2/87 Review of steering paper, solicited by DG Wilson for possible publication in Human Power

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DISCUSSION OF LE HÉNAFF'S PAPER

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Even a bicycle in a steady turn, the subject of Le Hénaff's interesting paper, is difficult to analyze. This is immediately revealed by a glance at the extremely complicated equations in ref. (2) below. Deep understanding can only come from the use of idealizations like those implied by Le Hénaff: assuming a concentrated mass at the CG point; and neglecting gyroscopic effects, deformation of the bicycle, and dissipative friction. Although Le Hénaff's assumptions cannot be used to predict all bicycle phenomena, they seem appropriate for estimating and understanding the potential energy or handlebar torque in steady turns.

However, such quantities calculated from steady equilibrium motions may have only *subtle* connections to what would be called "stability" — either by a bicycle rider or by a student of the bicycle as a *dynamical system* (where the change of lean and steer-angle with time are governed by differential equations).

Consider as a special case the dynamical stability of a bicycle with no rider. Call a bicycle "stable" if it recovers to steady straight-ahead motion after any small disturbance from that motion, and "unstable" otherwise. Suprisingly, for a range of speeds many bicycles <u>are</u> "stable" in this sense. Mathematical analyses and experiments both indicate, opposing some intuitions, that this "stability" is only possible for a bicycle if its Le Hènaff curves (modified as below) are "peaked" (concave down) in the center. For such a bicycle in a steady turn, the handlebars need to be held against further turning! (However, if there is a rider this torque is greatly altered when he/she bends sideways.)

The intuitive assumption that stability is synonymous with a potential energy minimum does not apply to a moving bicycle, which has finite kinetic energy, gyroscopic forces, and non-holonomic constraints (see refs (1),(3)). One must be wary of stability arguments that do not take into account the appropriate *dynamical* (time varying) equations of unsteady motion.

Incidentally, to calculate handlebar torque during steady turns, probably slightly different quantities should be plotted in Le Hénaff's figures 4 and 5. The vertical axis should plot (*the CG's distance from the line between the the front and rear wheel contact points)/(the cosine of the angle of lean of the plane containing the CG and the two contact points)*. The horizontal axis should plot the steer angle S. The handle-bar torque for a given steer angle S will be proportional to the slope of this curve at S, plus gyroscopic terms. (Such modified curves will be generally similar to those in figure 4. But in contrast to figure 4, the "peakedness" of the curves — the 2nd derivative evaluated at S=0 — should probably not depend on rider speed v.) Plotted in this fashion, for example, a bicycle with vertical head angle and non-zero trail would always make a "valley".

We appreciate Le Hénaff's work on bicycle equilibrium and stability, and hope he continues his investigations.

References

1) Gray, Andrew (1918) *A Treatise on Gyrostatics and Rotational Motion;* republished by Dover in 1959. (See pp.19-20 and 29-33.)

2) Man G.K. and Kane T.R. *Steady Turning of Two-Wheeled Vehicles;* in **Dynamics of Wheeled Recreational Vehicles**, SP-443, Proceedings of 1979 SAE Congress in Detroit, pp 55-75

3) Neimark J.I. and Fufaev N.A. (1967) *Dynamics of Nonholonomic Systems;* translated from Russian and published in 1972 by the American Mathematical Society, Providence R.I.. (See pp. 330-374.)

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