MAN-MACHINE DYNAMICS IN THE STABILIZATION
OF SINGLE-TRACK VEHICLES

by

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1. LITERATURE REVIEW AND THE SCOPE OF THE DISSERTATION

1.1 THE SINGLE-TRACK VEHICLE

The bicycle and motorcycle represent a unique form of road vehicle. Not only does the single-track vehicle depend upon forward motion for roll stability, but, depending upon vehicle design, the load carried, and the actual operating speed, stability often requires both forward motion and rider control. Also, because its roll angle is unconstrained and the ratio of rider mass to vehicle mass is high relative to other forms of ground transportation, the single-track vehicle is sensitive to motions of the rider with respect to his seat; thus, the rider can control the vehicle both by the handlebars and by his own body movements.

These and other dynamic peculiarities of the single-track vehicle have their advantages and disadvantages. On the negative side, the accident rate for motorcycles (which mingle more with traffic and operate at higher speeds than bicycles) appears to be greater than that for automobiles. Although the small size of the motorcycle makes it relatively invisible to the automobile driver and thus is probably the main reason for its greater accident rate, a poorly designed motorcycle can be dangerous by itself. For example, motorcycles and even bicycles have been observed to exhibit
dangerous oscillatory instabilities, the frequencies of which are sufficiently high that they cannot be controlled by the rider.

On the positive side, motorcycles and bicycles are enjoyed by many as a pleasant form of recreation and an inexpensive means of transportation. Their use in the United States has greatly increased in recent years, and they seem likely to remain important in this country as well as others in the future. A better understanding of single-track vehicle dynamics can lead to vehicles having more desirable handling properties, thus making them more pleasant to operate for a large number of riders.

In addition to possibly improving the safety and roadability of motorcycles and bicycles, an understanding of their dynamic behavior is an interesting academic study and provides information which may be of use in such fields as vehicle dynamics and manual control.

The research described herein is a mostly experimental expansion of previous theoretical research efforts. Thus, a discussion of the objectives and scope of the dissertation is best presented with respect to the existing literature, which is outlined in the next section.
Using physical quantities obtained from existing motorcycles, researchers found the equations of motion of the resulting equations, which modeled the motion of a vehicle. A one of the best known analyses was performed by Dr. Young, who developed a set of equations for the motion of a vehicle. The additional assumption of negligible yawing moments, in their textbook on dynamics, Yurashenko and other books, is that these make the point contact with the ground, in their textbook on dynamics, "The motion of a vehicle is normally studied for its stability. Work by Mann [3] was similar to Pearson's [2] in 1922, who included the geometric effect of the orientation of a point for motion of a point where derived other equations of motion for a point were developed stable without a further conclusion. In one narrow range of speed, 10-hp-12.5 mph, was the vehicle stable for the particular position indicated by Yurashenko, only speed. For the particular position obtained by Yurashenko, only equation 1) was found as early as 1899 (Yurashenko [1]), using point of view, as a man-machine system, from the theoretical point of view, has received considerable more attention than the vehicle, i.e., having a fixed rider whose hands are off the handlebars.

In the past, the uncontrolled single-track vehicle, the
However, the assumption that front and rear tires slip at the same
cyclic equations of motion were made by Struthn in 1964.

strung by an attempt to include the mechanics in motor-

All of the above analyses did not allow the tires to

analyzed by Dr. Ear.

Another with the Italian scooter, "Verdea" proved out by

damper added to the Italian scooter, "Verdea", for com-

strength and control [8] applied distributed's equations, with steering

approximations and the need for the tires to those of Dr. Ear.

formed a stability analysis of a single-track vehicle used

with the aid of a detailed computer, Collins [7] per-

assistance from the rider.

which the motorcycle is inherently stable, requirements on

acceleration.邓好ing found an intermediate speed range in

speeds, an instable slightly exceeds the form of an increase

slowly, but requirements and corrections from the drivers, at low

slightly unstable, the angle of lean increase very

speed range, according to Dr. Ear's results, the roll motion

for motorcycles than for bicycles. At the upper end of the

dynamic behavior, although the speeds are considerably higher.

ranges, each of which was characterized by a particular

Dr. Ear, like Dr. Mittring, the theoretical found several several speed

had previously performed [6]. It is interesting to note that

and compared his theoretical results with experimental he
equations of motion are based on assumptions like these made

[14] represent the most complete published to date. Sharpe's

It is believed that the theoretical analyses by Sharp

control in terms of steering torque and body lean.

freedom nonminimum-phase mode and has dealt with rider

have been released. This work involves a six-degree-of-

study is of a contrasted nature and few publications [13]

Cornell Aeronautical Laboratory, Inc. Unfortu nately, the

been performed at Cal Sp an Corporation (Corne 

A large amount of theoretical and experimental work on

Kondo [11, 12, 13]

set of linear equations with these states were prepared by

some experimental work on steady turning. A more General

and included some additional resistance effects. Pu also performed

equations were investigated in steering and the slip angles

more equations for the steady turning situation, which

[10] Pu

other analyses included the mechanisms in the manner

possible, since different vehicles were involved.

tative comparison between theory and experiment was not

road tests in addition to this stability analysis, quantit-

introduced unknown parameters, while sharp performed some

Evaluating accurately adequate results, to questionantable and

are proportional to the steering angle, while possibly
Although presented was an interesting discussion of the process of testing and developing models of slip angle and handbrake forces, also introduced some experimental work, which explored the mechanisms and the effect of the rider on the discussion. He also introduced some experimental work, which explored the mechanisms and the effect of the rider on the discussion. He also introduced some experimental work, which explored the mechanisms and the effect of the rider on the discussion. He also introduced some experimental work, which explored the mechanisms and the effect of the rider on the discussion. He also introduced some experimental work, which explored the mechanisms and the effect of the rider on the discussion. He also introduced some experimental work, which explored the mechanisms and the effect of the rider on the discussion.
Single-track vehicles have also been performed (Sarkar, [2]),

although detailed analyses of the steering geometry or the angle, deflection, and forces have been conducted on the step response, where the step force has been the step

unfortunately, during contact proportional to the steering angle,

and front wheel side force due to the step angle was detected

the interaction between front and rear frames was neglected

the ads of a simple one degree of freedom analysis, to which

taken about the steering axis of both the front and rear wheels of the steering geometrist [23] discussed the causes and cures of front wheel oscillation. Donning into the above categories have also been published, discussing results of two other types of studies which do not fit.

or stability

tries in general, in an attempt to identify basic mechanisms

and made some calculations with respect to steering geometries

of experiments with bicycles having altered steering geometries.

approach was taken by Jones [22], who carried out a series of tests such as steady turning and skidding. Another “basic physics”

of the behavior of the machine under various conditions,

present an interesting. Thoroughly subjective, description

situation of a vehicle with a vertical steering head, the did

metastable work, unfortunately restricted to the unstable

material [20]. Although Power [21] performed some math-

extensively were written about the same time (1979). By
of the steering system (unstable about six-nute axes) on the steering assembly.

Second, the mode studied by Bhurte [23] of the "weave" mode, an oscillator mode of lower frequency than the entire vehicle, and the "capsize" mode, a non-oscillation mode.

The weave mode, an oscillator mode of lower frequency than the entire vehicle, were also studied. For the complete model, three modes

case). The effects of changing motoroyde's parameter data

together being a degree of freedom (the so-called "fixed control"

and complete model with steering angle no

but with "instantaneous response of the side force; model with

version of not analyzed). Complete model; complete model; complete model;

located using data from an existing motoroyde; for our

assembly. The equations are linearized, and modal roots are

translational, rotational, and steering of the front fork

considered to possess four degrees of freedom: lateral

In Shap's analysis, the uncontrolled motoroyde is

as the most difficult to date.

From a review of the literature, the work of Shap emerges

accurately, especially for quantitative results are desired.

only be achieved through a mathematical model of reasonable

standing of the dynamics of the single-track vehicle can-

because of the complexity of the system, a complete under-

cussions and "basic physics" arguments, it is believed that.

Although much can be learned from qualitative di-

8
Figure 1.1 Root loci for BSA motorcycle (roots taken from Reference [14]).
Figure 1.2: Capsize mode roots for RISA motor vessel

[Graph showing capsizing characteristics with axes labeled: Forward Speed, m/s and Capsize Mode Roots, sec⁻¹]
This dissertation, illustrations of Equation (1.1) are discussed in Chapter 3.

The effective force "reaction tension" is the reaction and lateral stiffness, a the slip angle, f the time, and a is the lateral force, C the forward speed, C = \frac{n}{t}, where P is the lateral force, n = forward speed, C. (1.1)

\[ \frac{\partial \phi}{\partial t} + \frac{n}{\frac{\partial P}{\partial \phi}} = \frac{\partial \phi}{\partial \phi} + \frac{n}{\frac{\partial P}{\partial \phi}} \]

Represented this Fig. 1 a very simple manner, i.e.,

Impotant in the case of the single-truck vehicle, sharp
is often omitted from analyses of autopoddy dynamics, it is
while the Fig. 1 in the development of the lateral force

Damping of the wheel mode, particullarity at low speeds.
account for the steady degree of freedom increased the
development of the lateral force from the equations that
and with speeds (see Figure 1.1). Collision of the Fig. 1 in the

Entirely and drastically changed the wheel mode
of collision [7] and during [5], emphasize the wheel mode
straightening of motion, resulting in a vehicle model like those
with which the possibility of wheel or weave unstable at very
for higher speeds, the capsize mode becomes unstable
about 20-30 mph) there is a range of complete stability
unstable weave mode oscillation, while for medium speeds
motorcycle studied by sharp, low speeds are characterized by
A closed-loop control system is given by

\[ T(s) = \frac{K_p}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

where \( T(s) \) is the transfer function of the system, \( K_p \) is the proportional gain, \( \zeta \) is the damping ratio, and \( \omega_n \) is the natural frequency of the system.

A typical closed-loop control system is represented by a block diagram, where the input \( u(t) \) is fed through a transfer function \( G(s) \) to the plant \( P(s) \), and the output \( y(t) \) is fed back through a transfer function \( H(s) \) to the plant. The overall transfer function of the system is given by

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} \]

The dynamics of the system can be represented by a transfer function, and a random noise source is added to the system. In the study of human systems, this approach is common and is used in the study of man-machine systems.

Recently, some interest has been shown in studying the human-vehicle interaction. In particular, there has been a lot of interest in examining man-vehicle telematics, which has been developed in connection with the control of vehicles and single-track vehicle systems. Such studies tend to be difficult to maneuver, as the vehicle is not easily testable. However, several researchers have made designs recommendations for improving safety, for example, with the idea of the rider. For example, while several researchers know what dynamic properties are desirable from the point of view of the uncontrolled situation, it is at least as important to know how these changes in the design of a single-track vehicle affect the dynamics in

While it is important to know how physical changes in
Figure 1.3 Basic compensatory manual control system.

Symbols:
- \( r \): command signal
- \( e \): error perceived by operator
- \( n \): operator's "remnant"
- \( c \): operator's output
- \( y_p, y_c \): system output
- \( m \): linear transfer functions

Human Operator

Controlled Element
form of that described in Figure 1,3, are first investigated.
study of a number of possible closed-loop systems, having the
Werner [16] with the aid of the crossover model. In this
system presents an available way that performed recently
For the most complete analysis of the man-motion
control studies, E. E. Morner and Werner [29],
over model [28] and has proved useful in theoretical
shown as the cross-
for a near to the "crossover frequency", at which
\[ \frac{\omega_f}{\omega Ho} = (\omega f)^{\nu} \alpha (\omega f)^{\nu} \]
\[ e^{\frac{-1}{\omega Ho}} \]
control experiments can be the by a simple model, viz.
that a substantial amount of data collected in manual
experiments. These studies have shown
Many manual control studies have been concerned with

because real operators require a finite time to react to a
\[ (\omega f)^{\nu} \]
1, always appears in the operator transfer function, etc. Note that the time delay,
variables, ambient temperature, etc. for the operating temperature, level of
"environmental factors, such as motivation, level of
\( K e^{\frac{-1}{\omega Ho}} \)
where the constants \( K, I, T, L \), and \( N \), are functions of
\[ \frac{I + a_N I}{(I + a_L L)} = (\omega f)^{\nu} \]
\[ e^{\frac{-1}{\omega Ho}} \]
did not have forward motion, and the stimulator dynamics were
the task of stimulating a real airplane because the stimulator
control task that may have been considerably different from a
function were obtained while the operator was performing
transfer functions experimentally, these transfer
while this work represents a pioneering effort in obtaining
be performed by Van Huren, et al. [30], 31], 32, 33], 34]
Manuel control studies using a hypothesized stimulator have
for their findings.

motorcycle equations of motion, which served as the basis
and sharp are dependent upon the level of validity of their
presented. It should be noted that the conclusions of the
conclusions relating to vehicle design and operation were
and the design parameters of the motorcycle. A number of
were also investigated the effects of changing forward speed
sentation of the man–cycle systems described in Figure 1, 4,
systems were tested and used to create the overall repeat
The loop characteristics judged by were to be „good” control
published by Sharp.
dynamics were determined by the equations and parameter data
chosen to be a gain plus time delay, whereas the vehicle
for each possible system, the rider transfer function was
the outputs are steering torque and upper body lean angle.
roll angle, yaw angle or lateral position, and the possible
the inputs to the rider are considered to be, for example,
Figure 7.4 Man-Motorcyle System Proposed by Wert

Symbols: $x, y, z$: Lateral position
$\phi, \theta$: Rider body lean angle
$\psi$: Heading angle
$\gamma$: Rider-applied steering torque
$\phi$: Roll angle

Linear transfer functions: $\phi, \theta, x$, $y, z$
based upon the differential equations of Whipple, equations that were not subjected to experimental verification. Furthermore, Van Lunteren, et al., assumed the rider's control outputs to be body lean and steering angle, rather than body lean and steering torque. For motorcycles, steering-torque control is more likely than position control, due to the greatly increased instability of the capsize mode when the steering degree of freedom is omitted [14, 16]. It is likely that torque control is also superior to position control in the case of the bicycle, although this superiority is not firmly established.

It should be pointed out that Van Lunteren's major interest was the manner in which the performance of the human operator would be influenced by various factors such as the use of drugs, etc. He was not concerned with the dynamics of the bicycle. The bicycle stabilization task was chosen because the system is familiar to most people, thus ensuring that the test subject could be representative of a large population rather than a select skilled group, and because the frequently unstable bicycle forces the subject to pay close attention to his control task.

Another bicycle simulator has recently been developed in Japan, as described by Hattori, et al. [35]. The simulator is not as realistic as the machine of Van Lunteren, since the test subject applies nearly vertical forces to
the handlebars, no body movement is measured, and the experiments performed involve only a transient response to an initial imposed lean, rather than a continuous control task.

At the Massachusetts Institute of Technology, a vehicle was designed and built to test the feasibility of using an automatic device to control the rolling of a narrow track vehicle [36]. However, this work is not closely related to the stabilization of the single-track vehicle, since it involved a three-wheeled converted motorcycle in which rolling was induced by mechanically tilting the rear wheels and vehicle frame.

1.3 OBJECTIVES AND EXTENT OF THE DISSERTATION

This dissertation is intended to improve the understanding of the dynamics of the single-track vehicle by expanding upon previous research. In particular, the major objective has been to provide experimental information to relate to the theoretical work of Sharp and Weir. Chapters 2 and 3 deal with the uncontrolled motorcycle, that is, a vehicle in which the rider remains rigid with his hands off the handlebars. Chapters 4-6 discuss the man and motorcycle as elements of a closed-loop control system.

Chapter 2 presents a theoretical study of the effects of adding the self-aligning torques and overturning moments created by tire sideslip and inclination to Sharp's analysis
and also examines the influence of the lateral force and aligning torque deriving from the instantaneous curvature of the path of the tire contact patch. By means of an analog computer simulation, a comparison is made between viscous and coulomb friction steering dampers. Parametric data for these analytic and simulation studies were obtained from measurements made on an actual motorcycle with the rider seated on the vehicle.

Chapter 3 describes road experiments in which the measured motorcycle was subjected to two disturbances. For each experiment, the transient response of the motorcycle was recorded and compared to the response predicted by the applicable equations of motion.

After the degree to which the equations of motion were valid was ascertained, the man-motorcycle closed-loop system was studied. The particular rider control task researched was the stabilization of the vehicle roll angle by means of rider-applied steering torque. The objective was one of identifying the linear transfer function that correlates the rider's steering torque output with the vehicle roll angle, using data obtained in constant speed road tests, which were approximately one minute in duration.

In Chapter 4, the man-machine system is analyzed to determine what constraints must be placed upon the rider's transfer function, in the interest of good system characteristics, especially stability.
The roll stabilization experiments themselves are discussed in Chapter 5. In this experimental study, most of the excitation of the system was found to be the rider's remnant. Hence, although cross-spectral methods could be used to identify the transfer function that represents the motorcycle or "controlled element", it was necessary to use the method of Wingrove and Edwards [37, 38, 39, 40] to identify the rider's transfer function.

Chapter 6 presents the results of the roll stabilization experiments. Conclusions reached on the basis of the described research are summarized in Chapter 7.
2. THEORETICAL STUDIES OF THE UNCONTROLLED MOTORCYCLE

2.1 EQUATIONS OF MOTION

During the early stages of the research described in this dissertation, equations of motion were derived that are nearly identical to those recently published by R. S. Sharp [14]. Sharp's analysis of the dynamics of the linearized single-track vehicle, which analysis has thus been substantiated by two independent efforts, the present work and that of Weir [16], serves as the theoretical basis of this dissertation. The reader is directed to Reference [14] for a derivation of Sharp's equations. Appendix A presents the equations after a change in notation and coordinate systems has been made, and after some additions to the theory (mostly tire mechanics considerations omitted by Sharp) have been included.

It should be pointed out that the following assumptions were made in the process of deriving the equations that are presented in Appendix A:

1. Vehicle and rider are comprised of five rigid bodies linked together: engine (with transverse axis), rear wheel, rear frame and rider, front frame, and front wheel.

2. The roll angle, $\phi$, and front-frame steer angle, $\delta$, are sufficiently small that the sines of these angles may be approximated by the angles
in radians with the cosines of these angles set equal to unity.

3. Small disturbance motions prevail, permitting the equations to be linearized (except for coulomb friction in the steering head) by neglecting products of the variables \( v \) (lateral velocity), \( \phi, \psi \) (heading angle), and \( \delta \), and their derivatives.

4. For \( \delta = 0 \), both rider and vehicle are symmetrical with respect to the XZ plane as defined by the axis system placed in the vehicle (Appendix A).

5. Forward velocity, \( u \), is approximately constant, and the rates of spin of the wheels and engine are proportional to forward velocity.

6. The road is flat and level. Rotational motions about the Y axis (pitching) are negligible.

7. The wheels are rigid and make point contact with the road. Tire forces and moments may be approximated as linear functions of slip and inclination angles and instantaneous path curvature. Rolling resistance moments and tire tractive force are considered negligible.

8. Aerodynamic forces and moments are neglected, including their influence on the distribution of the loading on the front and rear tires, which vertical loading influences the cornering and inclination stiffnesses of the two tires.
shown, however, that the lag in side-force buildup on the
tire sides of a, and c, be equal. Analytic computer studies have
written, requires that the front and rear tire relaxation
The detailed program was developed first and, as

no interactions are required.
valuable tool in that it was very inexpensive to run since
vehicle, consequently, the detailed program proved to be a
mechanism has a negligible effect on the motion of the
head of the test vehicle (which has no damper in the steering
motion that the level of component interaction present in the steering
of inputs and system moduli. Analytic studies showed
of component interaction and to investigate a much broader range
Another computation was employed to analyze the effects

In the steering head could not be accommodated (θ = 0)
of the inner single-track vehicle, that is, component interaction
problem was limited to analyzing and predicting the behavior
functions, such as step and impulse functions. Thus the
time response of motion variables to a limited set of forcing
determine (with the aid of the Laplace transformation) the
equation yielded by this dynamic system, as well as to
lation was employed to find the roots of the characteristic
equations presented in Appendix A were solved both

SOLUTION METHODS
rear tire has a negligible effect on the response of the system. Hence, in digital computations, the relaxation length of the rear tire was assumed to be equal to that of the front tire. These parameters and all remaining coefficients in the equations of motion were determined from measurements made on a 1971 Honda CL 175, which served as the test vehicle in this study. Appendices C and D present a discussion of the measurement techniques that were employed and a listing of the resulting parameter data, respectively.

In order to identify the various natural modes of motion, the terminology introduced by Sharp [14] will be retained. The "wobble" mode refers to the natural mode that is predominantly characterized by an oscillatory motion of the front frame assembly. The "weave" and "capsize" modes refer, respectively, to an oscillatory and non-oscillatory motion of the entire vehicle. It should be noted that Sharp's calculations were restricted to a determination of the roots of the characteristic equation. Substitution of his motorcycle parameter data into the calculation procedure developed in this study produced roots that were identical to those reported in Reference [14].
contact patch. (See Appendix A.)

contact patch due to instantaneous curvature of path of the tire and instantaneous forces as well as lateral force and attitude torques and overturning moments resulting from slip.

the equations of motion were modified to include the tire completely. than had been done by Sharp [14]. In particular, for representation the mechanisms of the pneumatic tire more

this reason, an investigation was made to determine the need for a strong bearing on the accuracy of that analysis. For

moments are realistically described in a dynamic analysis of stability. Thus, the degree to which these forces and

interference for not only directional control, but also roll

it should be noted that single-track vehicles depend

2.3 Tire Model Studies
pushing the tire away from the instantaneous center of the path and (2) an aligning torque opposing the direction of the turn. Lateral force and aligning torques arising from path curvature are referred to in the remainder of this dissertation as "path curvature effects."

Whereas the overturning moment, namely, the moment caused by the lateral shift in the effective centroid of the vertical pressure distribution, is a negligible quantity on a two-track vehicle, it is readily seen that this lateral shift is significant on a single-track vehicle. The importance of this lateral shift in the vertical load, together with the other effects noted above, was produced with six different representations of the pneumatic tire. These six tire models are identified below:

1. Side forces caused by slip and inclination angle; no moments other than aligning torque as a function of slip angle; no path curvature effects.

2. Same as "1", with path curvature effects added.

3. Same as "1", with aligning and overturning moment dependence on inclination angle added.

4. Same as "3", with path curvature effects added (thus constituting the most complete tire model).
5. Same as "1", with aligning torque dependence on inclination angle added.

6. Same as "1", with overturning moment dependence on inclination angle added.

The modal roots, yielded by each of these six tire models, have been plotted in Figures 2.1, 2.2, and 2.3 as a function of forward speed. Only the wobble, weave, and capsize mode roots are plotted since the remaining modes of motion are judged to be physically insignificant, either because they are heavily damped or they cannot be excited to amplitudes comparable to the amplitudes achieved in the wobble, weave, and capsize modes. In making these calculations, it was assumed that the transmission gear would be selected in accordance with the following schedule:

- First gear, \(0 < u < 10\) mph
- Second gear, \(10 \leq u < 15\) mph
- Third gear, \(15 \leq u < 20\) mph
- Fourth gear, \(20 \leq u < 25\) mph
- Fifth gear, \(u \geq 25\) mph

Although the top speed of the test vehicle is approximately 80 mph, modal roots have been calculated for speeds up to 100 mph in order to locate the speed at which the wobble mode becomes unstable.
Figure 2.1  Root loci.
Figure 2.2: Weave mode roots, low forward speed.
Figure 2.3. Capsize mode roots.
An examination of the root loci plotted in Figures 2.1, 2.2, and 2.3 shows that (1) path-curvature effects primarily influence the wobble mode, (2) tire moments have an important influence on the capsize mode, and (3) the weave mode is influenced by the choice of a tire model only when speeds are below 20 mph. In particular, it is noted that the overturning moment due to inclination of the tire tends to destabilize the capsize mode (see Fig. 2.3) whereas the aligning torque caused by inclination angle has the opposite effect. Examination of the equations of motion shows that the overturning moment and aligning torque produced by inclination angle tend to counterbalance each other since their major influence on the vehicle system derives from the moments that are created about the steering axis. When these two tire moments are transformed into moments about the steering axis, they are seen to be of approximately equal magnitude but opposite sign. Without the existence of tire data and the equations of motion that have been derived for a single-track vehicle, such as a motorcycle, it is not obvious how these two properties of the tire combine to produce the effect noted.

Although not demonstrated in these root loci plots, calculations have shown that reduced inclination stiffness has approximately the same effect on the capsize mode as an increased overturning moment, namely, to cause the capsize
mode to become unstable at lower values of forward speed. It is clear that a tire of different construction than was used in this study, e.g., a radial ply tire with a higher cornering stiffness and a lower inclination stiffness, would have a marked influence on the modal roots of the motorcycle under consideration.

As noted earlier, the choice of a tire model influences the roots calculated for the weave mode only at low speeds (see Fig. 2.2). Aligning moments arising from inclination angles tend to increase the weave mode frequency in the 12-17 mph speed range, while overturning moments due to inclination angle and path curvature effects move the locus in the opposite direction, i.e., toward the real axis.

Two extreme examples of this frequency reduction result from the use of models "4" and "6", which cause the complex roots to break into two real roots for narrow ranges of speed, about 14.9 to 15.2 mph for model "4" and 12.4 to 13.2 mph for model "6". For these speed ranges, the weave mode, defined to be oscillatory, is replaced by three capsize modes. Thus, if the capsize mode is expressed as a function of speed, this function is continuous but multiple-valued for speeds at which the weave mode does not exist (see Fig. 2.3).
eventually the vehicle falls more quickly with a constant drag force relative to the friction force, but it is seen that the disturbance to the friction force is a function of the ratio of magnitude of the increased roll instability. The exact response in the contoured vehicle of parameter introduces the viscous damper hardness influences the rolling motion at a roll (at 42.5 mph), the dry friction damper introduces the viscous damper in two ways. First, it is seen that while steering need, these responses show the superposition of the levels of contoured and viscous friction present in the handballs about the steering axis. With a few different subject to a mathematical impulse of torque applied to the test motorgard, with a forward speed of 42.5 mph, Figures 2, 4, and 2.5 show simulated transient responses.

These dampers...

It was assumed that the test machine could be fitted with a viscous versus dry friction steering damper. In this study, accordingly the angular computer was utilized to compare friction is a nonlinear effect deemed worthy of study. The calculated motion of the cycle. Nevertheless, contoured steering need of the test vehicle had negligible effect on the small amount of contoured friction present in the system of angular computer simulation. It was found...

2.4 STEERING DAMPING STUDIES
Figure 2.4

Effects of viscous steering damper on the transient response of the uncontrolled motorcycle; u=42.5 mph.
Effects of coulomb friction steering damper on the transient response of the uncontrolled motorcycle; $u=42.5$ mph.
A second, perhaps more important, consideration is that coulomb friction dampers, being nonlinear, have different effects for different levels of disturbance. This fact indicates that a rider, encountering small disturbances, may set his damper to a level which could not handle large disturbances, should they occur. Also, if the damper's maximum capability is reduced by, say, oil or water contamination, it may not be possible to adjust the damper tightly enough.
3. EXPERIMENTAL STUDIES OF THE TRANSIENT RESPONSE OF THE UNCONTROLLED MOTORCYCLE

3.1 DESCRIPTION OF EXPERIMENTS

A major objective of this study was to compare the motion predicted by solutions of the motorcycle equations of motion with experimental data. To make this comparison, the test vehicle was instrumented and road tests were performed. During these road tests, the motorcycle was ridden at a constant forward speed, with the rider's hands off the handlebars, and his upper body prevented from leaning relative to the motorcycle by a rigid brace. The motorcycle was disturbed in such a manner that the disturbance could be simulated by the analog computer, thus allowing a comparison between the simulation and experimental data.

The test vehicle was instrumented to measure steering angle (δ), roll angle (ϕ), and yaw rate (r). A third wheel located at the end of a 27 1/2-inch pivoted bar was used to sense roll angle. (A soft model-aircraft tire was mounted on this wheel and proved to be very satisfactory for absorbing road roughness.) The angular displacement of the steering head and of the roll-sensing bar were both measured with rotary potentiometers that were geared up to increase their sensitivity, the effects of backlash proving to be negligible. Yaw velocity was measured with a rate gyro.
The outputs from these transducers were recorded on a multi-channel strip chart recorder. Both the recorder and the power supply for the potentiometers and the gyro motor were carried in an automobile that operated alongside the motorcycle during a test. Figure 3.1 shows the physical configuration employed in conducting the road tests—the beam supporting the wires between the vehicles, the rider restraining brace, and the third wheel arrangement.

Although straightforward in theory, in practice it proved to be difficult to directly compare the transient response of the motorcycle given various forcing functions, to the response predicted by the mathematical analysis. Forcing functions or disturbances were selected by considering both the information desired and practical limitations. An ideal set of forcing functions would excite each mode of motion independently while being workable from a physical standpoint.

In practice, it was possible to excite the various modes independently of each other to a very limited degree. The wobble mode, being nearly uncoupled from the other modes, especially at higher speeds, and being stable, was the easiest to excite and detect, primarily because of its oscillatory nature. The weave mode and the capsize mode were not so easily observed. The difficulty in identifying and observing these latter two modes derived from a number of factors.
Figure 3.1 Test motorcycle and instrumentation car.
Theory it is necessary to compare the theoretical predictions
obtained and observed independently. Hence, to test the
natural modes of motion cannot be

It is clear that the dynamics of the single-track vehicle

heading direction was the same as the accompanying car.

disturbance was such that after the disturbance, the new
cable, unless the motorcycle heading direction before the
run length was also jolted by the length of the instrumentation
instrumentation of the existence of instability or instable
form and jolt the duration of each run to a few seconds.
These instabilities made the vehicle tests difficult to per-
irreversible for the case for the weave mode, the capsize mode

the linear range of motion.

second peak, if it existed at all, was very large, far outside

irreversibility, the first peak was very small, while the
random disturbances produced by wind forces and road
which necessitated the use of data that for any disturbance,
the negative damping was so large that for any disturbance,

general, at low speeds, a weave mode oscillation existed, but
was approximated by 0.7, estimating a stable oscillation. In

to high speeds, the damping ratio of the weave mode

Thus, for moderate

Thus, for moderate

First, testing was jolted to a top speed of about

40 mph, due to safety considerations.
of interest. Front wheel load, etc. to a slight extent.
also changed the parameters of the vehicle (e.g., moments
of roll) torque when the weight was dropped, the fixed weight
acted side of the vehicle. Note that in addition to producing
positive torque (corresponding to dropping a weight from the
positive torque and 2216 lb-in (twelve-pound weights) a positive
weight by dropping the weights were 1080 lb-in (six-pound
release. (See Figs. 3.1. The magnitudes of the torques
means of a pull wire, a metal plate, and the instant of
weight was bolted in place, the other could be dropped
distance of 18 inches from the vehicle center plane. One
a weight was attached to each side of the motorcyclic as a
and control modes. To produce this input in the road tests,
the second forcing function consisted of a step function
not measured experimentally.
Although of unknown magnitude, since steering torques were
approximated mathematically as an impulse of steering torque,
until it was necessary to take control. This input can be
palm of the hand and allowing the system to more unrestricted
one side of the hand, the other to the steering wheel. This
consisted of a pulse of torque about the steering axis, This
road tests. The first, aimed at exciting the 
Two types of forcing functions were employed in the
These changes were estimated by treating the added weight as a point mass. On calculating the new parameters, it was found that changes in tire properties resulting from modified vertical loads on the front and rear wheels were extremely small and could be neglected.

It should be emphasized that the mathematical functions used to represent the physical inputs are approximations. It is not physically possible to apply a pure impulse of steering torque (infinite torque for infinitesimal time). Also, the step input of roll torque does not actually remain constant on the road, but changes as the vehicle rolls, due to a shortened moment arm. The amount of this variation, which depends on the position of the fixed weight relative to the vehicle XYZ axes as well as the roll angle, was found to be less than 3 1/2% for a roll angle of 10°, an angle larger than the roll angles encountered during the road tests. Thus, assuming a constant torque input appears to be valid for the small disturbance motion of interest here.

In practice, it was found that longer runs and consequently more information could be obtained by combining the steer displacement and roll moment inputs. Specifically, the rider would bring the vehicle as nearly as possible to "zero initial conditions" (i.e., v, r, φ, δ, and their time derivatives are equal to zero) and drop one weight. After a time interval, a, he would apply a steering pulse in the direction tending to correct the fall induced by dropping the weight.
Mobile oscillation was predicted using a time model with an increased time damping. As a result, very little low speed and the addition or path curvature effects were not noticeable. The model, and at low speeds the mobile mode as predicted by the model and at high speeds the effect was small, response curves, with the addition of path curvature effects were not noticeable in the example, the change in damping of the mobile mode brought about by path curvature influence the simulated responses. For curvature effects (models 2, 3, 6, and 7, Section 2.3) and torques due to the addition of the path curvature, and that the addition of the overturning moments and addition of steerer-torque impulse with the aid of the digital computer.

The vehicle response to the step roll moment and the model appears adequate, since it was found (Pb) studying the model number 11, as derived in Section 2.3, the use of equations of motion used in the simulation corresponded to the torque applied to the front frame assembly about the roll axis, applied at time t = 0, lowested by an impulse of the response of the motorcycle to a step moment about the roll axis. To compare the theoretical results with experimental data,
without path curvature effects. Further, it was found that a tire model including overturning moments due to inclination angles influenced the theoretical vehicle transient response in a manner opposite that of a model including aligning torques due to inclination angle. Taken together, the effects of these two moments approximately cancelled, as was the case with their influence on the weave and capsize mode roots. Simulation exercises that included coulomb friction in the steering head (as estimated to exist on the test vehicle) showed that these frictional effects were negligible. Accordingly, coulomb friction was omitted in the simulations conducted to correspond with test conditions.

Figures 3.2-3.5 show the transient response of the motorcycle as obtained from experimental data and as simulated, for forward speeds of 10.5, 20.0, 28.2, and 42.5 mph. The high frequency wobble oscillation can be seen in all of the measured time histories, except roll angle ($\phi$). The weave mode, with its heavy damping, is not visible in either the experimental or simulated results. The presence of the capsize mode is best observed in the data obtained at the two highest speeds, where it is seen that all of the dependent variables, most notably roll angle, are slowly diverging in the absence of any steering control.
the uncontrolled motorcraft; $v=10.5$ mph, second gear.

Comparison of experimental and theoretical responses of

Figure 3.2

Pulse

(right and left of steering)

Analog computer

Magnitude of Step

Roll Moment = 108 lb-in

Road Test

6.7°

25°/sec

1 sec

2.8°/sec
Comparison of experimental and theoretical responses of the uncontrolled motorcycle: u = 20.0 mph, second gear.

Figure 3.3

Analog Computer

Roll Moment = 108 lb-in
Magnitude of Step
Road Test:

5.7° 25°/sec 8.4° 1.0 sec 2.8 sec
Figure 3.4

Comparison of experimental and theoretical responses of the undercontrolled motorcycle: \( v = 28.2 \) mph, third gear.

\[ \text{Roll moment} = 108 \text{ lb-in.} \]

Magnitude of step:

Road test:

\[ \text{Angle \( \phi \)} \]

1 sec

2.84°

25°/sec

6.7°

14.3 in/sec

\[ \text{Angle \( \theta \)} \]
The uncontrolled motorcycle, \( u = 42.5 \) mph, fifth gear.

Figure 3.5

Analog Computer

Haul Moment = 216 lb-ft
Magnitude of Step
Road Test:

\[ \frac{1}{4} \text{ in/sec} \]

\[ \frac{6.7}{\text{sec}} \]

\[ \frac{250}{\text{sec}} \]

\[ \frac{3.84}{\text{sec}} \]
Many tests were made, and the variability between tests was small. The experimental results shown in Figures 3.2-3.5 are representative of the body of data that were obtained.

Figures 3.2 through 3.5 show that the high frequency wobble mode is accurately predicted by theory in the neighborhood of 40 mph, with the simulation becoming a poorer prediction of wobble behavior as forward speed decreases. Specifically, the oscillation of the actual steering assembly has less damping than that predicted by theory. In fact, the wobble mode was observed experimentally to have nearly constant damping and frequency throughout the speed range, even at zero speed.

The simulation of the vehicle response to a step moment about the roll axis also becomes less realistic as speed decreases. For speeds less than about 15 mph, the experimental data clearly indicate that the real vehicle rolls more quickly in response to a step roll torque than does the simulated vehicle, while for higher speeds, the agreement between theory and experiment is very good with respect to the roll response (as opposed to high frequency wobble response) produced by a pulse of steering torque.
In addition to conducting tests in which quantitative data were obtained, a number of qualitative experiments were performed, during which the motorcycle was ridden "hands off", with no disturbance applied and the rider's body restrained by the brace. During these runs, several observations about the dynamics of the vehicle were made. Specifically, the following points were noted in these qualitative experiments.

1. The test vehicle seemed to be unstable at all speeds, thus failing to confirm the range of complete stability (about 13-17 mph) predicted by the theory, using tire model "1". Other tire models, except model "6", also predicted a stable speed range.

2. No low frequency weave oscillation could be excited on the road, in spite of the undamped oscillations at low speed predicted by the theory, using every tire model except possibly "6". For the motorcycle to sustain such an oscillation, it was necessary for the front system to "automatically" steer into a fall (induced by a non-zero roll angle), thus providing a lateral force at the tires tending to right the vehicle. During the test runs, it was observed that the "automatic" steering did exist, but that it was not fast enough to provide a sufficient correction. Hence, instead of oscillating, the vehicle roll angle exhibited an exponential divergence in one direction.
The remainder of this chapter is devoted to discussing the possible reasons for the differences that were observed between the theoretical and experimental data. These discussions are given below, beginning with hypotheses that were rejected, and ending with the most probable explanations for the noted discrepancies.

The theoretical models are more realistic than model "tr" (tabular), since they include the experimental data. There is no evidence that any of the theoretical models that was much too unstable, as compared with model "tr" (the simulation is not shown here), indicated that a carribre mode that was much too unstable, as compared with model "tr" (the simulation is not shown here), indicated that the theoretical response of the motorcyle, namely, however, the simulated response of the motorcycle, namely, from stirp and interaction momenta, is the least realistic, while model "5", theoretical forces and attenuated momenta, is the most realistic. The theoretical forces and attenuated momenta from interaction momenta, and overestimated the theoretical model "5", theoretical forces from stirp and interaction momenta. The absence of any undesired negative oscillation would indicate that perhaps the speed range for complete stability is.

Ref. 2.3, carribre mode roots. (See as would be expected from the theoretical results. (See, unstable carribre root was decreased with increasing speed, as speed increased, apparentiy, the magnitude of the instability of the vehicle became less severe.)
the best experimental correlation was obtained. Since the

experiments, the lever of applied torque was adjusted until
disturbance, the level of applied torque was adjusted until

It should be noted that in stimulating the steering

of excitation, regardless it be too heavily damped to permit overshoot motion, its amplitude because the theoretical model was seen to be mode, simply because the experimental damping of the stimulus was not seen to explication, or the excessive damping of the stimulus model, which were not measured, is not at all likely the satisfactory steering torques are not exactly the same as the

The result that both the impulse and "truncated-step"

the torque over a longer period of time, the magnitude of torque, and the rudder was forced to spread

an impulse torque would require an impulsive act. Hence, disturbing the steering assembly with a step increase in speed increased, was strongly self-convergent.

step disturbance was applied to the real vehicle, which was approached complete steering with the experimental observation that the steering

Inadequacy of the impulse to torque at higher speeds is con- the association, would result in a better stimulation. The

tule, lasting less than a quarter-cycle of steering module

the simulation, indicated that a torque of constant mean-

It is less than about 35 mph, at higher speeds (e.g., 42.5 mph, a mathematical impulse function was employed for speeds

In stimulating the steering torque disturbance,
accuracy of the simulation with respect to experiment was the poorest at low speed, there was no "best" level of steering torque impulse in simulating the motorcycle at a forward speed on the order of 10 mph. Hence, Figure 3.2 shows two simulations, corresponding to two levels of steering torque impulse.)

2. Flexibility of the front wheel is not a likely explanation of the low speed wobble oscillation. This oscillation was observed closely at zero speed, and no deformation of the wheel could be noted. Rather, it was noted that tire flexibility was providing the necessary restoring torque.

Also, free-play in the front wheel bearings would cause a delay in the transmission of front tire forces and moments to the fork assembly. Such a delay could reduce the damping of the wobble mode. However, no wheel bearing play could be found to exist in the test vehicle.

3. During the road experiments, the values of the roll angle, steer angle, and yaw rate at time \( t=0 \) were not exactly as assumed in the simulation. (In most cases, these initial conditions were assumed to be zero.) Nevertheless, it is believed that the difference between the actual and assumed initial conditions was too small to account for the discrepancies under discussion, because the experiments were found to be readily repeatable. Tests performed at
Very rapidly, thus making it extremely unlikely that any appreciable amount of the calculated steering angles decay after the application of a steering pulse, the calculated steering angles are very small, and while accurate statements are obtained, the range of the motion variables is approximated because exactation may need to be taken into account, they are pro-

the contact patch upon application of the sharp steering increments of the steering geometry and slippage of the front surface with nonlinear equations of motion. Although non-
motorized the leverage at low forward speed can only be correctly

It is possible, but not very likely, that the zero, thus nonzero initial condition was included in the since the slope of the roll angle curve at time t=0 is not small initial roll velocity was estimated from the data,
a further. (Note: For the 20 mph experiment, PI: 3.3,

repeatedly could not have been obtained if initial con-

then the low speed tests agreed with the simulation. Thus the same speed agreed with each other to a much higher degree
Experiments in which a positive step or roll torque was applied
is probably a small amount of bias in the experimental data.

The center of mass position was not possible, there
motorcycle was imperfectly balanced. Since perfect adjustment
accordingly, no data was taken until the calibration of the
were released, the rider would then shift his position
drift from the straight-ahead direction when the handbrakes
of mass was not close to the X-Z plane, the motorcycle would
motorcycle was being ridden. If the overall vehicle center
the X-Z plane was determined by trial and error, while the
rider's hips required to bring the total center of mass into
heel to keep his legs from moving. (The lateral shift of the
the brace, while the rider's knees firmly against the seat
the motorcycle seat. The upper body was constrained by
mass to the plane by shifting his hips to the right, relative
to the X-Z plane. The rider could move the overall center of
symmetry of the seat, was approximated. The seat moved to the
such that its plane of symmetry coincided with the plane of
the actual vehicle center of mass, with the rider seated.

As was assumed in the determination of the equations of motion,
passage through the center of the rear tire contact patch
with respect to this X-Z plane (the plane of the rear wheel,
5, The test motorcycle, with rider, was not symmetrized.

Observation:
Throughout a large portion of a wooden
that a sliding condition could exist
In the motorecycle at very low speeds, without the rider, the motorcycle at very low speeds, without the rider, the motorcycle at very low speeds, without the rider, the motorcycle at very low speeds, without the rider. By theory, since a mobile oscillation could easily be induced, behavior of the motorecycle and the mobile response predicted be a cause of the differences observed between the mobile and fluorescent, compensation of the rudder's body did not appear to be a cause of the differences observed below the mobile mode-oscillations was observed to be considered as below the mobile oscillations, frequencies, and the frequencies of the mobile rudder, because (1) the tank was kept tilted to minimize the chance to be an inflection source of low speed steering oscillations of the gasoline in the fuel tank are judged.

The rudder and the gasoline in the fuel tank, respectively, determined from this assumption, especially of several rigid bodies. Several components of the man-motion, it was assumed that the motorecycle and rudder are complicated.

6. In the derivation of the equations of motion, the response to a step roll torque and that predicted by the no means all, of the difference between the measured roll and symmetry appears to have accounted for a slight part, but by then when it was positive. Thus, the lack of vehicle to that torque when the step of the torque was negative torque, the vehicle tended to roll more quickly in response to the vehicle (rather than the negative steps shown in Figure 3.2-3.5) indicates that there may have been a slight
seated on the vehicle, whereas the theory did not predict an oscillation under these conditions. (Moments of inertia, etc., were measured for the motorcycle without rider, as well as with rider.)

In spite of his restraining brace, it is possible that the rider unconsciously exercised a small amount of body control, which would influence the measured roll response to a step roll moment. It is likely that this body control occurred to a greater extent at low rather than high speeds, because (1) the vehicle tended to be more unstable at low speeds than high speeds, and (2) it was more difficult to correct the roll instability at low speeds by means of steering control. Both of these conditions tended to pressure the rider into taking some premature control action, i.e., body control. In riding the motorcycle "hands off", it was noted that body lean tended to oppose roll angle—that is, if the motorcycle started to roll to the left, the rider would tend to lean to the right. Thus, if the rider was making unconscious corrections to a negative step of roll torque, he would be leaning to the right. In that case, to keep the total vehicle center of gravity position unchanged, the motorcycle would roll further in the negative direction. Since the third wheel measured the lean of the motorcycle only (not the position of the combined rider-motorcycle center of gravity), it would indicate a faster roll than it would have if the rider had remained rigid.
The hypothesis that body movements performed unconsciously by the rider have influenced the low frequency response of the vehicle is a possible but not the most likely explanation of differences between theory and experiment. There are a few shortcomings to this hypothesis. First, care was taken to notice such movements if possible. Next, at 20 mph, the experimental roll response to a step roll torque appeared to diverge slightly faster than the simulated response, in the manner of 10 mph experiments, but, at 20 mph, the rider was under much less pressure to make a body movement correction. Finally, and most significantly, in the absence of disturbances, the rider was unable to sustain a roll oscillation, even while making body movements (within the constraint of the rigid brace) intended to reinforce such an oscillation.

7. Regardless of whether or not the rider was in fact influencing the roll response of the motorcycle to a step input of roll torque, it is very unlikely that he influenced the wobble mode, either actively or passively. Rather, it is felt that the reason the wobble mode is poorly predicted by the simulation is that none of the tire models investigated adequately represented the dynamic response of the tire lateral force and aligning torque to time varying values of slip angle. A finding in support of this hypothesis consists of the observation that the lightly damped wobble mode
can be obtained in the simulation by artificially adjusting the relaxation length of the front tire. Unfortunately, no single relaxation length was found to give valid results for all speeds.

The greatest advantage of the method in which tire dynamic effects have been included in the motorcycle equations (see Eq. (1.1)) is the ease with which tire dynamics may be incorporated into the vehicle equations of motion, and also the ease with which the resulting equations may be solved. It should be recognized, however, that tire dynamics models exist which have been found to predict transient lateral forces and aligning moments in a more realistic manner than the "point-contact" model of Equation (1.1). These more complete models are usually based on "string theory" [42, 43], in which the differential equations governing the lateral deformation of the tire tread are the same as equations governing the motion of a stretched string with an elastic lateral restraint. The tire dynamics model of Equation (1.1) can be derived from a string theory model by allowing the tire contact length, 2L, to become zero (hence the name, "point-contact" model).

While it is difficult to incorporate a string theory model with finite contact length into the motorcycle equations of motion, it can be shown that such an incorporation promises at least a more realistic theoretical prediction
of the wobble mode, i.e., reduced damping at low speeds. To show this, consider a tire and wheel constrained such that the wheel plane remains vertical and the hub center moves with constant velocity (speed \( u \)). Define the yaw angle, \( \psi \), of the wheel plane to be zero when the wheel plane is aligned with the velocity vector of the hub center, and further require the yaw angle to vary sinusoidally with a frequency of \( \Omega \) radians/second, i.e.,

\[ \psi = \psi_0 \sin \Omega t. \]

If path curvature considerations are ignored, the slip angle, \( \alpha \), is the negative of \( \psi \). Consider the lateral force response of the tire to the yaw angle input. Based on a one-dimensional string model, hereafter termed the "finite-contact" model, the sinusoidal lateral force output is expected to lag the yaw angle by the phase angle \([42]\),

\[ \phi_s = \tan^{-1} \left( -\sin \frac{2 \Omega}{u} \frac{\sigma}{u} \right) + \tan^{-1} \left( -\frac{\sigma_s \Omega}{u} \right), \quad (3.1) \]

where \( \sigma_s \) is the relaxation length associated with the finite-contact model. In the case of the point-contact model, the force lags the yaw angle by the phase angle \([42]\),
\begin{equation}
\phi_p = \tan^{-1} \left( \frac{\sigma_p \Omega}{u} \right), \tag{3.2}
\end{equation}

where \(\sigma_p\) is the relaxation length associated with the point-contact model.

The string model and the point-contact model also predict different lateral force \(F_y\) responses to a step change in slip angle. For the finite-contact model, this response is \cite{41}

\[
\frac{F_y(x)}{F_{yss}} = \begin{cases} 
\frac{(l + \sigma_s)x - x^2/4}{(l + \sigma_s)^2}, & 0 < x < 2l \\
\frac{\sigma_s^2 e^{-(x-2l)/\sigma_s}}{1 - \frac{\sigma_s^2 e}{(l + \sigma_s)^2}}, & x > 2l
\end{cases} \tag{3.3}
\]

where \(x\) is the distance rolled by the tire, and \(F_{yss}\) is the steady-state level of lateral force. For the point-contact model, Equation (3.3) reduces to

\[
\frac{F_y(x)}{F_{yss}} = 1 - e^{-\sigma_p/x}. \tag{3.4}
\]

For a given tire, the lateral force response to a step slip angle can be measured, and the values of \(\sigma_s\) and \(\sigma_p\) can
be estimated by fitting Equations (3.3) and (3.4) to the experimental data. In Appendix C, the value of $\sigma_p = \sigma_f$ is obtained in this manner (Fig. C.10). The value of the tire contact length, $2\ell$, was estimated to be about 4-5 inches for the front tire of the test motorcycle. Fitting Equation (3.3) to the data of Figure C.10, with $2\ell = 5$ inches, yields $\sigma_s = 1.15$ inches. From Appendix D, the value of $\sigma_p = \sigma_f$ is found to be 2.1 inches.

The phase lag of the lateral force behind the yaw angle can now be estimated from Equations (3.1) and (3.2) for the front tire of the test motorcycle. The wobble frequency of the test motorcycle was observed to remain essentially constant with forward speed, at a value of about 45 radians/second. Thus, in Equations (3.1) and (3.2), $\Omega = 45$ radians/second. Both $\phi_s$ and $\phi_p$ are functions of $\Omega/u$, which quantity is termed the "reduced frequency" ($\Omega_r$). Table 3.1 displays values of $\phi_s$ and $\phi_p$ for $u = 10$, 30 and 45 mph. The corresponding (approximate) values of $\Omega_r$ are also shown.

Notice in Table 3.1 that the lateral force output of the tire, as predicted by the finite-contact model, lags considerably more behind the yaw angle (and consequently the negative of the slip angle) than the force output as predicted by the point-contact model, especially at the lower speeds, where the simulation of the wobble mode based on the
there to a sinusoidal step angle input, as predicted by the
value of $\phi$. For example, the lateral force response of the
model are not the same as the effects of adjusting the
torque results. However, the effects of the finite-contact
improvement in the roll response to the step input of roll
direction of the workable mode at 10 mph, for example, no
value of $\phi$ is addressed appropriately to give a good pre-
the present motorcylote equations have indicated that all the
roll response to a step input of roll torque, solutions to
equations witht also improve the accuracy of the predicted
use of the finite-contact finite mode in the motorcylote
least, the workable mode.

Thus, it is reasonable to expect that the finite-contact
motorcyles equations of motion were found to be the poorest.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\phi'$</th>
<th>$\theta$</th>
<th>$\theta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-28</td>
<td>12</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>-10</td>
<td>12</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>-7</td>
<td>12</td>
<td>0.66</td>
<td>45</td>
</tr>
</tbody>
</table>

**TABLE 3.1**

Comparison of Finite-Contact and
Finite-Contact Models
Complete stability under the condition of sustained roll torque, the vehicle could not sustain an oscillatory motion. The oscillations are due to a step input of roll torque, and the oscillations may explain the inaccurate simulation of the steering assembly.

The steering assembly is low friction at the front fork, front fork assembly to steer. However, if the forward speed is too high, the steering assembly cannot rotate sufficiently, because the steering assembly is stationary and is even a small roll angle. The motor torque to steer in the direction of an impeding fail. It appears that the stability of the steering assembly depends upon the stability of the front fork assembly. The existence of roll stability and were oscillatory roll forward influence the finite-contact area of the real tire with influence the finite stability. Reasoning suggests a manner in which the finite-contact model does not depend on whether the step or impulse.

With Table 3.1, or by pure lateral translation, whereas angular is induced by pure yawing, as discussed in connection point-contact model, does not depend on whether the step
In summary, it appears that an improved static and dynamic representation of the pneumatic tire is required to adequately account for the dynamic behavior that is exhibited by the motorcycle at low forward speeds of travel.
4. THE MAN-MOTORCYCLE SYSTEM

4.1 MODELING THE MAN-MOTORCYCLE SYSTEM

Chapters 2 and 3 have dealt with the uncontrolled motorcycle. In the remainder of the dissertation, this restriction is relaxed, and the rider is allowed to control the vehicle by applying a steering torque to the handlebars. The man and motorcycle are assumed to interact in the form of the closed-loop control system diagrammed in Figure 4.1. In this model of the system, the rider, desiring to maintain a roll angle of zero, perceives an error signal equal to the negative of the actual roll angle ($\phi(t)$). He makes a correction in the form of a steering torque which is related to the error by the linear transfer function $Y_p(j\omega)$. His total steering-torque output ($t_\delta(t)$) is the sum of the torque correlated with the error plus an additive noise or "remnant" ($n(t)$). The motorcycle dynamics are represented by the linear transfer function, $Y_c(j\omega)$. Disturbances caused by cross winds and road irregularities are included in $i(t)$.

In this study, attention has been restricted to the roll stabilization task performed by the rider as the vehicle travels on a straight level road. The path following task is not being considered. Using analytical methods, Weir [16] has shown that the best closed-loop control system representing the stabilization task is the one shown in Figure 4.1, and that rider-body lean is more useful for path-following control.
Figure 4.1
Block diagram of the man-motorcycle system.
Recent body control studies to require strict- 
\text{For example, the author has observed that mild}
\text{operation and "breakout" path following maneuvers,}
\text{speed operation and "breakout" path following maneuvers,}
\text{body lean control is probably the most important for low}

\text{Function: $y(t) = \text{The remnant containted}$}
\text{is not related to the roll angle by the linear transfer}
\text{constants of all the steering torque output of the rider that}
\text{operating the vehicle on a straight road, the remnant, $u(t)$,}
\text{In modeling the stabilization task of the rider in}
\text{stabilization.}

\text{was purposed to maintain as being beyond the scope of this}
\text{although very interesting and important in many situations,}
\text{variations on the experimental findings, body-lean control.
\text{In order to simplify the problem and reduce the}
\text{allowed. In time and between different riders when body lean is}
\text{there is probably much more variation in control techniques
\text{easy operation, control by body lean is optional, and}
\text{when body lean was permitted. Appearance in situations of
\text{controlled on curved and straight roads just as easily as
\text{that, except for very low speeds, the motorcyle could be
\text{In this study with the rider's body plaque, it was found
\text{small. In fact, in the qualitative experiments performed
\text{use of steering torque, body lean would be expected to be
\text{stabilization can be accomplished most effectively with the
\text{the rider's center stability is one of stabilization, because
\text{By restricting the study to straight-path operations,'
in Table 4.1 (analytical expressions) and Figure 4.2 (pole
transfer function or $\lambda$ (ω)). The results of these calculations, shown
bar—was estimated to have negligible effect on the calcula-
addition instrumentation—a steering-torque measurement
instrumented for the man—motorcycle experiments. The
parameter data corresponding to the Honda test motorcycle and
calculated at forward speeds of 15, 30 and 45 mph, using
The transfer function of the controlled element was

In Appendix A, the equations of motion, control calculation is described
and roll angle as output, was calculated from the motor—
transfer function, $\lambda (\omega)$, with steering torque as input
In modeling the man—motorcycle system, the motorcycle

(\lambda (\omega) the controlled element

Any time variation in $\lambda (\omega)$
and although road shocks are with frequency phenomenon, and
front/rear mount shock absorbers (such as mechanical
fastery or involuntary (such as direct mechanical transmission
avoidance, etc.); 0) mass and instantaneous steering torques; volum-
road not being perfectly straight, obstacle or pump
roll angle (such as corrections needed due to the real
correction steering torques not linearly related to the
which is not necessarily related to the roll angle; (2) path
Figure 4.2
Controlled element; \( u = 15, 30 \) and 45 mph.
diagrams), were obtained from the motorcycle equations of motion having the simplest tire model studied, namely, tire lateral forces due to slip and inclination angles, and aligning moments arising from slip angles only. No steering damper was included, since the test vehicle did not have one.

The motorcycle, as described by the derived equations of motion, is stable at 15 mph, whereas it exhibits a capsize-mode instability at speeds of 30 and 45 mph. The Bode diagrams in Figure 4.2 show that the vehicle transfer function at 30 and 45 mph is nearly the same, but is considerably different from the transfer function describing the controlled element at 15 mph. Frequencies less than about 10 radians/second are controllable by the rider, and for these frequencies low-speed operation is dominated by the weave mode and the (stable) capsize mode, while for higher speeds, the frequency and damping of the weave mode increases and the motorcycle is dominated by the unstable capsize mode.

The domination of the dynamics of the controlled element by the weave and capsize modes provides some intuitive feel for the behavior of the motorcycle. For example, at low frequencies, and at 15 mph, the controlled element behaves approximately as a third order system, while, at 30 and 45 mph, the cycle can be approximated by a first order system of the form
have shown that the preferred form of the operator's
of interest to [28, 45, 46, 47]. Both refer to [28] and [47].
the form of a number of equations (4.1) have been studied by a number
of man-machine systems in which the controlled element has

4.3. THEORETICAL REQUIREMENTS FOR STABILITY OF THE MAN-

MACHINE SYSTEM

form

the transfer function of the controlled element has the same
form as that of the operator's transfer function. Thus, the
operator's transfer function is strongly dependent upon the
operator's systems (e.g., [28]).

Research in man-machine
form that it has at 30 and 45 mph. (4 mph,
the transfer function of the controlled element has the same
form that it has at 30 and 45 mph. (4 mph,

motorcyles normally operate. At speeds higher than 45 mph,

transfer function of the controlled element, while
the theoretical transfer function of the controlled element, while

From the preceding discussion, it is seen that the
factor corresponding to the weave mode,

factor corresponding to the weave mode.

expression. A more accurate approximation of
expression. A more accurate approximation of

(4 mph). 

\[ \frac{1 - e^{-\frac{\omega}{\tau}}}{\omega} \sim \omega_0^{2} \frac{\omega}{\tau} \]

Figure 4.3 shows such an approximation for the 30 mph case,

\[ \phi \]

\[ \phi \]
Figure 4.3 Controlled element transfer function, $u=30$ mph, and its approximation by a first order transfer function.
transfer function, \( Y_p(j\omega) \), is simply a constant gain, \( K_p \), and pure time delay\(^1\), \( \tau_p \), i.e., \( Y_p(j\omega) = K_p e^{-\tau_p j\omega} \).

The stability of the closed-loop system having the open-loop transfer function (Laplace transform notation),

\[
Y_p(s)Y_c(s) = \frac{K_p K_c e^{-\tau_p s}}{T_c s - 1},
\]

(4.2)

can be determined by the application of the Nyquist criterion, which states that

\[
Z = N + P,
\]

(4.3)

where \( Z \) is the number of zeroes of the closed-loop characteristic function \((1 + Y_p(s)Y_c(s))\) located in the right half of the complex plane, \( P \) is the number of poles of \( Y_p(s)Y_c(s) \) in the right half of the complex plane, and \( N \) is the number of clockwise encirclements of the \(-1\) point by the Nyquist map of \( Y_p(s)Y_c(s) \), or the number of clockwise encirclements of the \(-1/(K_p K_c)\) point by the Nyquist map of \( Y_p(s)Y_c(s)/(K_p K_c) \). A Nyquist plot of \( Y_p(s)Y_c(s)/(K_p K_c) \) is shown in Figure 4.4. For \( Y_p(s)Y_c(s) \) as defined in Equation (4.2), \( P = 1 \). If stability of the closed-loop system is to be achieved \((Z = 0)\), \( N \) must be \(-1\). From Figure 4.4, it is seen that there is one counterclockwise

\(^1\)It is shown in [28] and [45] that lead and lag equalization as part of the operator's transfer function did not significantly improve the closed-loop system performance.
A Nyquist plot of the open-loop transfer function also to be a gain and motor control model's transfer function as well. It is reasonable to expect the system stability due to the form discussed in the preceding paragraph. The controlled transfer function, \( \frac{d^2}{ds^2} + \frac{d}{ds} + 1 \), has a form with a steady-state behavior at 30 mph, at which speed the most of the remainder of the disturbance is considered.

Hence, only for \( I^+ = N \) (Fig. 1) and \( T_1 = \frac{d}{s} > f \), there is a closed-loop control point. There is one corner of the gain of the \( k^2 \frac{d^2}{(s^2)} \) for \( k^2 < \), and there are no other points on the Nyquist chart.
Figure 4.4: General Nyquist plot of $\frac{e^{-Ts}}{s - \frac{1}{T}}$. 

$s = 0$ 

$s = \pm j\omega$ 

$s = \frac{1}{T}$ 

$s = -\frac{1}{T}$
Figure 4.5 Nyquist plot of \( -100e^{\frac{\lambda}{2}}(s) \), 30 mpu.
38.2 \leq K_p \leq 425 \text{ lb-in/radian.}

These results, in the form of Nyquist plots, can be replotted as a Bode diagram, 
\(-Y_p(j\omega)Y_c(j\omega) = K_p e^{-\tau_p j \omega} Y_c(j\omega)\),
where \(Y_c(j\omega)\) is again the theoretical transfer function of
the controlled element at 30 mph; e.g., see Figure 4.6, for
which \(K_p = 1 \text{ lb-in/radian}, \) and \(\tau_p = 0.3\) second. For stability,
the minimum value of \(K_p\) is defined by

\[
(K_p)_{\text{min}} |Y_c(j\omega)|_{\omega=0} = 1,
\]
while the maximum value of \(K_p\) is determined by

\[
(K_p)_{\text{max}} |Y_c(j\omega)|_{\omega=\omega_1} = 1,
\]
where \(\omega_1\) is the frequency (other than zero) for which
\(\angle(-K_p e^{-\tau_p j \omega} Y_c(j\omega)) = -180^\circ\), or \(\angle(K_p e^{-\tau_p j \omega} Y_c(j\omega)) = 0.\)
Figure 4.6: Bode diagram of $G(s) = \frac{e^{-3f_0 s}}{s^{1/2}}$, $f_0 = 30$ rpm.

\[ \omega_1 = 3.0 \text{ radians/second} \]

\[ |\chi| = \frac{0.0235 \text{ rad/degree}}{0.0262 \text{ rad/degree}} = 0.0865 \]

\[ |\gamma| = 0.25 \text{ rad/degree} \]
5. ROLL-STABILIZATION EXPERIMENTS

5.1 OBJECTIVE AND DESCRIPTION OF EXPERIMENTS

The major objective of the manual control portion of the investigation described in this dissertation has been to identify a transfer function representation of the rider's task in stabilizing the vehicle. Experiments providing the necessary data have been performed using the test vehicle described in Chapter 3, with three test subjects, denoted Riders A, B, and C.

The test motorcycle was instrumented as follows. Roll angle and steer angle were measured as described in Chapter 3 (third wheel and geared rotary potentiometers). Yaw rate was not recorded, but the rate gyro was positioned to measure roll rate. To record steering torques applied by the rider, the rider steered one-handed with a torque bar (1/8" x 1" cross-section) attached to the handlebars (Fig. 5.1). This bar was designed to measure the component of applied force parallel to the plane of the front wheel and

Throttle control was relocated to the rear frame to replace the usual hand grip control on the handlebars. The effect of this unusual control arrangement upon the generality of the results is not known, but it is felt that any such effects were small and, at least, considerably smaller than those that would have been if the rider had been using position control rather than torque control.

2 Force was measured rather than moment directly to allow the sensitivity of the measuring bar to be adjusted by changing its position relative to the handlebars.
Figure 5.1: Steering torque bar and throttle control (arrows).
perpendicular to the steering axis. Strain gages (with temperature compensation) were attached to the 1" wide faces of the bar, which faces were parallel to the steering axis. The handle held by the rider was attached to the bar in such a manner that the rider could not apply moments to the bar, except about the axis parallel to the wheel plane and perpendicular to the steering axis. Due to the position of the strain gages and the bar itself, the gages were insensitive to moments about this axis and to force components along the longitudinal axis of the bar or parallel to the steering axis. Hence, the desired force was directly measured, and the steering torque was the product of that force and the distance from the front wheel center plane to the free end of the torque bar. Output from the four transducers, roll and steering-angle potentiometers, rate gyro and steering-torque bar, was recorded on analog magnetic tape. Power supplies and recording equipment were carried in an accompanying automobile in the manner described in Chapter 3. Appendix E presents a schematic of the instrumentation.

After the runs were made with each test subject, the analog records were filtered with a first-order filter having a break frequency of five radians/second, converted to digital form and stored on digital magnetic tape.

In each experiment, the motorcycle and accompanying automobile operated at constant forward speed over a section of essentially straight and reasonably smooth road, about
one-half mile in length. Riders A, B, and C operated the

cycle at a speed of 30 mph for about 50-60 seconds per trial
(test run), and about a dozen trials per rider were performed.
In addition, a few trials, lasting 65 seconds, were made with
Rider A at 15 mph.

The vehicle parameter data were measured with Rider A
don the motorcycle. To minimize the changes in these parameters
由于大小不同的骑手，它被描述为寻找与其他
riders approximately the same size as Rider A. It was also
desired to find experienced riders. Riders B and C fit
these characteristics, except that Rider B is a few inches
taller than Rider A, and Rider C is about 10 pounds heavier,
while several inches shorter than Rider A. Changes in
Yc(\omega) due to rider size differences were found to be
negligible (Chapter 6).

5.2 METHODS OF INTERPRETING DATA

Given records of the random input, x(t), and output,
y(t), for a linear system whose (unknown) transfer function
is G(\omega), it is possible to estimate G(\omega) by several
techniques. Two of these methods have been used in this study.

One method used herein is the cross-spectral method,
in which G(\omega) is estimated by

\hat{G}(\omega) = \frac{\hat{S}_{xy}(\omega)}{\hat{S}_{xx}(\omega)}.
time-intercept, \( b \) may be approximated by the discrete form of \( E(t) \) as then estimated by a standard

\[
E(x, t) = \sum_{k=0}^{\infty} \left( \frac{t}{\tau} \right)^k \frac{(1 - e^{-\tau t})}{\tau^k}
\]

The convolution integral

The input \( x(t) \) and output \( y(t) \) of the system are related by the inverse Fourier transform of the transfer function \( E(t) \), that is, sampled at a rate which has been determined, that is, sampled at a rate which has been determined.

The second method is termed the impulse response

\[
y(t) = \int_0^\infty x(t-\tau) \delta(\tau) d\tau
\]

where \( \delta(\tau) \) is the estimated cross-spectrum between \( x(t) \) and \( y(t) \)
Before the analysis techniques were applied to actual test data, they were checked out by using them to identify known systems. Data was first generated by driving an analog computer circuit with a low frequency random noise generator and digitizing the resulting signals. Cross-spectral analysis was used to identify a few transfer functions, such as an integration and a first-order Padé approximation to a pure time delay, $e^{-\frac{\omega}{\omega_0}} = \frac{1}{1 + \frac{\omega}{\omega_0}}$.

While these identifications were successful, it was found more convenient to use artificial data generated entirely digitally, since it was then easier and less costly to prepare and change the data. Also, with solely digital data, the pure time delay could be created exactly, rather than with a Padé approximation.

A more complete discussion of using both analysis methods to identify known transfer functions is given in Appendix C.

Consider next the closed-loop system shown in Figure 5.2, which system is of the same form as the one used to model the man-motorcycle system (Fig. 4.1). In the system of Figure 5.2, the transfer function belonging to the rider or operator is $G(s)$. In most, if not all, practical test situations, the output of $G(s)$ (that is, $x(t)$) is not known; only $c(t)$ is measured. If the disturbance, $i(t)$, is known, $x(t)$ is not needed to identify $G(s)$, which can then be estimated as
Figure 5.2
Block diagram of a typical manual control system.
\[ G(j\omega) = \frac{\hat{S}_{ic}(\omega)}{\hat{S}_{ie}(\omega)}, \]

where \( \hat{S}_{ic}(\omega) \) and \( \hat{S}_{ie}(\omega) \) are the estimates of the cross-spectra between \( i(t) \) and \( c(t) \), and \( i(t) \) and \( e(t) \), respectively. (See, e.g., Reference [26].)

Suppose now that \( i(t) \) is not known. In this case, it is necessary to estimate \( G(j\omega) \) by an open-loop method; that is,

\[ \hat{G}_m(j\omega) = \frac{\hat{S}_{ec}(\omega)}{\hat{S}_{ee}(\omega)}. \quad (5.2) \]

It may be shown [37, 38] that even if there is no error in estimating \( S_{ec}(\omega) \) and \( S_{ee}(\omega) \) (\( \hat{S}_{ec}(\omega) = S_{ec}(\omega), \hat{S}_{ee}(\omega) = S_{ee}(\omega) \)), there will be, in general, an error in identifying \( G(j\omega) \), and, in fact,

\[ \hat{G}_m(j\omega) = G(j\omega) + \frac{S_{en}(\omega)}{S_{ee}(\omega)}. \]

The term, \( \frac{S_{en}(\omega)}{S_{ee}(\omega)} \), is the error in identification (termed "bias error") and is seen to arise from correlation between \( e(t) \) and \( n(t) \). This correlation, and hence the bias error, will be reduced if \( i(t) \) is much larger than \( n(t) \). On the other hand, if \( n(t) \) is substantial relative to \( i(t) \), the bias error will be large. If \( n(t) \) is much greater than \( i(t) \),
cannot be used with the Winckove-Bawares method.

Contrast this with the standard physical readable or readable method, which is not

It will be shown that cross-spectral methods, which are not

In an experimental situation, it can be determined whether or not the measured data can be determined whether or not it is true.

\[ \text{denoted } G(t) \]

\[ \text{is calculated from} \]

\[ G(t) = \omega \]

\[ \text{By denoted } G(t), \text{ the estimated estimate of } G(t), \]

\[ \text{Finally, the estimated estimate of } G(t), \]

\[ \text{Next, the transfer function having } e(t-y) \]

\[ \text{is delayed in time by } \Delta \text{ an amount } \gamma \]

\[ \text{The time shifting method is applied as follows. First,} \]

\[ \text{the time shifting or Winckove-Bawares method,} \]

\[ \text{error reduction has been developed by Winckove and Bawares,} \]

\[ \text{plus error is needed.} \]

\[ \text{a method for accomplishing this} \]

\[ \text{identity } \]

\[ \text{and a method for reducing the identity } \]

\[ \text{rather, it will} \]

\[ \text{rather, it will be determined} \]

\[ \text{and } H(t) \]
The range of $\lambda$ is $0 \leq \lambda \leq \tau_p$, where $\tau_p$ is the time delay in $G(j\omega)$. Theoretical work was performed by Wingrove and Edwards only for this range of $\lambda$. For $\lambda > \tau_p$, the estimate in Equation (5.3) becomes less accurate, for the following reason. Let $G'(j\omega)$ be defined by

$$G'(j\omega) = e^{+\tau_p j\omega} G(j\omega).$$

The transfer function having $e(t-\lambda)$ as input and $c(t)$ as output is

$$G_\lambda(j\omega) = e^{\lambda j\omega} G(j\omega) = e^{(\lambda-\tau_p) j\omega} G'(j\omega) \quad (5.4)$$

From Equation (5.4), it is seen that if $\lambda > \tau_p$, $G_\lambda(j\omega)$ is not physically realizable. Hence, $G_\lambda(j\omega)$ cannot be accurately identified by a method which is constrained to identify a physically realizable system. The estimate of $G_\lambda(j\omega)$ is $\hat{G}_m(j\omega)$. Thus, the estimate of $G(j\omega)$ in Equation (5.3) is expected to decrease in accuracy when $\lambda > \tau_p$.

It can be shown [37] that if there exists a lag $\tau_o \leq \tau_p$ such that the autocorrelation function $r_{nn}(\tau)$ of the remnant $n(t)$ is zero for $\tau \geq \tau_o$, then the estimate for $G(j\omega)$ in Equation (5.3) will have zero bias error when $\tau_o \leq \lambda \leq \tau_p$. 

In a practical situation, however, $r_m(t)$ will not in general be zero for $t$ greater than some $T_0$, in which case it can be shown [37] that the bias error in identifying $G(j\omega)$ by Equation (5.3) is a minimum when $\lambda = T_p$. If $r_m(t)$ is small for all values of $t > T_p$, the bias error will also be small. The closer $r_m(t)$ is to a mathematical impulse function, or, equivalently, the nearer $r(t)$ is to being "white" noise, the greater will be the accuracy in identifying $G(j\omega)$.

When dealing with experimental data, the value of $T_p$ is known only approximately and is, in fact, one of the pieces of information desired from the data. Knowledge of $T_p$ is also important in the identification of $G(j\omega)$, since it is necessary to select $\lambda = T_p$ in order to minimize the bias error.

The value of $T_p$ is determined as follows [38]. For each value of $\lambda$, $\hat{G}(j\omega)$ is estimated by Equation (5.3). A transfer function of the form

$$\hat{G}(j\omega) = e^{-\lambda T_p} \hat{G}(j\omega)$$

is fitted to a Bode plot of $\hat{G}(j\omega)$. This process is repeated with a new value of $\lambda$ until the estimated time delay $T_p$ is equal to $\lambda$, in which case $T_p = \lambda$ is the estimated value of $T_p$. The value of $\lambda$ so obtained is the correct time delay.
\[ d = \gamma_{t=1}^{\infty} \]

The mean squared error (MSE) is given by

\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{y}_i)^2 \]

By reference to Figure 5.2, the remnant is defined as

\[ \text{Remnant} = \gamma_{t=1}^{\infty} \]

The transfer function \( G(s) \) has been poorly identified; a large estimated MSE does not necessarily mean that the transfer function is not identified, it means that a large portion of the operator's output is not linearly related to its error signal. Another measure of the remnant content of the operator's output is the mean square error, defined as

\[ M_{ \text{mse} } = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{y}_i)^2 \]

By reference to Figure 5.2, the remnant is defined as

\[ \text{Remnant} = \gamma_{t=1}^{\infty} \]

The normalized mean squared error (NMSE) is defined by

\[ \text{NMSE} = \frac{\sum_{i=1}^{n} (x_i - \hat{y}_i)^2}{\sum_{i=1}^{n} x_i^2} \]

and the MSE may be estimated after the impulse response and the remnant output. When test data are being analyzed, the remnant and is a measure of the remnant content of the operator's output.
cross-spectral method is combined with the time-shifting
methods combined (see G.6, Appendix G). If, however, the
to identify \( G(j\omega) \) with the time-shifting and impulse response
was less than \( 0.1 \) for \( t < 0.4 \) second, thus, it was possible

\[ (4) \text{ shown in Appendix G} \]

\[ \text{correlation function of } u(t) \]

\[ \text{situation in which the bias error is greatest, the } u(t) \]

\[ \text{the system was excited only by the random, } u(t), \]

\[ \text{the correlation function of the data, } u(t) \]

\[ \text{were assumed that } t \text{ } (t) = 0; \]

\[ \text{thus, on a detailed computer, as described in Appendix G. In } \]

\[ \text{data required for analysis was prepared artificially} \]

\[ \frac{H(j\omega)}{I} = (\omega) \]

\[ \text{and} \]

\[ G(j\omega) = e^{-\omega} \]

\[ (\omega) \]

With the following assumptions being made:

\[ \text{time-shifting method, the system shown in Figure 5.2 was analyzed} \]

\[ \text{in the time-shifting method in conjunction with the time-} \]

\[ \text{as an illustrative example of the results of attempting} \]

\[ \text{method.} \]

In the impulse response method, but not the cross-spectral
physically realizable systems. Such a constraint is inherent

to calculate \( \Gamma([j\omega]) \text{ must be constrained to identify only } \]

\[ \text{method requires that after delaying } e(t), \text{ the procedure used} \]

\[ \text{as mentioned earlier, application of the time-shifting} \]

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method, it is only possible to identify \(-1/H(j\omega) = -j\omega\). Figure 5.3 shows estimates \(\hat{G}_m(j\omega)\) for \(\lambda = 0, 0.1, 0.2, 0.3,\) and 0.4 second, using cross-spectral analysis. For each \(\lambda\),

\[
\hat{G}_m(j\omega) = -e^{\lambda j\omega / H(j\omega)} = -j\omega e^{\lambda j\omega} \quad (5.7)
\]

while the actual value of \(G_\lambda(j\omega)\) is, from Equation (5.4),

\[
G_\lambda(j\omega) = e^{(\lambda - \tau_p)j\omega}.
\]

From Equations (5.3) and (5.7), the estimate of \(G(j\omega)\) is

\[
\hat{G}(j\omega) = -1/H(j\omega) = -j\omega,
\]

regardless of the value of \(\lambda\).

The reason the cross-spectral method did not identify \(G(j\omega)\) is that it is capable of identifying the non-realizable transfer function in Equation (5.7), which transfer function represents a higher degree of correlation between \(e(t-\lambda)\) and \(c(t)\) than does \(G_\lambda(j\omega)\). The impulse response method, however, cannot identify the function in Equation (5.7), and thus is forced to identify \(G_\lambda(j\omega)\), even though \(G_\lambda(j\omega)\) represents a considerably weaker correlation between \(e(t-\lambda)\) and \(c(t)\) than does \(-e^{\lambda j\omega / H(j\omega)}\).
Since the impulse response and time shifting methods are not nearly as well documented in the literature as the cross-spectral method, some studies with prepared artificial data were carried out (Appendix G) to aid in interpreting Bode diagrams of \( \hat{Y}_p(j\omega) \), the estimate of \( Y_p(j\omega) \). Most of the rules set forth below are derived from the results of these analyses, using both artificial data and road test data.

1. Because the impulse-response function is estimated for a finite time interval, \((M-1)h\) (\(M\) is defined in Eq. 5.1b), systems having an impulse response function which does not decay with time can be difficult to identify. This difficulty is illustrated in Appendix G for the case of an integrator, the impulse response function of which is a step function. Identification of an integrator at low frequencies, where the error is greatest, is sensitive to truncation number, \(M\). Such a sensitivity is probably a good indicator that an unknown impulse response function is constant or growing unbounded with time.

2. When \( \hat{Y}_p(j\omega) \) is heavily biased (\( r_{nn}(\tau) \) is not close to zero for \( \tau \geq \tau_p \)), \( |\hat{Y}_p(j\omega)| \) tends toward \(|1/Y_c(j\omega)|\) at the upper and lower ends of the frequency range (near 1 and 10 radians/second, respectively).

3. When \( Y_p(j\omega) \) is of the form, \( Y_p(j\omega) = -K_p e^{-\tau_p j\omega} \), and \( \hat{Y}_p(j\omega) \) is biased,

\[
|\hat{Y}_p(j\omega)| < K_p ,
\]
increase in $M$ would also improve the accuracy of
is evidenced by increased oscillations in the power of the
increase $M$ tends to increase the variance of the estimator.

When a transfer function begins to be identified by the
impulse response method is of such a form that accurate
identification is possible, the effect of the transfer

function at the frequencies at which the
instabilities occur.

The recording at the frequencies at which the
instabilities
is based by $u(n)$ or possibly because there is little power
the impulse response function cannot be truncated, the estimate
identities, either because it is not physically realizable,
the impulse response, truncated, such that $\xi$ is difficult to

6. Instabilities in $\xi$ are evidenced by oscillations
not exhibit a local minimum at $\gamma = 1$.

NSF often increases monotonically with increasing $\gamma$ and does

Figure 5.1: When $\gamma$ is strongly based, however, the
true graph of the $\text{MSE}$ against $\gamma$ is shown in
but is always greater than the bandwidth of the actual remnant $u(t)$.

the estimated remnant, $u(t)$, decreases with increasing $\gamma$.

When $\gamma$ is based, the frequency bandwidth of

$$\omega_d(t) = \left(\gamma \xi \right)$$

and

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Figure 5.4: Typical variation of the MSE with $\lambda$. 

\[ \lambda \quad \text{d} \quad \lambda \quad 0 \]

\[ 0 \]

\[ Z_0^2 \]

\[ 1.0 \]
The estimate of the output is only possible if \( \theta \) (1) is achievable beyond these frequencies. However, this extra-accuracy beyond these frequencies is not possible for the impulse response method to extrapolate.

Unlike cross-spectral analysis, \( \phi \) is the power of the records being processed. It small is

\( g \cdot \text{The accuracy of } \chi(\phi) \) for frequencies at which

\[
0_{T_{\text{m}}} = (\phi \chi \chi)^{\phi} \chi^{\chi} \chi_{\text{m}}^{\phi}
\]

and

\[
(\phi \chi \chi)^{\phi} = \frac{\chi_{\text{m}}^{\phi}}{|(\phi \chi \chi)^{\phi} \chi_{\text{m}}^{\phi}}
\]

output is steered to zero, and from Equations (5.8) and (5.9),

5.9 (and (5.9))

the transfer function is known to which \( \phi \) \( T \) is the estimate of the unknown transfer

\[
\begin{align*}
0_{T_{\text{m}}} & \Rightarrow 0, \\
0 & \Rightarrow \frac{\chi_{\text{m}}^{\phi}}{|(\phi \chi \chi)^{\phi} \chi_{\text{m}}^{\phi}}
\end{align*}
\]

the impulse response method with

5.8 (and (5.8))

In any situation involving identification with

the impulse response method to pick out details in the

transfer function.
In practice, it cannot be determined whether either of these two conditions are met. Hence, estimates of the bandwidths of the records may be incorrect.

According to (7.2), the transfer function begins identified transformation, and (2) the impulse response function is accurate before Fourier.
6. RESULTS OF THE ROLL-STABILIZATION EXPERIMENTS

6.1 DESCRIPTION OF THE DATA

A total of fifteen trials were performed with Rider A (the author). These trials took place on two days about four months apart. In the first session, experiments began with a 30 mph test, followed by a 15 mph test, followed by a 30 mph test, etc., until eleven (total) were recorded, with three trials at each speed being suitable for analysis. The remaining nine were selected from a group of ten (all at 30 mph) performed on the second day.

On a single day, a total of thirteen trials were carried out with Rider B, with twelve of these being suitable for analysis. Similarly, on a single day, twelve trials were recorded with Rider C, eleven trials being suitable for analysis. Both riders were instructed to ride at a constant 30 mph while minimizing body movements with the aid of the brace attached to the motorcycle. A few practice runs were made before recording was begun.

Table 6.1 displays the rms levels and approximate maxima of the time histories that were recorded.

While the greatest emphasis has been placed upon 30 mph tests, the three trials at 15 mph provide some interesting comparisons with the 30 mph data. The most predominant

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<table>
<thead>
<tr>
<th>Roll Rate, deg/sec</th>
<th>Roll Rate, deg/sec</th>
<th>Roll Rate, deg/sec</th>
<th>Roll Rate, deg/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>30</td>
<td>0.5</td>
<td>2.25</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.5</td>
<td>2.25</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.5</td>
<td>2.25</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.5</td>
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<tr>
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<td>30</td>
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<td>2.25</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.5</td>
<td>2.25</td>
</tr>
</tbody>
</table>

**Approximate Maximum Values**

- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec

**Average Levels**

- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec

**Numerator of Fracts**

- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec

**Number of Fracts**

- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec

**Speed, mph**

- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec

**Day of Session**

- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec
- Roll Rate, deg/sec

**Table 6.1**

Data Description Roll-Stabilization Experiments
and [16], namely, that torque control of the motorcyle speed speeds are consistent with statements made in References [17] observed to occur during normal operation at moderate and high speeds than at low speeds.

The very low steering displacement which have been considered to be required at moderate and high

investigations of a given steering deflection, however, than 30 mph (approximately 45 mph below the highest speed.) Levels also do not vary significantly for speeds higher

levels that, during normal "casual" riding, maximum torque

steered by means of a torque wrench attached to the handgrips

6.1. Additional experiments in which the motorcyle was

Table torque being only about 20% lower for the lower speeds. (Table small at 30 mph than at 15 mph, steering-torque levels were

while the steering displacements were found to be much

free play of the clutching wiper within the potentiometer.

the possess some interesting artifacts from short coherence and the which was not designed to measure such small angles, did

then measured, since the steering potentiometer mechanism,

actual steering displacements may have been slightly higher

Great at 15 mph than at 30 mph. For the 30 mph tests, the

placement upon motorcyle speed. From Table 6.1 it is seen

Differences in the dependence of the level of steering dis-

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rad/s/second, the estimated spectra include the effects of
with a first-order filter, having a break frequency of 5
in Figure 6.1. Since the data were filtered prior to analysis
the filtering effect of YC Tiptred power spectra are shown
power spectrum of roll angle tended to be smoother, due to
frequency range from about 1.5 to 5.0 radians/second. The
characterized by a peak in the power spectrum. This dominant
characteristics of the time histories of steering
smaller at a low speed than at a moderate speed.
Suggestion is the roll angles are seen to be a little
tested. Similarly, the roll angles were also nearly speed independent for the speeds
rate levels were also nearly speed independent for the speeds
Reference to Table 6.1 shows that roll angles and roll
speeds.
more susceptible at low speeds than at moderate and high
assessing the superiority of torque control is considered
is probably still preferable. However, the basis for
equations of motion are of lesser validity, torque control
low speeds, although the steering angles are larger and the
and the small steering angles involved. At
steer-to-steer system with steering
steer-to-toe involved by a fixed-control system, the poor performance
steer angles are based upon the severe instability of the
roll angles is preferable to steering-angle control. These
Figure 6.1a  Estimated power spectra, 30 mph; Rider A, Day 1 (b = spectral window bandwidth; \( v \) = degrees of freedom; see Appendix F).
difficulty than to actually the case.

Without a control activity, a situation that would make the
vehicle unstable, the motor vehicle would have to be able to operate the vehicle for periods of time
where the motor vehicle is unstable, the motor
vehicle's instability forces the rider into continuous
tune that the motor vehicle represents an unstable system,

Since the remainder is the primary disturbance, it is for-

Mr. Greene and Edwards,

transfer function requires the time-stripping method of
statement is straightforward, while the identification of the controlled
disturbance to the system, identification of the controlled
road/with disturbance, i.e., when the remainder is the primary
source of excitation to the vehicle-motor vehicle system than is the
induced that the rider's remainder, u(t), is a much larger
factor accurately identified by a cross-spectral analysis
motions at low frequency. Thus, the fact that \( y(t) \) can be
shown in Chapter 3 to be a realistic description of vehicle
Recall that the motor vehicle equations of motion have been

(See Chapter 4)

equations of motion, (See Chapter 4.1)

of the controlled element, as calculated from the motor vehicle
cross-spectral analysis is close to the transfer function
steering torque. It can be seen that the result of this
cross-spectral analysis to records of roll angle and

Figure 6.2 shows a typical result obtained by applying

6.2 IDENTIFICATION OF THE CONTROLLED ELEMENT

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due to the fact that most of the power in the time histories of range of frequencies. This representation, in the context of a delta function related to a transfer function (a value of 1)

In Figure 6.2, the graph of estimated squared coherency

analytically induces mode is far above the range included in the spectral response experiments (Chapter 3), since the frequency of the candidate was found to be unrealistically in the transient

nothing can be concluded about the whodle mode, whose reper-

Note that, as a result of this cross-spectral analysis,
Good identification for frequencies at which there is little power of $t_\theta$, $\phi$, or both cannot be obtained with cross-spectral methods. It was found that higher levels of coherence and, usually, a wider "good identification" frequency range could be obtained by performing a cross-spectral analysis with roll rate and steering torque, since the power spectra of these quantities were much more closely aligned than were roll angle and steering torque (see Fig. 6.1).

A particularly accurate identification of $Y_c(j\omega)$ using $\dot{\phi}$ and $t_\theta$ in a cross-spectral analysis is shown in Figure 6.3. (For comparison, Figures 6.1a, 6.2, and 6.3 were prepared from the same experiment.) A correction factor of $1/j\omega$ has been applied after the cross-spectral analysis, since $Y_c(j\omega)$ relates steering torque to roll angle. Note the higher estimated squared coherence.

Figure 6.4 shows the identification of the controlled element for a vehicle speed of 15 mph. For frequencies of high estimated squared coherence, the agreement with the theoretical controlled element is very good. It is seen that for frequencies about 1-2 rad/sec, the agreement tends to worsen, although less certainty can be attached to the estimates at these frequencies, due to the low coherence. However, assuming that the remnant is again a much greater source of disturbance than wind and road irregularities, the estimates
Figure 6.3 Estimation of $Y_0(j\omega)$, 30 mph; Rider A, Day 1. Calculated from records of roll rate and steering torque.
Figure 6.4  Estimation of $Y_\theta(j\omega)$, 15 mph.
Calculated from records of roll rate and steering torque.
seem to indicate that the experimental controlled element amplitude is considerably less "peaked" than the theory predicts. That is, the weave mode of the real vehicle at low speed is apparently more heavily damped than is the theoretical weave mode, an observation which is consistent with the results of the uncontrolled motorcycle study (Chapter 3). It may also be observed from Figure 6.4 that the slope of the experimental $|Y_c(j\omega)|$, for $\omega > 2$ radians/second, is steeper than $-40$ db/decade, but not as steep as the theoretical $-60$ db/decade.

In spite of the size differences between the three riders, the controlled element, as identified experimentally, did not display a rider sensitivity. Figures 6.5 and 6.6 show identifications of the controlled element (30 mph), using cross-spectral analysis and records of Riders B and C, respectively, operating the test vehicle.

6.3 IDENTIFICATION OF THE RIDER'S TRANSFER FUNCTION

To identify the rider's transfer function using the time shifting method, one might work with either roll angle or roll rate. Before the data was taken, it was expected that the use of roll rate would be superior, because a small amount of friction present in the roll angle measuring apparatus caused the bar to bend very slightly when it was moved by the third wheel, creating a hysteresis effect.
Figure 6.5 Estimated 
Calculation from records of roll rate 
Estimation of \( \gamma'(\omega) \), 30 mph. Rudder B.

\( b = 1.26 \text{ rad/sec} \)

\( \frac{I}{b} \) 

Squared coherence

Experimental

Theoretical

\( \frac{1}{(b^2 \text{ rad}^2 \text{ sec}^2)} \), degrees
Figure 6.6 Estimation of $Y_c(j\omega)$, 30 mph; Rider C. Calculated from records of roll rate and steering torque.
However, subsequent analysis of the data has indicated that this bending is of negligible concern\(^1\), and, for a number of reasons, it became apparent that the use of roll angle, when it was available\(^2\), was superior to roll rate.

The main difficulty in identifying \(Y_p(j\omega)\) from records of roll rate and steering torque is that, in most cases, \(Y_p(j\omega)\) was found to have the form

\[
Y_p(j\omega) = -K_p e^{-\tau_p j\omega}. \tag{6.1}
\]

Thus, the transfer function relating \(\dot{\phi}\) to \(t_s\) is

\[
Y'(j\omega) = -K_p e^{-\tau_p j\omega} / j\omega,
\]

which, as shown in Appendix G, cannot be accurately identified by the impulse response method.

---

\(^1\) The results using roll angle agreed with results using roll rate. Possibly engine vibrations reduced the friction and consequently the bending; or, possibly the hysteresis, about 5% of the maximum roll angle, is negligible in these analyses.

\(^2\) Unfortunately, the roll angle measuring device, being in a hostile environment near the road, was not as reliable as the other transducers. No roll angle data are available for the last half of the tests with Rider "B" or for four tests with Rider "C".
estimates of these parameters could then be obtained by a
parameter estimation procedure, such as $X$ and $T^d$.

As an alternative, the transfer function model, thus approximating the values of the
time-constant and the parameters of interrest, such as $X$ and $T^d$, a more refined
and accurate alternative to indicate the form of
impulse response methods. In many cases, the impulse responses
resulting from the approximation of the transfer function and
the experimental data are used in conjunction with the experimental data.

Indeed, when $X=0$, the experimental steering torque
have already been removed to a large extent when $Y=0$.

Since the bias errors in using the roll angle
remain, the bias errors are probably better removed when
impulse, but in the practical case of a non-white
accuracy of both identification algorithms, after
the correlation factor of $J_s$ is applied, when $X=0$, the
identity cannot be identified, as a result, with $Y=0$ and $T^d$

Furthermore, it was found that the transfer function

\[ (s^d)^XJ_s / T^d \text{ or } T \]
follows:

The steering torque spectrum of Equation (6.2) was modeled as a mixture of radian/second first-order filters. Thus, the theoretical predictions, the experimental records were filtered with a 

\[ \sum_{n} \left| (m^f)^2 \frac{a(m^f)^2}{l} + I \right| = (m)^2 \]

Furthermore, the experimental records were filtered with a

\[ \left( \frac{(m^f)^2}{l} + I \right) = (m)^2 \]

Hence [19], the steering torque spectrum, is

The following expression:

\[ (m^f)^2 \]

With \( I = 0 \), simple closed-loop relationships [19] yield:

\[ \begin{align*}
\text{given the man-motorcycle system of Figure 4.1, the theoretical steering torque spectrum was found as} \\
\text{the experimental spectrum until the shape of the theoretical spectrum matched that} \\
\text{of the experimental spectrum then could be adjusted.}
\end{align*} \]

Theoretical and error procedures: Assuming a rider model, a
included only three test runs each and thus are not as

were determined, although two of the three conditions

each condition, a different form of the transfer function,

15 mph, and a speed of 30 mph on two different days. For

Rider A was tested under three conditions: a speed of

6.4 Rider A

Riders A, B, and C

Estimated transfer functions are presented below for

theoretical transfer function (3.6). The theoretical

was used to evaluate Eq. (5.3). For both 15 and 30 mph, hence, for convenience, the

and experimental conditions were in close agreement

mental spectra. For the values of the involved, the theoretical

only the shape of the assumed. The actual value of \( \tilde{s}_u \) was not of concern since

assumption supported by the data (section

white, remainder). It was assumed that \( \tilde{s}_u = 0 \) constant (a

spectrum of the filtered steering torque recorded. To evaluate

where

\[
(3.6) \quad \sum_n \left( \frac{z^n + \delta z^n}{z} \right) \mid \frac{\alpha(n)}{\tilde{y}_1} \mid = \tilde{y}_1 \tilde{y}_1 \quad \text{(m)} \quad \tilde{y}_1 \tilde{y}_1 \quad \text{(m)} \quad \tilde{y}_1 \tilde{y}_1
\]
behavior of the estimates at low frequencies are much apparent. Both the level of instability in \( \gamma \) and the
analyzing roll angles rather than roll rate are read.

Platy, the advantages of
Figure 6.7a than Figure 6.7b. Thus, the advantages of
Figure 6.7a than Figure 6.7b. Thus, the advantages of
which latter observation is considerably more evident in
values chosen with consideration towards the 6th spectrum.
towards high and low frequencies, particularly
usually tends |(1/\(1\))| = \(\sigma_i \), Notice that the
estimates of \( \gamma \) and \( \phi \) then those
toward high and low frequencies, particularly
usually tends |(1/\(1\))| = \(\sigma_i \), Notice that the
estimates of \( \gamma \) and \( \phi \) then those
in Sections 5.2 and 6.3 about the interpretation of these
Phytre 6.7 illustrates many of the observations made
Phytre 6.7 illustrates many of the observations made
Phase maxima was found to be 28.3°. The mean value of these three
values calculated for each\( \gamma \). The mean value of these three
values calculated for each\( \gamma \). The mean value of these three

Phase maxima of the resulting mean-moment system was
the theory. Based on the theoretical computation of \( \gamma \), the
Phase maxima of the resulting mean-moment system was
the theory. Based on the theoretical computation of \( \gamma \), the
Phase maxima of the resulting mean-moment system was
the theory. Based on the theoretical computation of \( \gamma \), the
Phase maxima of the resulting mean-moment system was
the theory. Based on the theoretical computation of \( \gamma \), the

(6.4)
\[ a = \gamma \]
Figure 6.7a Estimation of $Y_p(j\omega)$, 30 mph; Rider A, Day 1.
improved when roll angle is employed. The transfer function estimates when $\phi$ is used and $\lambda=0$ are not shown but were found to be closer to the rider transfer function than $1/Y_c(j\omega)$. With $\phi$, however, the estimates for $\lambda=0$ are seen to be close to $1/Y_c(j\omega)$.

As seen from Figure 6.7, the dominant frequencies in the $t_\phi$ power spectra were about 2.5 radians/second. When Rider A was tested on the second day, the dominant frequencies were considerably higher, ranging from about three to nearly five radians/second. If a transfer function of the form of Equation (6.4) is assumed to represent the rider, nearly all of the test data are "explained" by the assumed model, when $K_p = 200\text{-}320\text{ lb-in/radian}$ and $\tau_p = 0.14\text{-}0.21$ second. However, one trial, in which the dominant frequency in the steering torque spectrum was 4.8 radians/second, could not be fitted with a transfer function consisting of a constant gain and time delay, since the value of $\tau_p$ that would be required is unrealistically small, about 0.1 second. Also, difficulty was experienced in matching the $t_\phi$ spectrum for this test. In this case, it was found necessary to include a lead factor in the rider's transfer function, which then had the form,

$$Y_p(j\omega) = -K_pe^{-\tau_p j\omega} \frac{j\omega}{T_p j\omega + 1}. \quad (6.5)$$

The strong indication of lead found in one test suggests that the behavior of Rider A during all trials on the second
data could only be interpreted with a transfer function. The first example is the transfer function which the data, 9 and 6 spectra, represent. The first transfer function represents the data of a, b, and c, and were evaluated at 0.30 second. The value of t was represented by the equation:

\[ \frac{d}{dt} \alpha(t) \]

To fit the data, the transfer function was represented by the equation:

\[ \frac{d}{dt} \alpha(t) \]

The value of t is the transfer function. The second test day could be represented by the equation:

\[ \frac{d}{dt} \alpha(t) \]

Thus, the transfer function is the transfer function that the value of t was determined that the human operator's time delay is less acute. The average value of t was with a transfer function that was conducted with other spectra and on the first day with the spectra. However, the spectra, this was not surprising that the data of the spectra could be obtained with either the transfer function of 9 or the 6, a good fit to most of the data. The transfer function of 9 was found that nearly equaled the transfer function of 6.
time delays were about 0.25-0.35 second, and the gates were
second (second was required on the part of the reader. Here the
tool and lead equalization (break frequency), 5-10 radians/
but the results indicated that a combination of rate con-

.1. Only three test runs were performed at 15 mph;

about 150 to 350 1D radian.

were about 0.3 second for all the tests, and gates were
frequency, in neighborhood of 5-10 radians/sec. Time delays
appropriation intensification of lead equalization having a break
was found to be a constant 0.3 at 0 mph, the reader transfer function, $V(\omega)$.

.2. The time shifting method [37] can be used to

when than at 15 mph.

from the motocycle equations of motion is better at 30

and the transfer function for the power spectra

and the reader function of the reader in the
test, and the steetning for the power spectra

these were varying degrees of passes

.2. The time shifting method [37] can be used to

from the motocycle equations of motion is better at 30

and the transfer function for the power spectra

and the reader function of the reader in the

the results of the uncorrected motocycle experiments

analyzed are sufficiently large. As would be expected from

values of the power spectra of the time histories because

analyzed.
of the filter applied to the data.

From about 1.5 to 5.0 radians/second (frequency varied) at a dominant frequency. The value of this frequency varied.

steering torque and roll rate exhibited a pronounced peak of transfer functions. At this level of stability, power spectra

margin of about 0.71 to were impacted by the estimated transfer

margin of about 0.71 to 0.75 and Gahn

7. System phase margins of about 90-70° and Gahn

5 MHz data filter used (break frequency: 5 radians/second). Hence, n(t) appears to be "white", at least relative to the

same filter that was applied to the original analog records.

The same shape as that of "white" noise passed through the

remnant. The power spectrum of n(t) was found to have about

6. Much of the filter's steering torque output was

mode also decreased, resulting in lead and rate control.

as speed decreased, the damped natural frequency of the weever

mode decreased and natural frequency is much greater than \( \omega_0 \).

At 30 mph, lead is optional, since the weever

frequency \( \omega_0 \) starts of -20 dp/decade near the crossover

rate control in \( \Omega \) are needed to establish or extend a

motorvolute system [16]. Verifying amounts of lead and/or

model, which has been used to study theoretically the man-

the experimental results agree with the crossover

150
8. Body lean control was not studied here. Although Weir [16] has determined theoretically that body lean is more suitable for path following control than roll stabilization, riding the motorcycle with the upper body braced has indicated that body control is by no means necessary in normal maneuvers at speeds greater than about 15 mph. Hence, body control is optional and its use is probably determined by the "style" of the individual rider.

It would, however, be interesting to conduct roll-stabilization experiments in which body lean is the only means of control available to the rider.
REFERENCES


37. Wingrove, Rodney C., and Edwards, Frederick G., "Measurement of Pilot Describing Functions From Flight Test Data With an Example From Gemini X"


<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE A.1</td>
<td></td>
</tr>
</tbody>
</table>

**A.1 DEFINITION OF SYMBOLS**

Motorcycle Equations of Motion

Appendix A
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{I}<em>{fy}$, $\bar{I}</em>{ry}$, $\bar{I}_{ey}$</td>
<td>Polar moments of inertia of front wheel, rear wheel, and engine, respectively.</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of rear frame, including wheel, engine, and rider.</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Mass of front frame, including wheel.</td>
</tr>
<tr>
<td>$M_{xf}$, $M_{xr}$</td>
<td>Front and rear tire overturning moments, respectively.</td>
</tr>
<tr>
<td>$M_{zf}$, $M_{zr}$</td>
<td>Front and rear tire self-aligning moments, respectively.</td>
</tr>
<tr>
<td>$R_f$, $R_r$</td>
<td>Front and rear wheel rolling radii, respectively.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time.</td>
</tr>
<tr>
<td>$t_\delta(t)$</td>
<td>External moment about the steering axis applied to the front frame assembly.</td>
</tr>
<tr>
<td>$t_\phi(t)$</td>
<td>External roll moment applied to the rear frame assembly.</td>
</tr>
<tr>
<td>$W_f$</td>
<td>Front tire vertical load.</td>
</tr>
<tr>
<td>$u$, $v$, $w$</td>
<td>Velocity of point O (Fig. A.1) with respect to XYZ axes.</td>
</tr>
<tr>
<td>$XYZ$</td>
<td>Right-handed axis system fixed in vehicle rear frame with origin at point O. See Figure A.1.</td>
</tr>
<tr>
<td>$X_fY_fZ_f$</td>
<td>Right-handed axis system fixed in vehicle front frame with origin at front frame center of mass. See Figure A.1.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Constant such that $\bar{I}_{ey} au = \text{angular momentum of engine about its spin axis.}$</td>
</tr>
<tr>
<td>$\alpha_f$, $\alpha_r$</td>
<td>Front and rear tire slip angles, respectively [51].</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Steer angle of vehicle.</td>
</tr>
<tr>
<td>$Y_f$, $Y_r$</td>
<td>Front and rear wheel inclination angles, respectively [51].</td>
</tr>
</tbody>
</table>
The following symbols allow the equations of motion to be written in a more concise form:

\[
\begin{align*}
\frac{\ddot{x}}{J_R} &= \frac{\ddot{\gamma}}{J_K} \\
\frac{\ddot{y}}{J_R} &= \frac{\ddot{\gamma}}{J_K} \\
\ddot{\gamma} &= \gamma \\
\gamma &= \cos \theta \\
\theta &= \phi \\
\gamma &= \phi \\
\gamma &= \phi \\
\gamma &= \phi
\end{align*}
\]

<table>
<thead>
<tr>
<th>RESPECTIVELY</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front and rear tire relaxation tensions</td>
<td>( \tau )</td>
</tr>
<tr>
<td>Steering wheel angle (ψ, T, A, I)</td>
<td>( \rho )</td>
</tr>
<tr>
<td>yaw rate of vehicle</td>
<td>( \dot{\phi} )</td>
</tr>
<tr>
<td>yaw (heading) angle of vehicle ([\phi]_T )</td>
<td>( \phi )</td>
</tr>
<tr>
<td>Roll angle of vehicle ([\gamma]_T )</td>
<td>( \gamma )</td>
</tr>
</tbody>
</table>

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
DEFINITION & SYMBOL \\
\hline
\end{tabular}
\end{table}
Figure A.1: Single-track vehicle: dimensions, masses and axis systems.
Fourth equation

Steering column friction is an additional moment in the self-attending torque appear in the second and fourth equations. These appear in the third and fourth equations, while the torque about the steering axis. Thus, the three overturning moments is a balance of moments acting on the front frame assembly (A.6)

balances about the z and x axes; respectively. Equation (A.6) balances about the x-direction, the second and third bending moment, the first bending force, and moment.

It is seen from the equations of motion that the first equation

below.

Friction additions into the equations of motion are outlined. The methods of incorporating the tire mechanisms and steering

resulting equations are presented at the end of this section.

Furthermore, the tire mechanisms and steering

have been rewritten to conform to the notation and axes systems.

The equations of motion derived in Reference [14] have

A.2 CONSTRUCTION OR A MORE COMPLETE TIRE MODEL
Furthermore, lateral force and self-aligning torque are dependent on the instantaneous path curvature of the tire. For a linearized approximation, it may be shown that the instantaneous path curvature of a point p in a body moving in a plane with velocity components \((u_p, v_p)\) relative to axes fixed in the body is given by \((u_p = \text{forward velocity} = \text{constant})\)

\[
\frac{1}{\rho} = \frac{\dot{v}_p}{u_p^2} + \frac{r_p}{u_p}, \quad \text{(A.1)}
\]

where \(r_p\) is the angular velocity of the body.

The linearized lateral velocities of the front and rear tires are

\[
v_f = \dot{v} - e\delta - \dot{h}_o \hat{\phi} + \ell_f \dot{r} \quad \text{(A.2)}
\]

\[
v_r = \dot{v} - \dot{h}_o \hat{\phi} - \ell_r \dot{r}
\]

Thus, from Equations (A.1) and (A.2) the path curvatures for the front and rear tires are given by

\[
\frac{1}{\rho_f} = \frac{\dot{v} - e\delta - \dot{h}_o \hat{\phi} + \ell_f \dot{r}}{u^2} + \frac{(r + \dot{\delta} \cos \sigma)}{u}
\]

\[
\frac{1}{\rho_r} = \frac{\dot{v} - \dot{h}_o \hat{\phi} - \ell_r \dot{r}}{u^2} + \frac{r}{u}
\]
\begin{equation}
0 = \mu \frac{\dd^2}{d^2} - \frac{\dd^2}{d^2} - \mu \frac{d^2}{d^2} + \phi \frac{d^2}{d^2} w + \phi \frac{d^2}{d^2} w - \mu \frac{d^2}{d^2} w + \lambda^2 w
\end{equation}

The resulting equations of motion are given below:

\begin{equation}
\begin{aligned}
\text{rear tire} & : & \mu \frac{d}{d} & = \frac{d}{d} \\
\text{front tire} & : & \mu \frac{d}{d} & = \frac{d}{d}
\end{aligned}
\end{equation}

This requires that

\begin{equation}
\text{the inclination angle} = \gamma
\end{equation}

where \( \gamma \) = the rolling radius, and

\begin{equation}
\begin{aligned}
\left( \frac{0}{\lambda} + \frac{d}{d} \right) z & = z \\
\left( \frac{0}{\lambda} + \frac{d}{d} \right) \phi & = \phi
\end{aligned}
\end{equation}

Force and attitudes resulting from path curvature and

From reference [41] approximate expressions for lateral
\[
\frac{n}{\alpha} + \frac{n}{(\frac{\alpha}{\gamma} - \phi^0 u - \Lambda)} = \frac{n}{\lambda}
\]

\[
\frac{n}{(\phi + \lambda)} + \frac{n}{(\frac{\alpha}{\gamma} + \phi^0 u - \phi - \Lambda)} = \frac{n}{\lambda}
\]

\[
\phi = \frac{n}{\lambda}
\]

\[
\phi + \phi = \frac{n}{\lambda}
\]

\[
\frac{n}{(\frac{\alpha}{\gamma} - \phi^0 u - \Lambda)} = \frac{n}{\lambda}
\]

\[
\phi - \frac{n}{(\frac{\alpha}{\gamma} + \phi^0 u - \phi - \Lambda)} = \frac{n}{\lambda}
\]

\[
(\frac{\alpha}{\gamma} + \frac{\alpha_d}{\lambda}) (\frac{\alpha}{\gamma} + \phi^0 u - \phi - \Lambda) = \frac{n}{\lambda}
\]

\[
(\frac{\alpha}{\gamma} + \frac{\alpha_d}{\lambda}) (\frac{\alpha}{\gamma} + \phi^0 u - \phi - \Lambda) = \frac{n}{\lambda}
\]

\[
\phi = \frac{n}{\lambda}
\]

\[
\phi + \phi = \frac{n}{\lambda}
\]
and the form

\[
\begin{pmatrix}
(4)x \\
(4)\phi \\
(4)\psi \\
(4)\chi \\
(4)\lambda
\end{pmatrix} = (4)x
\]

\text{(4) constants)}

\[
\begin{pmatrix}
(4)x \\
(4)\phi \\
(4)\psi \\
(4)\chi \\
(4)\lambda
\end{pmatrix} = (4)x
\]

where the elements of the $4 \times 4$ matrix $A$ have the form

\[
A \begin{pmatrix}
(4)x \\
(4)\phi \\
(4)\psi \\
(4)\chi \\
(4)\lambda
\end{pmatrix} = (4)x
\]

be written in vector-matrix form as follows:

The motion equations of motion (RDS, A.3)-(A.6) may have the following form:

A.3 TRANSFER FUNCTIONS
\[
L^T \Phi + \frac{2p}{3} L^T \Phi e + \frac{2p}{3} \frac{L^T}{6} \Phi + \frac{2p}{3} \frac{L^T}{2} \Phi = L^T \Phi
\]

where the elements of the \(4 \times 4\) matrix \(A^T\) have the form

\[(A, 8)\]

\[
A = I + \frac{2p}{3} \frac{L^T}{6} \Phi + \frac{2p}{3} \frac{L^T}{2} \Phi = \bar{p}
\]

If \(q^0\) and \(q^0\) may be written

\[
\phi \phi \phi \phi \phi \phi
\]

where each \(E^k\) is a linear combination of \(\phi, \phi, \phi, L^T\).

\[
\begin{pmatrix}
I + \frac{2p}{3} \frac{n}{L^T} \phi & I + \frac{2p}{3} \frac{n}{L^T} \phi \\
I + \frac{2p}{3} \frac{n}{L^T} \phi & I + \frac{2p}{3} \frac{n}{L^T} \phi \\
I + \frac{2p}{3} \frac{n}{L^T} \phi & I + \frac{2p}{3} \frac{n}{L^T} \phi \\
I + \frac{2p}{3} \frac{n}{L^T} \phi & I + \frac{2p}{3} \frac{n}{L^T} \phi
\end{pmatrix} = \bar{p}
\]
\[
\begin{pmatrix}
(s)\phi \\
(s)\chi \\
(s)\eta \\
(s)\lambda
\end{pmatrix}
= (s)\bar{X}
\]

\[
\begin{align*}
(\hat{f}_J \times (s)\eta))_e + s(s)_p + \zeta(s)_c &= \hat{f}_e \\
\hat{f}_j + s(s)_e + \zeta(s)_p + \zeta(s)_c &= \hat{f}_e
\end{align*}
\]

where the elements of \( \alpha \) are

\[
(\alpha \cdot \alpha)
\]

\[
\bar{a} = (s)\bar{X} \quad \alpha
\]

Finally, with the assumption of zero values of \( \alpha \), \( \phi \), and their derivatives at \( t=0 \), substitute Equation (A.8)

\[
\begin{pmatrix}
(\hat{t}^{\theta} \hat{t})_\theta (1 + \frac{2n}{p} \frac{n}{J_D}) \\
(\hat{t}^{\phi} \hat{t})_\phi (1 + \frac{2n}{p} \frac{n}{J_D}) \\
0 \\
0
\end{pmatrix}
= \bar{a}
\]

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\[ \det = \begin{vmatrix}
\frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right) & \frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right) \\
0 & \frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right) & \frac{n}{J_D} \\
0 & \left(1 + s \frac{n}{J_D} \right) & \frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right) \\
0 & \frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right) & \frac{n}{J_D}
\end{vmatrix} = \frac{(s)\phi(t)}{(s)\phi(t)} = (s)^2 \phi(t) = \lambda^2 \]

Rule: For example, the transfer functions may be easily computed by Cramer's rule for a tenth order polynomial in \(s\).

The characteristic function of the system is \(\det(a)\).

\((s)I(t) = (s)^2 \phi(t)\)

\[\left(\begin{array}{cc}
\frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right) \\
\frac{n}{J_D} & \left(1 + s \frac{n}{J_D} \right)
\end{array}\right) = \bar{B}\]

\(\phi(t) = I(t) (t) H(s)\) etc.

\(L(\phi(t)) = L(\phi(t))\) etc.

\((s) = L(\phi(t))\) Laplace transform of \(\phi(t)\)
element used extensively in Chapters 4-6. Yields $X_g(\omega)$, the transfer function of the "controlled" torque. Furthermore, letting $s \rightarrow j\omega$ in Equation (4.10) the roll angle response, $\phi(t)$, to an impulse of steering $
abla$.

The inverse Laplace transform of Equation (4.10) gives

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APPENDIX B

SOLUTIONS OF MOTION EQUATIONS

B.1 FLOWCHART, LINEAR ANALYSIS DIGITAL COMPUTER PROGRAMS

Read Motorcycle Parametric Data

Calculate Differential Equation Coefficients

Calculate det(A), the Characteristic Function

Find Roots of det(A)=0 (Transfer Function Poles)

Calculate Coefficients of Transfer Function Numerator Polynomials

(Continued next page)

Figure B.1 Flowchart
Let \( s = j\omega \)

\[ \text{in} \frac{\psi(s)}{\delta(s)} \]

Calculate \( Y_c(j\omega) \)

Find Inverse Laplace Transforms:
- \( v(t) \), \( r(t) \), \( \phi(t) \), Responses
- to a Step Roll Moment or an Impulsive Steering Torque

Plot Responses Against Time

Plot Bode Diagram of \( Y_c(j\omega) \)

Calculate Zeros of Transfer Function Numerators

Figure B.1 (Continued)
positive

where the constants $a_0, a_1, \ldots, a_n, \ldots$ are all

$$
q_p(\theta_T - \theta_e + \sum_{p} \phi_p - \phi^2_p - \phi^3_p - \phi^4_p - \phi^5_p + \phi^6_p - \phi^7_p - \phi^8_p) = 0
$$

$$
q_p(\phi_c + \phi^2_c + \phi^3_c + \phi^4_c + \phi^5_c + \phi^6_c = \phi
$$

$$
q_p(\phi^7_c + \phi^8_c + \phi^9_c + \phi^10_c + \phi^11_c - \phi^12_c - \phi^13_c = \phi
$$

$$
q_p(\phi^14_c - \phi^15_c - \phi^16_c - \phi^17_c - \phi^18_c = \phi
$$

The following form: can be written in the preparation of figures 2.5, 3.5 and 3.2-3.5.

as speed range of interest, the motor velocity, or mation, the motor velocity parameter.

An Appendix D, it was found that, for at least the

B.2 ANALOG COMPUTER CIRCUIT
A toe computer simulation of the motorcycle, dimensionally,
and the equations following equation (A.6), Appendix A. In the
attitude moments were simulated (Figure B.2b) directly from
equations on the analog computer. The lateral forces and
Figure B.2a shows the implementation of the above.

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Figure B.2d: Simulation of the lateral force

and a future moment.

\[ \frac{y}{y_0} = \frac{x_1}{J_1} \]

\[ \frac{n}{J_0} = \frac{x_1}{J_1} \]

\[ \frac{J_1}{J_1} \]

\[ \frac{J_{\text{CMZ}}}{J_{\text{CMZ}}} \]

\[ \phi \]

\[ \phi \]

\[ \phi \]

\[ \phi \]
APPENDIX C
MEASUREMENT OF VEHICLE PARAMETERS

C.1 MEASUREMENT OF MASSES, CENTER OF GRAVITY LOCATIONS, AND MOMENTS OF INERTIA

The overall vehicle/rider mass and the mass of the front system was determined by simple weighing. Although a second identical vehicle was available for disassembly so that the road vehicle would always be available for tests, total vehicle/rider measurements were made with the road vehicle.

With the single-track vehicle, the rider is a significant part of the total mass and its moments. To measure mass distribution, then, it is necessary to exercise much care to keep the rider's position relative to the vehicle as nearly constant as possible, both during an experiment and from one experiment to another. Because the rider was assumed in the theory to be a rigid body rigidly attached to the rear frame, it is also desirable to restrict his movements during the road tests. It is readily recognized that, even if the rider were encased in a plaster body cast, he would not be completely rigid, and his flexibility is always a source of error. To reduce this error, the motorcycle was fitted with a strong brace (see Fig. C.1) to help the rider maintain his position. While the brace cannot
two cables and finding their point of intersection.
and wheeled assembly was found by suspending the assembly from
different supports, and support positions provided experimental
throughout angles between 150°. Taking force measurements for
and the vehicle was then rotated about the pitch (y) axis
position, one angle was positioned directly below the c.e.
force required for equilibrium. In the case of c.e. vertical
force required for equilibrium, one of which loaded a scale to measure the
positions were found by restating the vehicle on the edges of
vehicle center of gravity vertical and longitudinal
and rider mounted in this frame.
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and rider mounted in this frame.
Figure C.2 Measurement of yaw moment of inertia.

Figure C.1 Measurement of roll moment of inertia.
Figure C.1 shows the experimental arrangement for measuring vehicle roll moment of inertia. The vehicle is resting on knife edges at the front and rear. A pair of calibrated coil springs located near the bottom of the frame was needed to ensure oscillatory motion, as the vehicle center of gravity was near the axis of rotation. It is worth noting that rider flexibility had a greater effect on roll measurements than on others; in fact, without the rider restraining brace, only a useless one or two cycles of oscillation would result. With the brace, ten to fifteen cycles were easily obtained. As mentioned before, the brace was used for all total vehicle measurements.

Yaw moment of inertia was found using the arrangement shown in Figure C.2. Here the vehicle is suspended at the front and rear by two cables. The vehicle was caused to oscillate about a vertical axis through its center of gravity.

Vehicle product of inertia was obtained by inclining the roll axis upward 37° from the horizontal. Use of shorter cables allowed the measurement of the vehicle moment of inertia about a vertical axis through the c.g. Use of the values obtained from all three experiments permitted the calculation of the vehicle product of inertia by means of axis rotation relationships.
A maximum value was estimated.

As would be expected, since the test vehicle does not have any form of steering damper, the friction level in the steering head is very low.

C.2 ESTIMATION OF COULOMB FrICTION IN THE STEERING HEAD

To the appropriate axes system.

the frame was subjected, and the result expressed relative to the system. The contribution to the moment of inertia due to the linear differential equation describing the experimental function was found from all known quantities. This function was written as a function of mass, geometry, spring constant and frequency of oscillation about the axes of rotation was written as a function for each moment of inertia measurement, the moment of inertia-dependent moment of inertia for the engine.

The moment was estimated from geometry. Taking all gear and product ratios of inertia of actuator, camshaft and transmission were obtained by the manufacturer. Moments and product of inertia of engine crankshaft and J axes. Moments of inertia of engine crankshaft and J axes in the assembly, thus permitting calculation of three axes in the assembly shown in Figure C.3. The front assembly was mounted below the transverse and caused to oscillate about pendulum shown in Figure C.3. The front assembly was

Moments of inertia of the wheels and front fork.
Picture 0. 4 Motorcycle tire mounted for testing.

Picture 0. 3 Torsional pendulum for measurement of moments.

Picture 0. 2 Inertia of wheels and front frame assembly.
the Honda OHV. The tire parameters were measured for the tires used on
the last four quantities were measured for the tires used on

Turning moment, and self-attaching torque is possible, and
resistance moment, lateral force, vertical force, over-
mounted. Measurement of the longitudinal force, steering
loads, slip and traction angles and is restricted by load
mounted allows the tire to be tested at various vertical
per second (rpm, g). The framework in which the tire is

the tire attained a simulated roadway moving at about 2 feet
per second using the HSH test bed. The tester, which holds
measured using the HSH test bed. The tester, which holds

C.3.1 Method of Measurement. The parameters were

C.3 Measurement of tire parameters.

An attempt was made to obtain more accurate values.
Friction had not affected the wheel on the computer results, no
steering head. Since the effect of this type of wheel on the

to be the maximum constant friction that could be in the
torque (about 1.2 Id-in) was taken

"breakaway" torque (about 1.2 Id-in) was taken

axle and gradually increased until the steering

a small spring scale, a torque was applied about the steering

equal the normal operating front wheel load. By means of
were applied to cause the vertical force from the cables to
zero. The vehicle X-axles were kept horizontal and weights
torque about the steering axles while the steer angle was

In the following way. The front of the vehicle was sus-

183
Figures C.5 and C.6 show lateral force as a function of slip inclination angle for the vertical loads existing in the road tests. Self-aligning torque is shown in Figure C.7 as a function of slip angle. Self-aligning torque dependence on inclination angle and overturning moments, due to their low levels, could not be accurately measured. Reference [24] contains some data on self-aligning torque as a function of inclination angle; these measurements were combined with information obtained from the tires tested at HSRI to provide estimates of tire coefficients. Overturning moments arising from slip angles were found to be less than 20 lb-in/degree and had a negligible effect on the analytical results; hence, they were taken to be zero. In Figure C.8, circular tire cross-sections are rotated through an inclination angle of 10° to estimate the amount of lateral motion of the vertical load (center of pressure in the contact patch). This distance, multiplied by the vertical load, approximates the tire overturning moment to a degree consistent with measurements, as shown in Figure C.9.

C.3.2 MEASUREMENT OF TIRE RELAXATION LENGTH. The gradual build-up of tire side force due to a step input of slip angle is shown in Figure C.10. These curves were obtained by loading the tire against the bed with a fixed slip angle and manually cranking the bed to record lateral force versus
Figure C.5: Measured tire lateral forces as functions of slip angle.

Slip Angle, degrees

Lateral Force, lb

-15 -10 -5 0

200 lb. vertical load
25.5 psi
5.75 x 18
Front tire

31.3 deg = 1.790 rad

300 lb. vertical load
28.5 psi
3.25 x 18
Rear tire

55.9 deg = 3.200 rad

185
Figure C.6 Measured tire lateral forces as functions of inclination angle.
as functions of slip angle.

Figure C.7 Measured tire self-aligning moments

Degrees

Slip

200 lb vertical load

25.5 psi
Front tire

300 lb vertical load

28.5 psi
Rear tire

52.0 deg 10^-1 in/deg = 2980 10^-1 in/rad

10^-1 in/rad = 1750 10^-1 in/deg

30.0 deg 10^-1 in/deg
Figure C.8 Geometrical estimation of tire overturning moment due to inclination angle.
Figure C.9 Measured overturning moments due to

Inclination angle, degrees

-238 to +14/\text{rad}

Inclination angle, degrees

Measured

Moment, 1p-t\text{in}

Overturning

\text{80} \quad \text{72} \quad \text{64} \quad \text{56} \quad \text{48} \quad \text{40} \quad \text{32} \quad \text{24} \quad \text{16} \quad \text{8} \quad \text{0} \quad \text{-8} \quad \text{-16} \quad \text{-24} \quad \text{-32} \quad \text{-40} \quad \text{-48} \quad \text{-56} \quad \text{-64} \quad \text{-72} \quad \text{-80}
Figure 0.10 Tire lateral force response to a step slip angle.

Distance Rolled, inches

Front

Rear
observed to follow the same curve as the lateral force.

Self-Attenuating torque arising from slip angle was

the equation of motion.

the validity of the assumption would affect the accuracy of

the results to relaxation tendon; thus, it was not felt that

any influence on the modes of motion that were

was found that the forces and moments due to interaction

may not be the case. However, from the computer results, it

response is identical to that due to a step slip angle; this

matched analaysis of the single-track vehicle that this

step interaction angle, while it was assumed in the mathe-

force, self-attenuating torque and overturning moment due to a

ments, it was not possible to observe the build-up of lateral

the test device and the low levels of some of the measure-

It is worth noting that, due to the construction of

Jensen for the tire.

a curve of the form $x = \frac{e^{-\frac{x}{a}} - x^2}{2a^2}$, where $a$ is the relaxation
distance trailed (x). The data points were then fitted with
For the vehicle without the weight, the root locus plot in Chapter 2 uses the values for no weight. The values for the twelve-pound weight for the twelve-pound weight, data for the six-pound weight, and data for the twelve-pound weight described in Chapter 3 were added to the vehicle. Data for the six-pound weight are not given; parentheses indicate the values (where changed) after the torque bar, the effect of which is very small. Numbers in parentheses indicate the values of measured parameters for the Honda CR 75 with rider, rider restraining brace, full-
<table>
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<tr>
<th>SYMBOL</th>
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</thead>
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<td>$I_{xz}$</td>
<td>800 (867) lb·sec$^{-2}$·in</td>
</tr>
<tr>
<td>$I_{zx}$</td>
<td>58 (55.5) lb·sec$^{-2}$·in</td>
</tr>
<tr>
<td>$I_{sz}$</td>
<td>3.83 lb·sec$^{-2}$·in</td>
</tr>
<tr>
<td>$I_{sxz}$</td>
<td>-0.06 lb·sec$^{-2}$·in</td>
</tr>
<tr>
<td>$I_{zy}$</td>
<td>3.22 lb·sec$^{-2}$·in</td>
</tr>
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<td>$I_{txy}$</td>
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</tr>
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<td>$K_F$</td>
<td>4.29 lb·sec$^{-2}$·in</td>
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<td>$K_r$</td>
<td>0.086 in·l</td>
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<td>$m_t$</td>
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<td>$l_{198.5}$ (197)</td>
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<td>$g_f$</td>
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<tr>
<td>$g_r$</td>
<td>2.1 in</td>
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<tr>
<td>$g_f$</td>
<td>1.5 in</td>
</tr>
</tbody>
</table>

<table>
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<td>$C_s$</td>
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<tr>
<td>$C_{af}$</td>
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<tr>
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</tr>
<tr>
<td>$C_{myt}$</td>
<td>0</td>
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<tr>
<td>$C_{mx}$</td>
<td>1720 lb·in/rad</td>
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<tr>
<td>$C_{mxr}$</td>
<td>1720 lb·in/rad</td>
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<tr>
<td>$C_{mxt}$</td>
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<tr>
<td>$C_{mzr}$</td>
<td>-421 lb·in/rad</td>
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<tr>
<td>$C_{mzt}$</td>
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<tr>
<td>$C_{zy}$</td>
<td>50 lb·in/rad</td>
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<td>$C_{zyr}$</td>
<td>26.5 lb·in/rad</td>
</tr>
<tr>
<td>$C_{zyt}$</td>
<td>2.97 in</td>
</tr>
<tr>
<td>$C_{zr}$</td>
<td>27.3 in</td>
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<tr>
<td>$C_{zr}$</td>
<td>5.37 in</td>
</tr>
<tr>
<td>$e$</td>
<td>-60 (-.54) in</td>
</tr>
<tr>
<td>$h_0$</td>
<td>10.1 lb·in</td>
</tr>
<tr>
<td>$h_s$</td>
<td>40.3 in</td>
</tr>
<tr>
<td>$h_t$</td>
<td>4.975 in</td>
</tr>
<tr>
<td>$l_s$</td>
<td>2.13 in</td>
</tr>
<tr>
<td>$l_t$</td>
<td>386 in/sec$^2$</td>
</tr>
<tr>
<td>$I_{xt}$</td>
<td>332 (342) lb·sec$^{-2}$·in</td>
</tr>
</tbody>
</table>
APPENDIX E

INSTRUMENTATION DIAGRAMS, ROLL-STABILIZATION EXPERIMENTS

Figure E.1 Instrumentation schematic.
Figure 6.8a  Estimation of $Y_p(j\omega)$, 30 mph; Rider A, Day 2; the test in which $Y_p(j\omega)$ evidenced the most lead.
containing lead equalization.) The $\hat{Y}_p(j\omega)$ Bode diagrams derived from data taken on the second day seemed to contain less bias and a lower level of instability than other sets of calculated results for $\hat{Y}_p(j\omega)$, possibly because the relatively high frequency content of the records obtained on the second day pushed the range of "good" identification to higher frequencies, where bias and instability effects are generally most noticeable.

The transfer function obtained for Rider A using data collected during the second test day was found to be (on the average)

$$\bar{Y}_p(j\omega) = -261 e^{-\cdot3j\omega} (0.155j\omega + 1)(\frac{lb-in}{radian}). \quad (6.6)$$

The transfer function of the controlled element, as computed from theory, together with the estimates of $Y_p(j\omega)$, yielded an average system phase margin of 48.6°.

The three 15-mph trials performed with Rider A on the first day were obviously not intended to be the basis for a comprehensive study of rider control activities at low speeds. However, the results of these tests do give a fairly good indication of how the rider must change his method of control as speed decreases.

In estimating $Y_p(j\omega)$ from the data collected at 15 mph, it was found that (1) the same results could be obtained regardless of whether roll angle or roll rate was employed in the analysis, and (2) when $\lambda$=0, the estimated transfer function tended toward $-Y_p(j\omega)$ rather than $\frac{1}{Y_c(j\omega)}$. Figure
were predicted from the phase diagram. However, in much better agreement with the values that were obtained are the parameters of about 30°. The resulting parameter values for the system are given in section 6.2, and the parameters of the cross-spectral estimate of \( \alpha Y_d \) were constructed where the errors were indicated by the phase diagram. Because of this, several corrections were made to the phase values of about 30°. However, the phase values necessary to fit the expected data were obtained from the experiment. The spectrum of the data was chosen to fit the data. It was possible to fit the theoretical

\[
\alpha Y_d = (\alpha Y_d) Y_d
\]

function of the form, lead at relatively high frequencies. Hence, a transfer function also indicates the usual time delay and possesses a derivative in the frequency range of interest here. Rate control is impacted well below 0.5 radian/second, but it exists at all for the frequency range, the break frequency of which is approximately 6, as shown in Figure 6.9 shows a sample estimate of \( \alpha Y_d \) when \( Y = 0 \).
Figure 6.9. Estimation of $V^p(f \omega)$, 15 mph.

$\omega$, radians/second

t

$\frac{T_0(f \omega)}{\phi(f \omega)}$, degrees

$\frac{T_0(f \omega)}{\phi(f \omega)}$, lb-in

$\frac{P_f(f \omega)}{\phi(f \omega)}$, db
are shown in Figure 6.10 for each of the three estimates of

\[ \frac{\overline{V}}{m} = \frac{\overline{V}}{m} \]

The average estimates, calculated for the crossover test, estimated values of \( m \) and \( t \) for the crossover test, and the crossover test results are consistent with the theoretical analysis of the model. The results of the preceding two sets of 30 mph tests based on the experimental \( (\overline{V}) \) are shown in Figure 6.10. A result that indicates a system phase margin of about 25° at 15 mph was obtained for the rider.
Figure 6.10a: Cross-over model approximations to

\[ \gamma \left( \frac{1}{s} \right) = \frac{10}{s^2 + 5s + 10} \]

\[ \gamma \left( \frac{1}{s} \right) = \frac{10}{s^2 + 5s + 10} \]

15 mph

30 mph

\[ \gamma \left( \frac{1}{s} \right) = \frac{10}{s^2 + 5s + 10} \]

\[ \gamma \left( \frac{1}{s} \right) = \frac{10}{s^2 + 5s + 10} \]
From the point of view of the crossover model, the ideal break frequency of the second degree torus is reduced to between 0.5 radians/sec and 1.5 radians/sec. The average value of $T/\omega$ can be made to be 8.5 radians/sec. It may be that the reader tends to select this amount of lead on the average.

For such a frequency (e.g. $\varphi$), $\frac{dx}{dt} = \alpha x$ would be required for a crossover frequency greater than the damped natural frequency of the wave mode, whereas the slope

$$\alpha x \frac{dx}{dt} = \varphi$$

that a richer transfer function closer to the form,

to be useful for the theoretical $\alpha$ at 15 mph, it would be expected rate control and lead were required. In fact, from consideration of the effect of lead control and Lead were required. It was seen that lead equalization was optional. For 15 mph, both rate control on the part of the richer, and frequency decreases, requiring various degrees of lead and frequency decreases, however, the wave mode damped natural frequency will have this slope for a wide range of frequencies. Thus,

$$\alpha x \frac{dx}{dt} = \varphi$$

In $T/\omega$ is reduced to 20 dB/decade for a broad range of frequencies. Thus, in speeds such as 30 mph, since the slope of the crossover frequency requirement is easily met for moderate and high model error, it is necessary for the slope of

$$\alpha x \frac{dx}{dt} = \varphi$$

to be close to

$$\alpha x \frac{dx}{dt} = \varphi$$

for the crossover model, it is
2 radians/second). However, the experimental $|Y_c(j\omega)|$
slope seems to be less steep (between -60 and -40 dB/decade),
perhaps due to different locations of the capsize mode break
frequency and the damped natural frequency of the weave mode
than those locations predicted by the theory. For the
experimental $Y_c(j\omega)$, a rider transfer function of the form
of Equation (6.7) is sufficient to bring the slope of
$|\hat{Y}_p(j\omega)\hat{Y}_c(j\omega)|$ close to -20 dB/decade at the crossover
frequency.

The constant gain and time delay form of $Y_p(j\omega)$ is a
good description of the motorcyclist's method of controlling
the roll angle throughout a speed range in which the motor-
cycle usually operates (speeds greater than, say, 25 mph).
This transfer function also lends itself readily to intuitive
interpretation. Basically, if the rider wishes to change
the roll angle, he applies a steering torque to the handlebars
of opposite sign to the direction of the desired change. For
example, if the machine is falling to the right ($\phi>0$), the
rider applies a positive (right) steering torque, which
causes the tires to sideslip and produce positive forces on
the vehicle, forces which roll the vehicle in the negative
$\phi$ direction. Although negotiating turns was not studied here,
experience [19] has indicated the same behavior at least in
a qualitative sense: to enter a right turn, the rider first
applies a negative (left) steering torque to "set up" the needed lean to the right. Likewise, leaving the right turn requires a positive (right) torque to zero the roll angle.

6.5 TRANSFER FUNCTIONS FOR OTHER RIDERS

Riders B and C were tested at 30 mph only, on one day per rider. In general, the results of testing these two riders were very similar to each other and were more like the results obtained in the 30 mph tests conducted with Rider A on the first day rather than on the second day.

Specifically, the dominant frequencies in the power spectra of steering torque were relatively low, both for Riders B and C. For Rider B, these frequencies ranged from about 1.5 to 2.5 radians/second. For Rider C, they ranged from about 1.7 to 2.7 radians/second. Also, the $t_\hat{\omega}$ spectra of both riders tended to be more flattened (the peak less pronounced) than those of Rider A.

From Bode diagrams of $\hat{Y}_p(j\omega)$ and the $t_\hat{\omega}$ spectra, it was determined that a constant gain and time delay was a good fit to the transfer functions exhibited by Riders B and C. Sample Bode diagrams of $\hat{Y}_p(j\omega)$ and $t_\hat{\omega}$ spectra are shown in Figures 6.11 and 6.12. As expected, the identification of $Y_p(j\omega)$ tended to be poorer when roll rate was used in the analysis,
Figure 6.1la  Estimation of $Y_p(j\omega)$, 30 mph; Rider B.
Figure 6.11b  Estimation of $Y_p(j\omega)$, 30 mph; Rider B. Extreme example of excessive high frequency phase lag in $Y_p(j\omega)$.
Figure 6.12b  Estimation of $Y_p(j\omega)$, 30 mph; Rider C.
b reduced the error level for simply attenuating its remnant. Hence, it is felt that rather than using different values between tests, this did not lead to much higher error levels. It was found to remain consistent choice of \( V \). It was, therefore, decided to some extent upon this.

While this error level depended to some extent upon the torque bars, which did not bother the other riders. Apparently, the steering angles of the test runners, etc. (Table 6.1). Furthermore, the mean of the phase margins f g 3.9° ± 10°. and the average phase margin for rider \( g \) was 3.9° ± 10°.

\[
\theta_{\text{average}} = 206^\circ \pm 30^\circ
\]

Average transfer function exhibited by rider \( g \) was

Similarly, the transfer function obtained for rider \( d \) was 4.9° ± 10°. Estimates for rider \( b \) was 4.4° ± 10°. The mean of the phase margins resulting from the individual tests:

\[
\theta_{\text{average}} = 167^\circ \pm 33^\circ
\]

The average transfer function obtained for rider \( a \), rather than roll angle, although the difference was not as great as was noted with respect to data produced by rider \( a \).
It is interesting to note that most of Rider B's motorcycle experience was trail riding, while the other riders received their experience on the road. Due to large ground disturbances, trail riding probably requires "tighter" control of the motorcycle than road riding. Hence, Rider B was apparently tending to carry his off-road techniques over to on-road riding.

While system gain margins were not calculated for all of the tests, the gain margins that were calculated give an idea of their relative sizes. The lowest gain margin calculated for the 30 mph tests were about 4 db (Rider A, first day). Rider A, on the second day, and Rider C produced gain margins of about 6 db. Rider B, with gain margins of about 9 db, probably was the most conservative rider, even though his phase margins were lower than those of Rider A on the second day.

Table 6.2 summarizes the rider transfer functions obtained. In this table, the confidence intervals for the means of the $Y_p(j\omega)$ parameters and the phase margins assume that the estimators of those means are unbiased. The variability in transfer function estimates implied by the

\footnote{If X is a random variable and $E(X) = \mu$, then $\bar{X}$ is an unbiased estimator of $\mu$ if $E(\bar{X}) = \mu$. ($E(X)$ means "expected value of X."
### TABLE 6.2
SUMMARY OF RIDER TRANSFER FUNCTIONS

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<thead>
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<th>Day of Test</th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>Speed, mph</td>
<td>15</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Rider</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Form of</td>
<td>( -K_p j\omega e^{-\tau_p j\omega} )</td>
<td>( -K_p j\omega e^{-\tau_p j\omega} )</td>
<td>( -K_p j\omega e^{-\tau_p j\omega} )</td>
<td>( -K_p j\omega e^{-\tau_p j\omega} )</td>
<td>( -K_p j\omega e^{-\tau_p j\omega} )</td>
</tr>
<tr>
<td>( \hat{y}_p(j\omega) )</td>
<td>( \frac{1}{(T_x j\omega + 1)} )</td>
<td>( \frac{1}{(T_x j\omega + 1)} )</td>
<td>( \frac{1}{(T_x j\omega + 1)} )</td>
<td>( \frac{1}{(T_x j\omega + 1)} )</td>
<td>( \frac{1}{(T_x j\omega + 1)} )</td>
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#### Mean Values of Estimated Quantities

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<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
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<tr>
<td>( K_p ), lb-in</td>
<td>75.6</td>
<td>277</td>
<td>261</td>
<td>167</td>
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<tr>
<td>( \tau_p ), seconds</td>
<td>0.300</td>
<td>0.298</td>
<td>0.300</td>
<td>0.328</td>
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<td>( \frac{1}{T_x} ), radians/second</td>
<td>9.04</td>
<td>6.45</td>
<td>48.6</td>
<td>41.4</td>
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<tr>
<td>( \phi_m ), degrees</td>
<td>25.0</td>
<td>28.3</td>
<td>48.6</td>
<td>41.4</td>
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</tbody>
</table>

#### 90% Confidence Intervals

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<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p ), lb-in</td>
<td>(53.6, 97.6)</td>
<td>(219, 335)</td>
<td>(236, 286)</td>
<td>(152, 182)</td>
</tr>
<tr>
<td>( \tau_p ), seconds</td>
<td>(.216, .384)</td>
<td>(.260, .336)</td>
<td>(.287, .369)</td>
<td>(.276, .322)</td>
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<tr>
<td>( \frac{1}{T_x} ), radians/second</td>
<td>(3.37, 11.42)</td>
<td>(5.27, 7.41)</td>
<td>(42.4, 54.8)</td>
<td>(36.7, 41.5)</td>
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<tr>
<td>( \phi_m ), degrees</td>
<td>(15.2, 41.4)</td>
<td>(42.4, 54.8)</td>
<td>(36.7, 41.5)</td>
<td>(36.7, 41.5)</td>
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</table>

#### Parameters for Crossover Model, Based on Average \( \hat{y}_p(j\omega) \)

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<th>Day 2</th>
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<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_c ), radians/second</td>
<td>3.2</td>
<td>2.0</td>
<td>1.9</td>
<td>1.3</td>
</tr>
<tr>
<td>( \tau_c ), seconds</td>
<td>0.36</td>
<td>0.53</td>
<td>0.39</td>
<td>0.66</td>
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</table>

#### Average Estimated Linear Coherence

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<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\rho} )</td>
<td>0.61</td>
<td>0.57</td>
<td>0.335</td>
<td>0.17</td>
</tr>
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</table>
the confidence intervals lumps together (1) actual changes in rider behavior between tests under the same condition, (2) errors in identification and spectral estimation, and (3) errors in curve-fitting.

The confidence intervals were constructed as follows. Let \( x \) be a random variable that is measured in an experiment (such as \( K_p, \tau_p, \) etc.), and let \( \mu = E(X) \). If there are \( n \) such measurements, \( x_1, i=1, 2, \ldots n, \) the \( x_i \) are independent and normally distributed, and \( \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \) and
\[
s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2
\]
are the sample mean and variance, respectively, then it may be shown [50] that \( \frac{(\bar{x} - \mu)\sqrt{n}}{s} \) follows a \( t \)-distribution with \( n-1 \) degrees of freedom. To apply this fact to the motorcycle test results, it is assumed that the \( x_i \) are independent. To test for a normal distribution, the \( x_i \) and functions of the \( x_i \) were plotted on normal probability paper, a straight line indicating normality [49]. The following random variables were found to approximately follow a normal distribution: \( K_p, \tau_p, \) and \( \text{antilog}_{10}(\frac{1}{10T_L}) \). Phase margin (\( \phi_m \)) followed a normal distribution except for the Rider B data. When the sample size was only three, it was assumed that \( K_p, \tau_p, \text{antilog}_{10}(\frac{1}{10T_L}) \) and \( \phi_m \) were normally distributed.

Usually, when human operator transfer functions are determined experimentally (e.g., [26]), the disturbances
correlation function or power spectrum of the estimated
of the estimated, however, the more biased the estimated
and the wider the bandwidth.

The remnant spectrum gives an indication of the degree
functions for these records were more difficult to identify.

b and c, especially b, and may be a reason why transfer
about 0.4 to 0.8. The RMSE tended to be higher for others
where the tests performed, the average RMSE is very large.

In the area under the W power spectrum (which is remnant),
the estimated RMSE = \[ 1 - \frac{d}{2} \text{ when } X = T \text{ indicates the }

6.6 Remnant Estimates

of \( z \) are on the order of 0.15.

approximately created data is possible, even though the values
identical to the transfer function from the reference values of \( p \) shown in Table 6.2 are considered. Thus,

expected that \( p \) should approach unity, since a value of one
the greater disturbance to the system, it would not be
present study, however, when the operator's remnant is
output is linearly correlated to his error signal. In the
external to the operator, represent the largest fluctuation

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remnant does not in practice give a good indication of the
degree of bias in $\hat{Y}_p(j\omega)$, but the bias can be qualitatively
estimated by comparing the parameter estimates based on the
Bode diagrams of $\hat{Y}_p(j\omega)$ with those estimates based on the
t₆ spectra.

On the basis of the remnant estimates and subsequent
spectral analysis of these estimates, it was found that the
power spectrum of $\hat{n}(t)$ was approximately

$$S_{nn}^{\hat{n}}(\omega) = \frac{\text{Constant}}{25 + \omega^2}, \quad (6.9)$$

especially when roll rate has been used in the analysis.
Some example remnant spectra are shown in Figure 6.13. Only
the constant in Equation (6.9) was obviously rider-dependent
or speed-dependent. Thus, it appears that with respect to
the analog filter applied to the data, the remnant is indeed
"white". Since a "white" remnant is ideal from the point
of view of removing bias from $\hat{Y}_p(j\omega)$, it is possible that,
say, a 10 radians/second filter would have produced better
results. On the other hand, permitting higher frequencies
in the data degrades the accuracy of the impulse response
method. It appears that a study of the effects of data
filtering on the use of the time shifting method would be
very useful.
Figure 6.13: Estimated remanent spectra.

$\omega$, radians/second

$\frac{25 + \omega^2}{25}$

\[ \text{SN} : 10^{-2} \text{ in}^{-2} \text{sec}^{-1} \text{ rad}^{-1} \]

Legend:
- C
- B
- A
- V
- △

80% confidence intervals

\[ \omega = 38 - 30 \]

Recorded analysis in used

Symbols and indices
of such speed ranges in the neighborhood of 15 mph. Cycle equations of motion, however, predict the existence of the damper of the weave mode was near zero. The motor-speed range of complete stability or a speed range in which

3. Experimentally, it was not possible to locate a better agreement between theory and experiment.

c characteristics of the pneumatic ride is needed to produce a more realistic representation of the dynamic descrtive steering motion below about 35 mph. Most of these equations of motion do not account for 0 to 10 mph

these modes of motion, except at very low speeds (on the order of the equations of motion derived by Sharp [74].

The summarizing remarks can be made.

- On the basis of the experimental and theoretical studies which have not been previously investigated.
- (2) Study aspects of the dynamics of the motorcycle and the man-motorcycle system, and relate them to the theoretical studies of both the descriptive results presented are the results of the research described in this

7. CONCLUSIONS
4. Inclusion of tire lateral force and aligning moment arising from instantaneous curvature of the path of the contact patch chiefly affects the wobble mode, but gives no significant improvement in the experimental correlation.

5. Tire overturning moments and aligning torques due to tire inclination tend to destabilize and stabilize, respectively, the capsize mode. If both are included in the tire model, the effects almost exactly cancel each other, at least for the motorcycle and tires studied in this investigation.

6. A theoretical evaluation of the relative merits of coulomb friction and viscous steering dampers shows the advantage to be with the latter. The viscous damper does not decrease roll stability, while the coulomb friction damper does. Also, the effectiveness of the coulomb friction damper is dependent upon the magnitude of the steering disturbance, and the effectiveness of the viscous damper is not.

The analysis of roll-stabilization experiments, in which the three experienced riders stabilized the vehicle by applying a steering torque to the handlebars, yielded several important results, which are listed below.

1. During normal operation on a paved road in good condition, the primary source of excitation to the man-motorcycle system is the rider's "remnant". This fact allows accurate identification of the transfer function of the controlled element \( Y_c(j\omega) \) by open-loop cross-spectral
Figure E.2 Strain gauge circuit for the measurement of steering torque.

Temperature compensation:

- d: dummy strain gauges
- a: active strain gauges

Gain=100

Amplifier

Differential amplifier

Adjustment

Zero
In this discussion, a random process $w(t)$ possesses a linear function of the data. New time series were derived as follows:

where $h$ is the constant time interval between samples, $k = 0, 1, \ldots, K + 1$, $x$ is the sampled version of $x(t)$ and $y_i(t)$ is the sampled version of $y_i(t)$. In the sampled system, the input and output, respectively, are the experimental data.

Power spectra, cross-power spectra, and transfer functions from the estimation of auto- and cross-correlation functions, were used for the estimation of auto- and cross-correlation functions. A program prepared by the Statistical Research Laboratory (SRL) of the University of Montana, based on the program prepared by the Statistical Research.

Appendix A

IDENTIFICATION OF TRANSFER FUNCTIONS: COMPUTER PROGRAMS

P.T. CROSS-SPECTRAL ANALYSIS
\[ x(\text{k}h) = x'(\text{k}h) - a_0 - a_1 \text{k}h, \]
\[ y(\text{k}h) = y'(\text{k}h) - b_0 - b_1 \text{k}h, \]

where \(a_0, a_1, b_0\) and \(b_1\) were determined by performing least squares linear regression analyses on the input and output series.

The quantities of interest were then estimated as follows.

1. Autocovariance function:

\[ \hat{C}_{xx}(\text{l}h) = \frac{1}{K-k_0+1-k} \sum_{k=k_0}^{K-l} x(\text{k}h)x[(k+\text{l})h], \quad l=0,1,2,\ldots L \]

2. Autocorrelation function:

\[ \hat{r}_{xx}(\text{l}h) = \frac{C_{xx}(\text{l}h)}{C_{xx}(0)}, \quad l=0,1,2,\ldots L \]

3. Cross-covariance function:

\[ \hat{C}_{xx}(\text{l}h) = \frac{1}{K-k_0+1-k} \sum_{k=k_0}^{K-l} x(\text{k}h)y[(k+\text{l})h], \quad l=0,1,2,\ldots L \]

\[ \hat{C}_{xy}(-\text{l}h) = \frac{1}{K-k_0+1-l} \sum_{k=k_0}^{K-l} x[(k+\text{l})h]y(\text{k}h) \]
4. Cross-correlation function:

\[ r_{xy}(\ell h) = \frac{\hat{C}_{xy}(\ell h)}{\sqrt{\hat{C}_x(0)\hat{C}_y(0)}} \]

5. Auto-spectrum:

a. Unsmoothed estimate:

\[ \hat{S}_{xx}(\frac{n\pi}{Lh}) = \frac{2h}{\pi} \sum_{\ell=0}^{L} \epsilon_{\ell} C_{xx}(\ell h) \cos \frac{n\ell\pi}{L} \], \( n=0,1,...,L \),

where \( \epsilon_{\ell} = \begin{cases} 1, & 0 < \ell < L \\ 1/2, & \ell = 0, L \end{cases} \)

b. Spectral estimate as smoothed by "hamming":

\[ \tilde{S}_{xx}(0) = 0.54 \, S_x(0) + 0.46 \, S_x(\frac{\pi}{Lh}) \]

\[ \tilde{S}_{xx}(\frac{n\pi}{Lh}) = 0.23 \, S_x \left[ \frac{(n-1)\pi}{Lh} \right] + 0.54 \, S_x \left( \frac{n\pi}{Lh} \right) \]

\[ + 0.23 \, S_x \left[ \frac{(n+1)\pi}{Lh} \right], \quad 0 < n < L \]

\[ \tilde{S}_{xx}(\frac{\pi}{h}) = 0.54 \, S_x(\frac{\pi}{h}) + 0.46 \, S_x \left[ \frac{(L-1)\pi}{Lh} \right] \]
Transfer Function:

The estimate of smoothed estimates \( x_{s} \) and \( \hat{x}_{s} \) averaged as in the case of the auto-spectrum, to

"hamming" as is the case of the auto-spectrum to

The estimates \( x_{s} \) and \( \hat{x}_{s} \) were then smoothed by

where the \( a_{e} \) are as previously defined and \( n=0,1,2,\ldots \).

\[
\frac{T}{u} \cos \left[ \left( \frac{\pi}{2} - \left( \frac{\pi}{2} \right) \right) \right] = \frac{T}{u} \cos \left[ \left( \frac{\pi}{2} + \left( \frac{\pi}{2} \right) \right) \right] = 0
\]

\[
\frac{T}{u} \cos \left[ \left( \frac{\pi}{2} - \left( \frac{\pi}{2} \right) \right) \right] = \frac{T}{u} \cos \left[ \left( \frac{\pi}{2} + \left( \frac{\pi}{2} \right) \right) \right] = 0
\]

And the quadrature spectrum \( x_{s} \) were calculated by

where the unsmeothed estimates of the co-spectrum \( I \) of

\[
\left( \frac{u}{u} \right) x + \left( \frac{u}{u} \right) x = \left( \frac{u}{u} \right) x
\]

Cross-Spectrum:
spectrum detail (resolution of peaks) and less bias can be
variance and resolution of the spectrum estimation. Greater
window employed in a spectral analysis influences the bias,
as discussed in [55], the bandwidth of the spectral
is no great handicap.

Hence, restriction to a "hamming" window
the shape of a spectral window is much less important than
"hamming" spectral window. However, as indicated in [55],
the spectral analysis program restricts the user to a
parameters is now discussed.

The methods of selection of these
parameters, L, the number of samples, h, the time step, and
parameters of the program, these important
To use the spectral analysis program, three important

\[ \begin{align*}
& \left( \frac{\partial^2}{\partial \nu^2} \right)_{xx} + \left( \frac{\partial^2}{\partial \nu^2} \right)_{xx} = \left( \frac{\partial}{\partial \nu} \right)_{xx} \\
& \left( \frac{\partial}{\partial \nu} \right)_{xx} \\
& \left( \frac{\partial}{\partial \nu} \right)_{xx}
\end{align*} \]

Squared Coherence:

\[ \text{frequency} \frac{\partial}{\partial \nu} \text{radians/second} \]

System having x as input and y as output, for the
phase, respectively, of the transfer function for the
where and \( \left( \frac{\partial}{\partial \nu} \right) \) and \( \left( \frac{\partial}{\partial \nu} \right) \) are the estimated gain and
achieved by the use of a narrow bandwidth. However, a wide bandwidth gives the spectral estimator less variance. In general, selection of the window bandwidth depends upon the nature of the spectrum (degree of smoothness), plus the features of the spectrum which are of interest.

The bandwidth of the "hamming" spectral window is given by [55]

\[
b = \frac{1}{\int_{-\infty}^{\infty} w^2(u) du},
\]

where [56]

\[
w(u) = \begin{cases} \frac{.54 + .46 \cos \frac{\pi u}{Lh}}{Lh}, & |u| \leq Lh \\ 0, & |u| > Lh \end{cases}
\]

Hence,

\[
b = \left[ \int_{-Lh}^{Lh} \left( \frac{.54 + .46 \cos \frac{\pi u}{Lh}}{Lh} \right)^2 du \right]^{-1} = \frac{1.256}{Lh}
\]

(F.1)

Once \( h \) has been chosen, the bandwidth \( b \) is seen to be determined by the choice of \( L \), the number of lags used in the estimation.

The choice of the time step \( h \) is also important in a spectral analysis, and must be chosen with reference to the analog filter which has been applied to the signal before digitizing. The highest frequency (the Nyquist or
\[
\sum_{n=0}^{\infty} \frac{e^{-n^2}}{n!} = 2
\]

A window with a smoothed spectral estimator window is given by

\[\text{The number of statistical degrees of freedom associated} \]

respectively.

Angular frequencies are 2.5 and 2.0 cycles/second, for which the
frequency amplitudes, hence \( \beta = 0.5 \) or 0.25 second were
considerably smaller than those 2 cycles/second. Hence, angular
frequencies were considered greater than that in the angular Fourier
transforms that the spectral amplitudes at frequencies greater

a visual inspection of the unfiltered and absolute real data.

of the amplitudes of an unfiltered spectrum at that frequency.
a filtered spectrum at 2 cycles/second would be about 1% at 5 radians/second (199 cycles/second) the amplitude of
amplitude, with a first order filter having its break point

quency at the filtered spectral having a substantial

it is necessary to choose \( T/2 \) to be larger than any fre-

is termed "attenuation" of frequencies, To avoid attenuation,
would be identical as a frequency less than \( T/2 \), this
orthogonal stein after filtering, the higher frequencies

If frequencies higher than \( T/2 \) were present in the
in seconds apart is \( T/2 \) cycles/second (in radians/second).

"Folding frequency" which can be detected from data spaced
From Equation (F.1), for a hamming spectral window, this number is

\[ \nu = 2.512 \frac{T}{Lh}. \]

To detect a detail of width \( a \) in a spectrum, it is shown in [55] that the bandwidth \( b \) of the spectral window should be less than \( a \). Also, increasing the record length \( T \) decreases the variance of the spectral estimator, which means that peaks in the estimated spectrum are more likely to indicate peaks in the real spectrum, rather than variance in the estimator. From [55], the record length is determined by

\[ T = \frac{\nu}{2a} \quad (F.2) \]

In this study, \( T/Lh \) is usually about 10, so that \( \nu = 25 \). From Equation (F.2), a record length of 50 seconds would allow the identification of detail of width 0.25 cycles/second, or 1.57 radians/second. (Road tests were approximately 50-55 seconds in length.) Smaller details could be the result of variance in the estimator.

It is shown in [55] that

\[ \frac{\nu S_{xx}(\omega)}{S_{xx}(\omega)} \]
then related by the convolution integral
that \( E(\tau) = 0 \) for \( \tau > 0 \). The processes \( x(t) \) and \( y(t) \) are
system, which is assumed to be physically realizable, so
let \( E(\tau) \) represent the impulse response function of the
Given the random processes \( x(t) \) as input and \( y(t) \) as output,
Consider again the problem of identifying a system.

and are described here.

Equations were taken directly from the literature [36', 39',
such a program was written for this study. The necessary
identification was found at the University of Michigan,
impulse response method of Tufano, time-invariant system
since no existing computer programs employing the

P.2 IMPULSE RESPONSE METHOD

chi-squared distribution table.
estimate at a particular frequency, using a conventional
used in selecting up a confidence interval for the spectrum
distribution with degrees of freedom. When least can be
is a random variable having approximately a chi-square

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From experimental records, it is desired to estimate \( p_0, q_1 \) and \( E(\eta) \) with \( w = T, 2, \ldots, M \).

\[
\begin{align*}
\sum_{k=0}^{n} \sum_{k=0}^{n+k} & \left[ \sum_{k=0}^{n+k} E(\eta) \right] \sum_{k=0}^{n+k} E(\eta) \\
& = \sum_{k=0}^{n+k} E(\eta) \sum_{k=0}^{n+k} E(\eta)
\end{align*}
\]

[38]

Drift and offset may be added to Equation (p.4) as follows:

As when the vehicle is "out of trim," a compensation for vehicle situation, by operation about a non-zero value, such as when the vehicle is "out of trim," a compensation for vehicle situation, by operation about a non-zero value, may be caused by instrumentation errors or in the main drift. Drift is an instrumentation error, while offset voltages which may contain errors due to drift and/or offset voltages which may contain errors due to drift and/or

In a practical situation, \( x(t) \) and \( y(t) \) are transducer outputs.

\[
\sum_{k=0}^{n+k} \sum_{k=0}^{n+k} E(\eta) \sum_{k=0}^{n+k} E(\eta)
\]

Approximation to Equation (p.3) is

That there are \( k \) samples per series, a discrete sample that have been sampled at time instants in seconds apart, and

Further, assume that \( E(1) = 0 \) for \( t < \frac{M}{w} \), that \( x(t) \) and \( y(t) \)
\[
\begin{pmatrix}
\varphi \nu \\
\vdots \\
\varphi (k+1) \\
\varphi (k+2) \\
\varphi 
\end{pmatrix} = \Lambda \\
\begin{pmatrix}
\varphi \nu \\
\vdots \\
\varphi (k+1) \\
\varphi (k+2) \\
\varphi
\end{pmatrix} = \Phi
\]

where

\[
\Phi \nu (x, x) = \Phi
\]

It is desired to minimize the error function

\[
\Phi \nu (x, x) = \sum_{k=1}^{K} (A(k) - \epsilon_k)^2
\]

It is easily recognized that equation (P.5) is of the
desired form and defines the estimate of \( \Phi \nu \).
The terms for plus and drift correction in the above are:

\[
\begin{pmatrix}
\psi(w-k)x & \cdots & \psi(w-k)x \\
\vdots & \ddots & \vdots \\
\psi(t-k)x & \cdots & \psi(t-k)x \\
\psi(k)x & \cdots & \psi(k)x \\
\psi/\tau & \cdots & \psi/\tau \\
\end{pmatrix}
\]

\[\tau = w\]

[38] Reference [38] and [44], but these terms were not shown in Reference [38] and [44], with the reference by including the independent variables \(x\) and \(k\), in addition to \(x\) and \(w\).
a) The time shifting method

The relationship between \( x(\text{k}) \) and \( y(\text{k}) \) is expressed as

\[ \text{NMSE} = \sum_{\text{k}=0}^{\infty} \frac{\text{NMSE}_2}{2} \]

where \( \text{NMSE}_2 \) is the normal mean squared error, \( \text{NMSE}_2 \) is the output of the system having the estimated transfer function is known given the original input \( x(\text{k}) \), the output of the estimated output \( y(\text{k}) \).
without needing to invert the \((X^TX)\) matrix for each value, thus saving a considerable amount of computation time.

Shifting the output series and applying the impulse response method yields the discrete impulse response function estimate, \(\hat{g}_m(\ell h)\), \(\ell=1,2,\ldots M\), and the transfer function \(\hat{Y}_m(j\omega)\). The estimate of the rider's transfer function is then \(\hat{Y}_p(j\omega) = e^{-\lambda j\omega} \hat{Y}_m(j\omega)\), which may be calculated directly from \(\hat{g}_m(\ell h)\) by

\[
\hat{Y}_p(j\omega) = e^{-\lambda j\omega} h \sum_{\ell=1}^{M} \hat{g}_m(\ell h) e^{- (\ell - 1) h j\omega}.
\]

In applying the Wingrove-Edwards and impulse response methods to the identification of rider transfer functions from experimental data, \(h\) was chosen to be 0.1 second, and \(M\) was usually 15.
The series $u_n$ is known as a discrete first order auto-
regressive series [55]. The theoretical autocorrelation
function of $u_n$ is

$$r_k = \frac{\sigma_z}{\sigma_u} = 0.3,$$

where $\sigma_z$ is the standard deviation of the random variable $z(n), n \geq 0$.

A second series of numbers $u(n)$ was calculated using the equation

$$u_n = r_1 u_{n-1} + z(n), n > 0,$$

where $r_1$ is a constant and $z(n)$ is a random number distributed with a mean of zero and a standard deviation of 0.1.

For any $k$, the random variable $z(n)$ was taken to be 0.1 second. For any $k$, the random variable $z(n)$ was formed from a sequence of random numbers where $k = 1, 2, 3, \ldots, N < 500.$

A subroutine was generated to simulate the data as described in Appendix F. The test data were created of the impulse response and cross-spectral analyzation of the methods, known transfer functions were identified by means of

To test the validity of the methods employed in

6.1 IDENTIFICATION OF KNOWN TRANSFER FUNCTIONS

APPENDIX G
For each system, the open-loop transfer function was given by

\[ \frac{\theta}{\theta_p(t)} = \frac{G(j\omega)}{1 + H(j\omega)} \]

by Figure G.2. The data were employed to test the data processing procedures.

More specifically, two systems of the general form of

\[ \dot{y}(t) = A x(t) + B u(t), \quad \dot{x}(t) = C x(t) + D u(t) \]

were assumed to arise from the test system of the Pioneer Rover system, the entire analogous to the case of the Pioneer Rover system, the entire "remnant" of the system assumed to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to represent the Pioneer Rover system, assuming to 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Figure G.1 Theoretical autocorrelation function.

Smooth curve (dashed points connected with a)
of the artificial test remanent, $a_1 = 0.3$.

1' seconds
Figure C.2: Control system representation of the artificial test data.
\[ \begin{aligned}
&\{((\eta + c)(\eta) + [\eta(\eta - 1)](\eta) - [\eta(\eta - 1)](\eta)\} \\
&\quad \times (\eta) x = (\eta) y
\end{aligned} \]

\[ x = (\eta) x \]

\[ \eta > \eta > \eta > \eta \]

\[ \eta > \eta > \eta > \eta > \eta \]

\[ \eta > \eta > \eta > \eta > \eta \]

The following equations were used:

\text{For the first system, the approximation to integration of a more complex model was assumed.}

\[ \begin{aligned}
&\dot{x} = (\eta x)^2 \\
&\frac{m}{m} + \frac{\eta}{\eta} = (\eta x)^2
\end{aligned} \]

\text{To allow the identification of the system, the first system was chosen to be}

\[ \begin{aligned}
&\dot{x} = (\eta x)^T \\
&\frac{m}{m} + \frac{\eta}{\eta} = (\eta x)^T
\end{aligned} \]

\text{Therefore, the transfer function of the system, the transfer function of the open-loop transfer function of the system, was chosen because of the similarity to the form suggested to radian/second, with a crossover frequency of 1.}
9.2 IDENTIFICATION OF TRANSFER FUNCTIONS FROM INPUT-OUTPUT RECORDS

had been removed.

seconds (500 data points) after the "start-up" data points
value of \( N \) was adjusted to give a total record length of 50
for which \( x(\eta n) \) or \( e(\eta n) \) were zero were omitted. The
spectral analyser computer program, the first few values of
etc. When the data was read into the impulse response of
Note that it was necessary to set the first four data points

\[
N > K > 0 \text{ for } n = 0,1,2, \ldots
\]

\[
\begin{align*}
\{ [p(3-K)]e + [p(4-K)]e \} \frac{z}{p} + [p(1-K)]x &= (x)_{\eta n} \\
\frac{z}{p} &> K > 0, \quad n = 0,1,2, \ldots
\end{align*}
\]

For the second system, the following equations were used:
which the results of applying the Impulse Response method to
this sensitivity is clearly demonstrated at the lower 6.3%.
response method is very much transfer function sensitive.
function being determined, with the accuracy of the Impulse
spectral method is independent of the particular transfer
and Impulse response methods to that the accuracy of the
most striking difference between the cross-spectral

\( m = 10, 15, 20 \).

estimated for ten, fifteen and twenty time instants
chosen to be 0.1 second. The Impulse Response function was
for the Impulse Response method, the time step was also
total number of data points.
value of \( T \), at least for values of \( T \) from 5 to 15% of the
spectral methods was not found to be very sensitive to the
to be 50. Determination of transfer function was done
step of 0.1 second was used. The number of 1, 2, 3, was taken
accommodate the standards with the highest bandwidth. This
increment used in the spectral analysis was small enough to
intercept \( H(\omega) \). To avoid aliasing or frequency shift the time
amplitude at frequencies greater than 1 radian/second of the
\( e(\omega) \) and \( x(\omega) \) (about 0.5 c/s), due to the attenuation of
\( c(\omega) \) (about 2.5 c/s) were considered. The factor larger than those of
and \( H(\omega) = \frac{1}{\omega^2} \), the half-power bandwidth of \( \omega \) and
8.7 for the first data set investigated (8.7) = e.\( \omega \)
Figure G.3 Identification of a differentiator and an integrator with the impulse response.

\[ \phi = \frac{(\omega f)^{T_w}}{(\omega f)^{T_o}} \]

\[ \theta = \frac{(\omega f)^{T_e}}{(\omega f)^{T_c}} \]

\[ |\frac{m_f}{T}| = \left| \frac{(\omega f)^{T_w}}{(\omega f)^{T_e}} \right| \]

\[ |\omega f| = \left| \frac{(\omega f)^{T_e}}{(\omega f)^{T_c}} \right| \]
the integrator, \( H_I(j\omega) = 1/j\omega \), and the differentiator,
\( H_D(j\omega) = -j\omega \), are shown. To identify \( H_I(j\omega) \), \( c(kh) \) and \( -e(kh) \) were taken to be the input and output, respectively, and to identify a differentiator considerably more accurately than it can identify an integrator. Also, in Figure 6.4 it is shown that the impulse response method can very accurately identify a pure time delay. The gain and phase of the pure time delay were identified with accuracy to at least three significant digits.

It is not difficult to understand the reasons that the accuracy of the impulse response method is system-dependent.

Basically, the accuracy of the method depends upon the degree to which the discrete system output at time \( k\) can be approximated by a weighted sum of the values of the digitized input a time \( k\) and a finite number of previous time instants. The output at time \( k\) of a pure time delay of \( k \) seconds can be exactly predicted by applying a weight of 1 to the input at time \((k-k)h\) and a weight of 0 to the input at any other time instant. However, for an integrator, the impulse response of which is a step function (a constant for all \( t > 0 \)), the output at time \( k\) depends on all values of the input at times previous to and including \( k \). The impulse response method attempts to
Figure 9.4
Identification of a pure time delay.

- Impulse response method
- (λ = 0)

\[ |e^{-4j\omega}| = 1 \]
approximate this impulse response function by a truncated estimate, and the approximation reduces the accuracy of the transfer function identification, especially at low frequencies. It would be expected that the accuracy of the method in identifying transfer functions having impulse response functions which do not decay to zero would be improved by increasing the truncation number, M.

Figure G.5 shows the identification of a more complex transfer function, $e^{-\cdot3j\omega/j\omega}$, actually the product of two types of transfer functions previously identified. The impulse response function corresponding to $e^{-\cdot3j\omega/j\omega}$ does not decay to zero as time increases without bound; thus, the impulse response method does not give accurate results. However, increasing the value of M from 10 to 20 does, in fact, significantly improve the accuracy of the identification.

G.3 IDENTIFICATION OF $G(j\omega)$ WHEN $x(\text{kh})$ IS NOT KNOWN

The data obtained from actual road experiments differ from the artificial test data in that, for the rider/cycle system, the time series analogous to $x(\text{kh})$ is not known. In order to identify the rider's transfer function, then, the method outlined in the text and Appendix F (the Wingrove-Edwards or time shifting method) must be used in conjunction with the impulse response method. (Cross-spectral methods cannot be used.)
Figure 4.5 Identification of $e^{-3f_0/\mu}$.

Graph showing impulse response method with $M=20$ and $M=10$. Cross-spectral method also plotted.
Thus, it appears that the errors in identifying the impulse input of the actual transfer function when x(kn) is unknown is seen to be not much less accurate than when x(kn) is known. Also, identification of the impulse response method depends on system dependent. Hence, identification in the same way that the accuracy of the impulse response G(s) and G(s) presents evidence that the actual identification using the Winrove-Dawkins technique and the impulse response G(s) and the controlled element H(s) = (s) = 1. Thus, Figure 7.3 shows that the controlled element H(s) = (s) = \frac{e^{-0.3 impressed e^{-0.3 impressed}}}{jw} in which the "true" transfer function is G(s) = (s) = \frac{e^{-0.3 impressed}}{jw} is more accurate than the identification shown in Figure 6.7. In Figure 6.7, the "true" transfer function is G(s) = (s) = \frac{e^{-0.3 impressed}}{jw} is seen to be the controlled element \frac{e^{-0.3 impressed}}{jw}. In Figure 6.7, the "true" transfer function was then taken to be G(s) = (s) = \frac{e^{-0.3 impressed}}{jw}. For x(kn) to (kn), the estimated "true" transfer function \frac{e^{-0.3 impressed}}{jw}. The result was a transfer function G(s) = (s) = \frac{e^{-0.3 impressed}}{jw}. By an amount \alpha, was treated as the input series in the impulse response analysis, and the test data shown in Figure 6.7 is taken to be the output. These results, the "true" transfer function was delayed in time. The results of applying the Winrove-Dawkins technique and the impulse response method to both steps of identification the results of applying the Winrove-Dawkins technique
Figure 6.6: Identification of transfer function.

\[ G(f) = G_T(f) = e^{-\frac{\theta}{T}} \]

(\text{Note: Graph showing impulse response and cross-spectral method.})

\[ |m\| = |\frac{(m\omega)H}{T} - 1| \]

\[ T = |m\| \cdot e^\frac{\theta}{T} \]
method

\[ g(f) = e^{-\frac{3\omega}{f}} \]

Figure 4. Identification of a "first" transfer function:

\[ \theta = \left( \frac{m_f}{\omega} \right) H \left( \frac{1}{L} \right) \]

\[ \tau = \left( \frac{m_f}{\omega} \right) H \left( \frac{1}{L} \right) \]

\[ E^2(f) \]

\[ \frac{E^2(f)}{T(f)} \]
due to the limitations of the impulse response method and not due to the additional complication of employing the Wingrove-Edwards technique.

Once an impulse response function, $\hat{g}(mh)$, $m=1,2,\ldots,M$, has been estimated for the "rider" transfer function, the unknown quantity $x(kh)$ may be estimated by

$$\hat{x}(kh) = \sum_{m=1}^{M} \hat{g}(mh)e^{[(k-m+1)h]}.$$ 

The series

$$\hat{r}_e(kh) = c(kh) - \hat{x}(kh)$$

is an estimate $\hat{n}_t(kh)$ of the "remnant" $n_t(kh)$ when $\lambda = t$, the time delay of the "rider". The autocorrelation function, $r_{n_t}n_t(\tau)$, of the remnant, $n_t(t)$, was estimated (for integer multiplies of the time step $h$) from the series $n_t(kh)$ and $\hat{n}_t(kh)$, the latter series being estimated from both sets of artificial data (Fig. G.8). Both estimates are close to the theoretical autocorrelation function, although the estimate based upon $n_t(kh)$ shows more scatter for larger values of $t$ than do the estimates based upon either $\hat{n}_t(kh)$ series. If this scatter is not due to errors in estimating the autocorrelation function, it would indicate that the identification of $G_1(j\omega)$ and $G_2(j\omega)$ through use of the Wingrove-Edwards
Theoretical \( r_{n_tn_t}(k') = 0.3^{k'} \)

- Estimated from original \( n_t(kh) \) series
- Estimated from \( c(kh) - \hat{x}(kh) \), where \( \hat{x} \) was calculated from \( \hat{g}_1 \)
- Estimated from \( c(kh) - \hat{x}(kh) \), where \( \hat{x} \) was calculated from \( \hat{g}_2 \)

Figure G.8 Identification of \( r_{n_tn_t}(\tau) \).
\[ \gamma(n_{x}\chi) + \gamma(n_{y}\chi) = n_{x}\chi + n_{y}\chi. \]

\[ \gamma(n_{x}\chi) = 0. \]

Hence, \( \gamma(n_{x}\chi) \) are considerably smaller than those of \( \gamma(n_{y}\chi) \).

For frequencies greater than 1 radian/second, the amplitudes for this series are not the same as \( \gamma(n_{y}\chi) \) due to the attenuation of the integrator.

Instead, rather, they indicate that \( \gamma(n_{y}\chi) \) is approximately the same for both \( \gamma(n_{x}\chi) \).

For \( x > 1 \), the NMSE again increases. The large values of \( \gamma(x) \) at these ranges were also nearly independent estimates for \( x \) in these ranges were also nearly independent.

And it was found that the corresponding transfer function for which the NMSE is not very sensitive to the value of \( x \), ranges of \( x \) which ranges depend upon the form of \( \gamma(x) \), as \( x \) increases, the NMSE increases rapidly. There exists \( \gamma(x) = 0 \), since \( \gamma(x) \) are highly correlated with \( 1 \).

Thus, the NMSE is very small when the degrees of \( x \) is correlated with \( \gamma(x) \).

In Figure G.9 as a function of \( x \), the NMSE for a measure of\( \gamma(n_{y}\chi) \) as shown in Figure G.1. The quantita function shown in Figure G.1. The quantita would have been more accurate at the autocorrelation.
Figure G.9 Normalized mean squared error as a function of \( \lambda \); artificial test data.

Normalized mean squared error

\[ \text{identification of } e^{-4j\omega} \]
\[ \text{identification of } e^{-3j\omega} \]

\( M = 15 \)
\( M = 20 \)
\( M = 10 \)
G.4 IDENTIFICATION FROM DATA CONTAINING OFFSET AND DRIFT

The actual experimental data further differed from the artificial test data in that the former could contain offset and/or drift\(^1\). The artificial test data contained such low levels of offset and drift that no correction was used in the impulse response program for the results shown in Sections G.2 and G.3.

The method of minimizing the effect of offset and drift in the impulse response computer program was tested in the following manner. First, the artificial test data with \(G(j\omega) = G_3(j\omega) = e^{-3j\omega}\) and \(H(j\omega) = H_1(j\omega) = 1/j\omega\), were modified somewhat arbitrarily to include offset and drift by defining new series as follows:

\[
\begin{align*}
e_m(kh) &= e(kh) + 100 - 0.5k, \\
c_m(kh) &= c(kh) + 250 - 0.65k.
\end{align*}
\]

Series \(e_m(kh)\) and \(c_m(kh)\) are analogous to the output of transducers employed in the road tests, which output may be the desired quantities (voltages proportional to roll angle, steering torque, etc.) corrupted by offset and drift. Next, \(^1\)Appendix F discusses offset and drift and the method in which their effect is minimized.
with $e_m(kh)$ and $c_m(kh)$ as transfer function input and output, respectively, the impulse response analysis, including the Wingrove-Edwards technique, was performed. Figure G.10 shows the results of including an offset and drift correction versus the results of not including such a correction, for $\lambda = 0$ and $\lambda = 0.3$ second. Note that the offset and drift, when not compensated for, tend to reduce the accuracy of the estimate of $-1/H_1(j\omega)$ ($\lambda = 0$) for low frequencies, while not having significant influence upon the estimate for $G_3(j\omega)$ ($\lambda = 0.3$ second).

G.5 IDENTIFICATION OF "RIDER" TRANSFER FUNCTIONS WHEN THE REMNANT BANDWIDTH IS SMALL

Figure G.11 shows the theoretical autocorrelation functions of $n_t(kh)$ when $\alpha_t = 0.8$. For a "rider" time delay $\tau$ of 0.4 seconds, it can be seen that there is an appreciable bias error in identifying the "rider" transfer function when the closed-loop system is excited by this remnant.

An identification of $G(j\omega)$ is shown in Figure G.12, with $\lambda = 0.4$, and

$$G(j\omega) = G_4(j\omega) = 1.5 e^{-0.4j\omega}.$$  

and

$$H(j\omega) = H_4(j\omega) = 1/j\omega.$$
Figure 6.10 Correcting for offset and drift in the data records (impulse response method).

\[ m_{\frac{H}{T}}(s) = \frac{(m_j)(H)}{1-s} \]

\[ t = |m_{\frac{H}{T}}|e^{-\theta} \]

\[ |m_j| = \left| \frac{(m_j)(H)}{1-s} \right| \]
Figure G.11  Theoretical autocorrelation function of the artificial test remnant, $\alpha_1=0.8$ (discrete points connected with a smooth curve).
Figure G.12 Identification of "rider" transfer function ($\alpha = 0.8, \lambda = 1.0$ sec).
the actual bandwidth of \( u(t) \). Further, there was no local white decrease with increasing \( \gamma \), was always greater than several values of \( u(t) \) indicated that the bandwidth of \( u(t) \).

These identifications, however, were estimated from the results of

The remnant \( u(t) \) was estimated from the results of the phase difference from the true ethan and phase. Notice also that

\( \hat{\gamma} \) is underestimated at high

\[ \frac{\mathcal{H}(\omega)}{\mathcal{I}} \]

Purported "towards" at the high and low frequencies are

In Figure 2.12, notice how the \( \mathcal{I}(\omega) \) estimates are

\( \hat{u} \) to estimate \( \mathcal{I}(\omega) \).

As output. The other was calculated in a manner usual manner, using \( e(t) \) as transfer function input and

Two identifications are shown.
minimum of the NMSE with respect to \( \lambda \), for \( \lambda = \frac{1}{\tau_p} \), as was the case (Fig. G.9) when \( n_t \) was closer to "white" noise.

Rather, an increase in \( \lambda \) always increased the NMSE.

If new data is created, using the same \( n_t \) (\( \lambda = 0.8 \)),
but decreasing \( \lambda \), more bias will result. Figure G.13 shows
such an identification, for \( \lambda = 0.1 \) and 0.2, where

\[
G(j\omega) = G_5(j\omega) = 5 \cdot e^{-0.1\omega}
\]

and

\[
H(j\omega) = H_1(j\omega) = \frac{1}{j\omega}.
\]

Notice that \( |\hat{G}(j\omega)| \) shows less "pulling" toward \( \frac{1}{j\omega} \)
and that \( \hat{G}(j\omega) \) appears to be a constant gain and a pure time
delay. However, the estimated values of gain and delay are
considerably in error, gain being underestimated and the
delay being overestimated. Increasing \( \lambda \) from 0.1 to 0.2 did
did not affect the accuracy significantly, and the estimator
displayed more variance or instability.
Figure G.13: Identification of \[ \frac{\text{transfer function}}{\text{function}} \text{ (at } \omega = 0.8) \].

\[ \begin{align*}
\frac{\text{Phase}}{\text{Phase}} &= \omega = 0.8 \\
\frac{\text{Gain}}{\text{Gain}} &= \frac{5 \angle 50^\circ}{10 \angle 0^\circ} \\
\frac{\text{Gain}}{\text{Gain}} &= \frac{5 \angle 50^\circ}{10 \angle 0^\circ} \\
\end{align*} \]