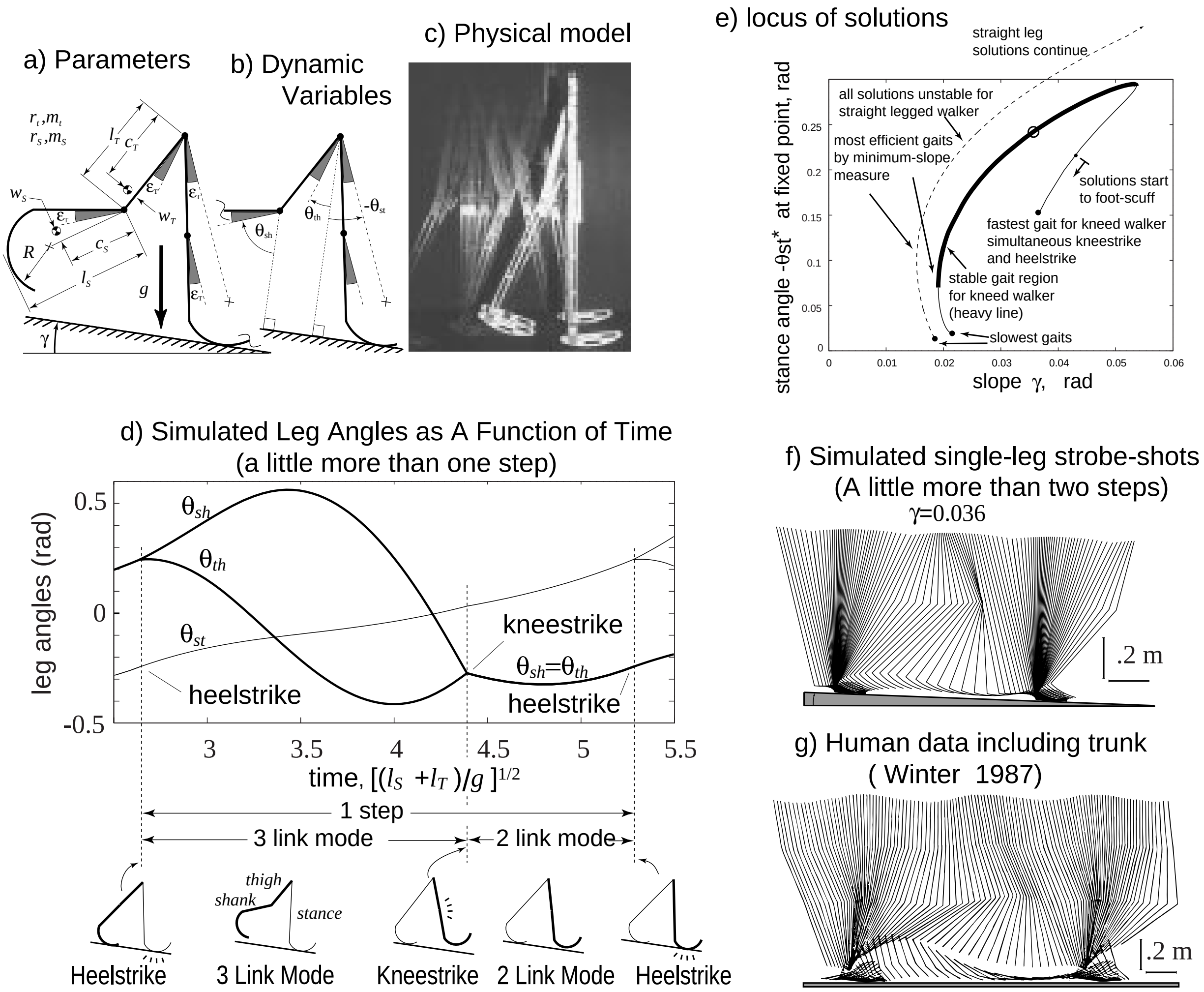

Figure 3: **Kneed 2-D Passive Dynamic Walker.** **(a) Theoretical model parameters**, not drawn to scale, include radii of gyration and masses of thigh and shank, denoted by r_T, m_T, r_S, and m_S, respectively. The circular foot is centered at the '+'. ϵ_T is the angle between the stance thigh and the line connecting the hip to the foot center. **(b) Dynamic variables** are θ_{st}, θ_{th}, and θ_{sh} which are measured from the ground-normal to lines offset by ϵ_T from their respective segments. **(c) Our physical model** walking down a shallow ramp with strobe exposure (approximately one step). The visible double leg-set constrains the physical model to 2-dimensional motion ($\ell_t = 0.35\text{m}, w_t = 0\text{m}, m_t = 2.354\text{kg}, r_t = 0.099\text{m}, c_t = 0.091\text{m}, \ell_S = 0.46\text{m}, w_S = 0.025\text{m}, m_S = 1.013\text{kg}, r_S = 0.197\text{m}, c_S = 0.17\text{m}, R = 0.2\text{m}, \gamma = 0.036\text{rad}, g = 9.81\text{m/s}^2, \epsilon_T = 0.197\text{rad}$). **d) Computer simulated steady gait cycle** (from [18]). Angles of leg segments are shown from before a heelstrike to after the next heelstrike in a stable gait. The heavy line on the upper graph corresponds to the motion of the heavy-line leg on the 5-frame cartoon under the graph. The angular velocities have discontinuities at kneestrike and heelstrike, which appear as (barely visible) kinks in the curves. The parameters for the simulation correspond to measured values from the physical model in (c). **e) Locus of solutions** for a kneed walker (solid) and the same walker with knees locked (dashed). Both stable (heavy line) and unstable (light line) periodic motions are shown. The solution in (d) is marked with an open circle. **f) strobe line drawing** of the positions of one leg, spaced evenly in time, over a little more than two steps of the simulation in (d). **g) Human subject data** from [56] (taking bigger steps, and shown at a smaller scale).

ground normal. Further, the shank center of mass must be directly below the knee. This efficient mass distribution correlates with humans being such that when standing on one foot the relaxed hanging leg and stance leg nearly coincide (when viewed from the side).

Like the point-foot walker, the kneed walker can also exhibit complex motions. If parameters are appropriately adjusted (but symmetrically), period-two gait cycle motions (limping) can be found as well as chaotic (stumbling) gait [18].

The kneed model and real human beings

In humans, double-support (two feet on the ground at once) accounts for about 20% of a gait cycle. In most of our theoretical and physical models double-support is an instant. Thus we cannot, without adding more complexity to our models, address the details of the propulsion from ankle flex in double support, though this may not be a critical flaw, as argued by [55]. Another theoretical modeling approximation is the locking of the stance leg. Experimental data (e.g. [56]) shows that the stance leg flexes slightly at mid-swing (as careful inpection of 3g reveals). Finally, the model is only two-dimensional and lacks all upper body parts.

A Physical Model With Knees

Using information from our simulation, John Camp and Yan Yevmenenko, undergraduate NSF REU students in my lab, along with several other undergraduates, constructed working kneed walkers very similar to McGeer's, figure 3c. At the one slope for which we made a detailed comparison, the physical walker's stride period and step length matches our simulation to within 5%. The development of our simulation occurred simultaneously with tests of this physical model. The simulations were used to adjust physical model parameters to achieve stable gait in the lab. The physical model's performance also helped uncover errors in the early simulation programs.

Proposed Work

We propose to elucidate basic principles of human walking by studying mechanics-based theoretical and physical models. Some of the broad questions we would like to address include: What strategies might the body or prosthetic devices use to generate walking motions? What strategies make for efficient walking? What aspects of walking motions make them easy to stabilize or, somewhat conversely, to control in a versatile manner? What are the trade-offs between stability, efficiency, and controllability? Because *unstable* passive-dynamic limit cycles can always be be stabilized (in principle) with essentially zero energy cost, is passive-dynamic stability actually a concern in nature's design of biological machines (we are already confident that efficiency is important)? How well can uncontrolled or crudely controlled mechanics explain human locomotion?

Procedure of Study

Most of our planned work is based on using simulation, non-linear dynamics theory, and simple physical experiments with a variety of physical models. Simple analytic approaches will also be used when possible.

The approach we have been following and plan to continue following is based on the parametric design recipe used by McGeer. We plan to use the approach with a sequence of theoretical and physical models. These models include straight legged walkers in 3-D; 2-D models with upper bodies and powered ankles; and possibly multi-segment 3-D models. The models are described in more detail after this list of methods. Not all aspects of the program below need be applied to all models, and various elaborations are required for some models.

In short the procedure involves theoretical model selection, finding governing equations, dynamical systems interpretation, stability and efficiency analysis, physical model construction and experimentation, and evaluation of results. In more detail:

1. **Determine an appropriate mechanics based theoretical model.** We plan to look primarily at theoretical models based on linked rigid bodies. Design choices involve the nature of connections, and contact. The theoretical models will be based in part on previous results and imitation of human design. There are numerous subtle judgements that we will need to resolve in this process. For example, at present we have a physical walker that balances in 3-D [14] yet we do not yet know exactly which aspects of its physical description are needed to theoretically predict its stability with computer simulation.
2. **Write equations of motion for the theoretical models.** The motion is determined by the differential equations and jump conditions of classical mechanics. Equations will be generated either by hand, using symbolic algebra (eg. Maple or Mathematica), or with a special purpose dynamics-equation generator such as AUTOLEV. If time allows, simulations will be generated two ways to assure accuracy (we have caught many mistakes this way).
3. **Set up a solution scheme.** Generally we will use standard numerical integration schemes or packages (e.g. MATLAB) to solve the differential equations and algebraic jump conditions.
4. **Treat a step as a function.** The solution of the equations, from the state at one step to state at the next step, can be thought of as a function $\mathbf{f}$, termed the "stride function" by McGeer. All of our theoretical and physical models, even to the extent that we add power and control, will be autonomous processes. Thus much information about a step will be encoded in the function $\mathbf{f}$. This function will take as input the list of values of the various angles and rates (the state variable vector $\boldsymbol{\theta}$) just after ground collision (or any other well defined point in the motion) and will return the values of $\boldsymbol{\theta}$ after the next ground collision.

   For a given set of initial conditions, the solution of the governing differential and algegbraic equations over the period of time corresponding to one step yields one evaluation of the function $\mathbf{f}(\boldsymbol{\theta})$. In the language of dynamical systems, the stride function is a Poincaré map. Many of our questions about the dynamics of a given theoretical walking model will then be reduced to questions about the function $\mathbf{f}(\boldsymbol{\theta})$.

   Other steps below depend on making a single evaluation of $\mathbf{f}$ routine and fast.
5. **Find steady, possibly unstable walking.** A simple (period-one) *gait cycle*, if it exists, corresponds to a set of initial values for the angles and rates which lead back to the same angles and rates after one step. This set of angles and rates $\boldsymbol{\theta}^*$ is a fixed point of the Poincaré map $\mathbf{f}(\boldsymbol{\theta})$, i.e., $\mathbf{f}(\boldsymbol{\theta}^*) = \boldsymbol{\theta}^*$. This cycle corresponds to a zero of the difference function $\mathbf{g}(\boldsymbol{\theta}) \equiv \mathbf{f}(\boldsymbol{\theta}) - \boldsymbol{\theta}$. A *period-two* gait cycle returns the same variable values after *two* steps, and so on. Period-one motions are our central interest because they correspond to the important task of steady walking.

   We will find fixed points of the function $\mathbf{f}$ using homemade and standard (e.g. Matlab) root finding functions on the difference function $\mathbf{g}(\boldsymbol{\theta})$. We will use systematic methods (e.g., multidimensional Newton-Raphson) in combination with other random and guided searching methods such as monte carlo and simulated annealing, as needed.

   There is no guarantee that we will find gait cycles (roots of $\mathbf{g}$) for any given theoretical model and set of parameters. Although finding the limit cycle involves solving n equations for n unknowns, not all parameter combinations lead to solutions. Also, for parameter combinations which do produce gait cycles, there is no guarantee of a numerical routine finding them. The root finding aspects of the work will involve a mixture of intuitively based theoretical model definition, on starting new searches on known solutions, and on various numerical methods.

6. **Evaluate performance.** For each steady motion we need to evaluate the stability and other performance indices (e.g. measures of speed or efficiency) using analytically guided numerical methods. A simple and useful measure of stability comes from the eigenvalues of the derivative matrix $\mathbf{J}$ of the map $\mathbf{f}$

$$\mathbf{J} = \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}} \qquad \text{with components } J_{ij} = \frac{\partial f_i}{\partial \theta_j} \tag{2}$$

The *linearization* J generally characterizes the dynamics when motion is close to a periodic walking cycle. Small perturbations $\hat{\boldsymbol{\theta}}$ to the limit cycle state vector $\boldsymbol{\theta}^*$ at the start of a step will grow or decay from the kth step to the $k+1$th step approximately according to $\hat{\boldsymbol{\theta}}^{k+1} \approx \mathbf{J}^k \hat{\boldsymbol{\theta}}$. We plan to evaluate $\mathbf{J}$ by numerically evaluating $\mathbf{f}$ a number of times in a small neighborhood of $\boldsymbol{\theta}^*$. We then numerically evaluate the eigenvalues λ_i of the linearization J. If all of the eigenvalues are small enough, $|\lambda_i| < 1$ all sufficiently small perturbations will decay to $\hat{\boldsymbol{\theta}} = \mathbf{0}$ and the system will asymptotically approach its limit cycle. If the Jacobian has any eigenvalues outside the unit circle, any perturbation with a component along the corresponding eigenvector will bump the system divergently off the limit cycle — the cycle is unstable and can not be realized in an uncontrolled physical model. If an eigenvalue has magnitude of one, then the cycle is neutrally stable for infinitesimal perturbations along the corresponding eigenvector and such perturbations will neither shrink nor grow (to first order). Inevitably eigenvalues of magnitude 1 generally appear and do not affect balance stability. For example, the indifference of most of the 3-D devices to direction of travel generates an eigenvalue of 1 in the map.

We have found the eigenvalues of the linearization J to be a suprisingly useful characterization of stability. We will be on the lookout for how well this measure correlates with other possible stability indicators, such as measures of the size of the basin of attraction.

The important essence of passive dynamic research may end up in finding limit cycles without need for exponential stability. Because humans do have control and need to exercise this control to go where they want, slow instabilities may not be important. For example, many bicycles are passively stable in a limited range of speeds [24]. This stability is lost by a very slow instability at high speeds (starting typically at about 18 mph). Bicycle riders, on the other hand, only sense *increased* stability at higher speeds due apparently to the one decreasing eigenvalue. Passive instabilities that are easily controlled and have long time constants may have little cost of any kind to controlled biological systems.

On the other hand, the design of physical, absolutely-uncontrolled passive-dynamic walkers does depend on both motions *and* stability (finding fixed points of $\mathbf{f}$ and having the eigenvalues of the Jacobian $\mathbf{J}$ inside the unit circle on the complex plane). Besides stability, a theoretical model may need to satisfy other performance criteria such as acceptable foot clearance, sufficient knee-locking torques, minimal collisions, etc., high efficiency, high speed, etc. These criteria require that, for some purposes, we study the whole motion as a function of time $\boldsymbol{\theta}(t)$ associated with periodic heel-strike values (the fixed points of $\mathbf{f}$).

7. **Tune the system parameters.** Given a limit cycle and various measures of performance, parameters will be tuned to improve performance (which also changes $\boldsymbol{\theta}^*$). For example, the successful final construction of our kneed robot depended completely on this numerical tuning.

A superficial counting analysis predicts that if the number of adjustable system parameters exceeds the number of degrees of freedom one should be able to decrease the magnitudes of all eigenvalues simultaneously. However there is no assurance that this generically possible process will not terminate at a local minimum above the stability threshohold ($|\lambda_i| = 1$), or a parameter boundary before a stable parameter set can be found.

8. **Build a physical model.** Building physical devices may seem unneeded if we have good simulations. However, we have found that watching physical models is essential. We have found both errors in theoretical modeling (i.e. mistakes and bugs) and simulation opportunities (e.g. the tinkertoy walker) by looking at our physical models. Although the physical models we study are relatively simple, designing, building, and measuring the properties of prototypes often takes noticeable effort.
9. **Compare and analyze.** We will compare the results of the simulations and physical experiments with the design goals and with human walking data in the literature. For instance, do stable passive models have similar slope vs. speed scaling laws as compared to humans? We will also generate small simple analytical approaches such as those which led us to zero-slope walking and to our efficiency scaling results.

Proposed Models for Investigation

We plan to study the models below using the methods outlined above. They are roughly in order from the most well-defined to the most open-ended.

1. Theoretically model our physical 3-D walker.

We have a working physical 3-D walking mechanism, as described in [14], but we do not yet have a theoretical/computational model that predicts its stability. Although this mechanism is not particularly anthropormorphic, we should understand what features make it stable before we go on to other 3-D theoretical or physical models. The theoretical model we plan to initially investigate is shown in figure 1b. Because our completed point-foot, no-hip-spacing, no-scrub torque, frictionless-joint calculation does not predict stability, we do not know if the physical model's stability essentially depends on foot curvature, hip spacing, foot scrub friction, or hip-joint friction. These technical questions about this simple mechanism need to be addressed in order to investigate how passive-dynamic mechanisms can contribute to the side-to-side balance of human wallking.

2. Put our 2-D walkers onto 2 legs.

Our physical model of our 2-D walker uses 4 legs to keep side to side balance. More convincing demonstration of the passive-dynamic concept depends on making more human-like mechanisms. At present we have no assurance that we can make a staticallly unstable 3-D mechanism that balances from side to side without the use of strange mass distributions like that of the 3-D physical toy described above. There are ways to achieve side to side balance without 4 legs, however. The German patent (Bechstein and Uhlig, Dec 5, 1912, #7453 — see especially figure 8 therin) describes a means of controlling the side to side wobble of walking toys using polygonal feet bottoms that vaguely resemble the bottoms of human feet. Unlike our 2-D walker, the toys described in the patent do not have knees and did not need dynamics for fore-aft balance. But the idea might still be useful for our physical models. Further, maybe a mechanism like that described by Allison (U.S. Patent # 1,207,464, 1916) can be used to keep the ground contact foot trajectories more in line (as for human gait).

The combination of these two ideas, 1) guiding the walkers body using ground contact feet that are developable surfaces and 2) guiding the leg pivot at the hip so that the stance leg passes more under the body, may allow us to build a two legged robot with the anthropomorphic gait and the reasonable mass distribution of our present four-legged robot.

Construction of such a mechanism would help demonstrate the relevance of 2-D analysis to 3-D gait and enhance our ability to communicate the utility of the passive-dynamic approach to more skeptical observers.

2-D walker with an upper body.

An obvious short-coming of all passive-dynamic modeling to date is in the lack of an upper body. Chopped-at-the-waist theoretical and mechanical models may represent the motions of a more complete mechanism, but this has yet to be demonstrated. McGeer [36] intended - but did not pursue - an idea to use an ingeneous actuator and simple feedback to control the motion of the upper body relative to the legs. But there seems every reasonable hope that a theoretical model with an upper body, or even with arms, could have at least *unstable* limit cycle motions.

Given the success of previous McGeer-like theoretical models, one can hope for finding a passive stabilization strategy. Recently Goswami et al. (private communication) have shown that adding torsional dampers significantly enhances stability of body-less theoretical models (at some cost in efficiency presumably). There might be a combination of torsional springs and dampers that would stabilize walking motions even with an upper body.

Such a theoretical model might not have too great a relevance for healthy humans because the simulation of springs is most accurately accomplished with tiring co-contraction (which is often avoided by humans). But it does point towards the utility of passive measures for prosthetics and towards simple, spring or damper simulating control laws.

3-D walker with more body parts

Depending on the successes and insights from the above theoretical modeling and physical construction, we plan to move on to a three dimensional theoretical model with more body parts. Some ideas to pursue in three dimensional theoretical modeling and possibly physical construction include knees, upper body, arms, and a head with hip spacing, mass distributions, etc. based on existing human data (e.g. [50]).

What are the phase relations of the body parts in the passive motions and how do they compare to human motions? Although we do not expect to find stable passive motions with so many degrees of freedom, the similarities and dissimilarities of any limit cycles with human motion would be informative. Good agreement would point towards stable controlled human motion being controlled unstable passive motion.

Ankles

Human walking is typically not powered by gravity but primarily by ankle flex as a foot is leaving the ground at the end of its stance phase.

Tendons and muscles can act like springs to store and release energy [3] [41]. Springs might also be an effective energy-recovery mechanism in theoretical walking models, as they have been found to be in the theoretical running models of [34]. The mechanism by which springs could improve the efficiency of low-speed walking has not yet revealed itself to us, however.

Preliminary investigations suggest that the energy scaling law (1), in which the cost of locomotion increases with the fourth power of speed, is a lower bound on energetic cost for rigid-body springless theoretical models. Investigation of a 2-D theoretical model with springy ankles might be a starting point for learning the possible role of springs in walking.

Powered walking.

Ultimately, actuated theoretical models (e.g. [53]) possibly using sophisticated muscle descriptions (e.g., [59]) will probably yield the most accurate predictions of gait. Airplanes need engines and people need muscles. In the passive models gravity is used as a proxy for muscle power (see page 3). We propose to more directly investigate the utility of the fully passive approach by trying simple actuation strategies instead of gravity. Simple powered physical models might also be built.

My student John Camp [10] recently simulated straight leg 2-D walking without knees but with a primitively powered and controlled ankle and found stable gait on level ground [10]. A more anthropomorphic powered theoretical and/or physical model could probablyy be made stable and efficient with primitive control.

Because there are options for how power is added, the introducion of power is *necessarily* the introduction of control. However, we would like to preserve the spirit of the passive-mechanics approach by keeping the adjustable power parameters to be extremely simple. Such simple powering parameters could be switch-on or switch-off conditions that only depend on the internal configuration and not on sensed orientation, for example.

The videos of the recently publicized 4000 watt, \$10,000,000+ Honda Humanoid Robot (about which little is officially known) provide insight into the extreme cost, difficulty, and inefficiency of attempting control-based walking (for both robots and humans, presumably). It seems likely that that the human body's design and control scheme(s) are more like finding and operating near passive motions, and then adding small amounts of actuation. Perhaps passive dynamics, coupled with a negative-work minimization strategy [48], may simply explain much of the nervous system's coordination scheme for locomotion. We believe that an energy-minimization approach may lead to efficient and human-like theoretical models of walking as well as more realistic physical models.

Various studies have postulated optimization schemes for human gait - e.g., [15]. Can a minimum-muscle-work optimization scheme lead to a realistic but nearly passive theoretical human walking model? We do not plan to implement 3-D theoretical models with realistic tendon geometry and muscle constitutive laws. Such more-realistic theoretical models will be more ably assembled by other research teams. However, the relation between energy optimization and the strengths and shortcomings of the passive approach may be highlighted by our simple theoretical models.

Pathological gait.

There may be some correlation between parameter effects in passive or simply powered theoretical walking models, and in some cases of pathological gait where the causes are mechanical (and not neurological) in nature.

We would like to better understand the effects of mass distribution on the existence and characteristics of possibly awkward passive kneed and straight-legged gaits. One possible parameter study uses a pointfoot walker, and keeping the legs symmetric, introduces two parameters w and ℓ to locate the center of mass of the leg. (These parameters are labelled w_S, w_t, ℓ_S, ℓ_T in figure 3a, but for straight-leg walkers only w and ℓ are necessary.) We imagine that as w is incremented, passive solutions will cease to exist at some value w^*. Does this value relate to the morphology of gait-impaired subjects? Can the analytic solution predict the existence or non-existence of gait? Our experience thus far has indicated that gait is highly sensitive to w. If this is also true in humans, then subtle changes in morphology might account for some gait irregularities. A natural extension of his project would be to collect normal and abnormal gait and subject morphology data from the literature and test our analytic predictions.

We can then repeat this procedure (1)-(3) for height ℓ of the center of mass along the leg or other parameters in other theoretical models, as seems appropriate.

Running.

In the spring of 1996, two undergraduates, Michel Maharbiz and Pedro Felzenswalb, began a numerical investigation of passive dynamic running in 2D using a theoretical model like that simulated by Raibert [47] but using the nonlinear-dynamics approach. Interestingly, they found, as had [34] for a different theoretical running model, that unstable passive running motion on level ground at finite speed. That is, with a massless foot idealization, 100% efficient locomotion is possible for Raibert-like hopping machines. Although running is not our main focus, a possible goal is to find a single theoretical model that, with the tuning of a single parameter, is capable of both walking and running gaits. One possibility would be to more carefully explore a theoretical model like the walk/run model proposed by Alexander [1].

Fundamental questions about efficiency and stability

The results from theoretical running and walking models raises a fundamental theoretical question. Is it possible to have an asymptotically stable locomotion mechanism that is also perfectly efficient? The theory of Hamiltonian systems does not apply to walking machines because, by virtue of their intermittent contact, they are non-holonomic [49]. We know from our study of bicycle stability and the like that non-holonomic systems can have asymptotic stability even without dissipation. Can legged mechanisms also be made stable without dissipation? Although this is partially a question in pure mechanics it also pertains to humans. We are used to thinking of efficiency and stability as design trade-offs. But, as far as we know, this is *not* a fundamental restriction. Insight into these issues is relevant to understanding healthy humans and also to prosthetic corrections.

Goals, potential impact, and biomedical relevance

Use of passive or crudely controlled theoretical and physical models to gain understanding of locomotion may lead to other long term applications in rehabilition and orthotics.

Functional Neuromuscular Stimulation and Prosthetics

One of the more direct applications of our work could be in the area of Functional Neuromuscular Stimulation (FNS). FNS offers a way to restore some motion to paraplegic patients by applying external electrical stimulation to muscles which, because of injury or other reasons, have become paralyzed. In addition, FNS can improve limb range of motion, muscle strength, and bone mineralization [32].

Some drawbacks to FNS, from [58], include the following: **a)** the low strength of electrically-stimulated muscle, **b)** the difficulty in fine-tuning resulting muscle forces, **c)** a heavy reliance on orthotics for balance, and **d)**) a lack of knowledge regarding the mechanics of the muscles, joints, and body.

Our approach addresses these issues as follows:

a) Research in prosthetic design by [5, 16] has shown that the mass distribution of the prosthesis has an effect on the oxygen consumption of the user and on their gait. Operating near a passive gait cycle is energetically efficient as compared to other control strategies. If parameters in the legs are tuned properly as in [18] passive gait cycles can exist with arbitrarily low energy demand. These cycles might be used as a basis for FNS using smaller muscle forces and reducing muscle fatigue.

b) Passive gait cycles can be asymptotically stable; that is, small disturbances decay over time. We believe that simple powering schemes on level ground will produce stable gait as well. With this type of control, minor variations in muscle force or duration will not destabilize the gait. Thus both the size of muscle force needed for balancing purposes and the fineness with which it needs to be controlled might be reduced using passive-dynamic strategies.
c) Use of passive-dynamic stability could potentially reduce the need for awkward balancing paraphernalia. Some time in the future we invisage clinical use of dynamically stable, 3-D, anthropomorphic theoretical models, adjusted to the subject's parameters. Prosthetics and braces would then be designed using the theoretical model so as to gain efficient and stable motions as easily as possible. Similarly, prosthetic designs could be better tuned, by using the passive dynamic approach, to provide stability with a minimum of awkward hardware.
d) Stable passive-dynamic models remain stable when mechanical parameters are only changed slightly. Thus designs based on very stable limit cycles may make less demand on exact knowledge of system parameters.

Educational mission

The view of coordination as being neuro-muscular is deeply implanted in the consciousness of the medical community. We believe our work can have an impact on a variety of medical practicioners and researchers. Better appreciation of the *fact* that pure mechanics governs much of how humans move might have many useful subtle consequences. We believe our work, well communicated, will have a positive effect by contributing to a change in the way therapists, doctors, and medical researchers think about coordination and locomotion. Some of the methods we are using and developing are applicable to powered and controlled prosthetic designs as well.

The nonlinear-dynamics parametric-design approach provides a systematic way to tune control parameters. We believe that as reserchers come to understand the approach which we propose to use (but do not claim as original), it will largely replace real-time continuous feedback controls in both robotics and human gait synthesis. Many researchers doing numerical simulations of human motion, for example, are not yet aware of the utility of the non-linear dynamics tools to help them find stable motions (such as the interpretation of a limit cycle as a fixed point, and linearization of the fixed point as a measure stability) . Our work, properly communicated, should help spread the use of these tools.

I generally work with 1-5 undergraduate students together with the graduate students. Undergraduates typically receive class credit for their work. Accomplishments of the undergraduate in my lab include the construction of working passive walkers, writing walking simulations, developing methods to accurately measure walker parameters, and theoretical modeling of straight-legged and kneed walkers using Working Model simulation software. I believe I have had a big impact on the intellectual growth of some of these students.

Summary

I believe that deeper understanding of passive strategies will lead to an understanding of the need for, and efficient strategies for the use of, nerves and muscles in healthy humans. Similarly I think that learning the limits of passive solutions can guide the design of controllers and motors in orthotics and robotics.

These are the basic reasons that our studying theoretical and physical passive-dynamic models of human locomotion will be relevant and useful.

1. Passive-dynamic models can have three key features which healthy people have, and which are desirable for both prosthetics and robotics

   (a) existance of human-like gait,
   (b) efficient gait, and
   (c) stable gait.

2. Human coordination strategies may well be close to passive-dynamic strategies. Thus insight gained in this research increases understanding of humans.
3. Passive-dynamic models are simple enough so that deeper insights are possible.
4. Stable passive-strategies provide an approach to more robust synthesis of stable gait.
5. Better understanding of passive strategies will be useful in the design and fitting of both low-tech and high-tech prosthetic devices.

My background and abilities in mechanics and modeling are well suited to this research. My lab's work is now, as far as I know, the most advanced research on these topics in the world. We are in a good position to make significant further contributions with the help of NSF funding.

Results from previous funding

The undergraduate research group described above has been mostly funded recently through a three-year grant from the NSF Research Experience for Undergraduates (REU) program (award # 9300579, 8/15/93-7/31/96, total funding: $150,377). This program is designed to give research exposure to qualified undergraduates who are considering graduate school. There are about 10 students selected to work each summer among several research groups in the department, including my lab. Special consideration is given to applications from minority and/or underrepresented students in engineering.

A 1984-90 NSF PYI award allowed the hiring of post-doc Jim Papadopoulos and the start of my biomechanics education.

Human resources

Graduate students

At present, I am advising two graduate students. Mike Coleman who defended his dissertation in May 1997, and is completing final revisions [11]. Coleman's describes his study of McGeer's rimless wheel and preliminary investigations of the point-foot walker in 3D.

Mariano Garcia, my other graduate student and an NSF fellow, has been reproducing and extending McGeer's research in kneed walking, as described above. He will be working closely with me on the initial parts of this research. It is expected that Garcia will complete his PhD dissertation by the end of the first year of this grant and that a new student will join the research effort for the final two years.

Anindya Chatterjee finished his PhD at the end of 1996: It was entitled 'Algebraic Collision Laws For Rigid Bodies.' He also recently co-authored 3 papers on basic issues in rigid body collisions and is still working with us on walking issues.

Other less recent students have included Frank Horowitz and Jeff Nussbaum, who worked on friction in theoretical earthquake models, Suresh Goyal, who studied friction laws for robotics, and Scott Hand, whos master's thesis was about the "passive-dynamics" of a bicycle [24].

Undergraduate education

Outside of the laboratory I am heavily involved in undergraduate teaching, especially in dynamics. In the last 4 years I have co-authored a class-test draft of a dynamics textbook for Oxford University Press which is expected to be complete in August 1998.

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E. Biographical Sketch: Andy Ruina

Education

- 1981 PhD., Brown University, Division of Engineering *Friction and Earthquakes* Advisor: J. R. Rice
- June 1978 ScM., Brown University, Division of Engineering *Hydraulic Fracture* Advisor: J. R. Rice
- June 1976 ScB., Brown University, Division of Engineering Major: "Mechanical Systems"

Experience

- 8/97 - pres. Visitor, Automation and Technology Lab, Helsinki University of Technology, Helsinki, Finland.
- 11/87 - pres. Associate Professor, Theoretical and Applied Mechanics, Cornell Univ.
- 6/81 - 11/87 Assistant Professor, Theoretical and Applied Mechanics, Cornell Univ.
- 9/88 - 7/90 Visiting Professor/Sabbatic, Newman Laboratory of Biomechanics, Mechanical Engineering, MIT, Cambridge, MA
- 9/80 - 6/81 Visiting Asst. Prof., Theoretical and Applied Mechanics, Cornell Univ.
- 8/79 - 5/80, 7/79 - 8/79 Geophysicist, US Geological Survey, Menlo Park, CA

Biographical Sketch

My background is in continuum mechanics, dynamics, geophysics and friction. I was a primary developer of Dietrich-like state variable friction laws and was the first, with my student Horowitz, to show that a smooth theoretical model of the earth could generate rough earth-quake behaviour. I have been increasingly interested in macro-scale bio-mechanics. I spent two years at the Newman Laboratory of Biomechanics at MIT (where, among other things, I was the subject of an experiment to record bone kinematics during gait using protruding bone screws). I have advised graduate studies of friction mechanics, dynamic bicycle stability, human power production, rigid-body collisions, and passive dynamic walking.

I also oversee various undergraduate biomechanics projects in my lab, some for course credit and others as part of our NSF Research Experience for Undergraduates (REU) program. Undergraduate projects have included developing rowing simulations, improving oar designs, constructing an excercise machine to maximize human power output, adding constraints to bicycle pedals, studying the effects of mechanical impedance on pedalling efficiency, and building various passive dynamic walking devices.

Over the past four years, I have been writing an undergraduate dynamics text. During this time my research has been in passive-dynamic walking (as detailed in this proposal), the mechanics of rigid body collisions, and collaborations with C. Y. Hui on various topics in solid mechanics.

Awards

NSF Presidential Young Investigator Award, 1984.
Dean's Teaching Award: 1992, 1997

Some Relevant Papers:

1. Coleman, M., Chatterjee, A., and Ruina, A., *Motions of A Rimless Spoked Wheel:A Simple System With Collisions,* Dynamics and Stability of Systems, vol 12, no 3, pp139-160, 1997

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Some of the papers above are accessible over the World Wide Web, from http://tam.cornell.edu. A more complete resume with descriptions of these and other papers is available from http://tam.cornell.edu/faculty/ruina/ruina.html

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