A Statically Unstable Passive Hopper: Design Evolution

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I have designed a sequence of gravity-powered passive-dynamic toys. These explore locomotion in general and hopping in particular. As with walking, running, crawling, etc., for animals, locomotion in these devices is a horizontal translation by means of approximately periodic patterns of motion. These toys were developed using intuitively guided trial-and-error design iteration based on live viewing, sound sequences, and review of slow motion video. A series of statically stable mechanisms is described. A progression of designs led to the central result: a monopod hopper that repeatedly hops more than 70 steps down a ramp, without conventional feedback control, fast spinning parts, or sensing means, yet unlike the previously statically stable designs, it cannot stand still stably. This free hopping was facilitated by a special mass distribution, and a spring that allowed relative translation and rotation between the body and leg. A retrospective evaluation reveals similarities to the morphology and gaits of hopping bipeds. These toys, interesting dynamical systems in any case, highlight the possibility of a significant role of mechanical structure in locomotion. [DOI: 10.1115/1.4035222]

1 Introduction

Some recent literature on walking locomotion and even some on running locomotion can be traced to a simple ramp-descending toy, the "Wilson Walkie" [1] (Fig. 1(a), and Appendix C). This toy consists of a body and two hinged legs with spherical feet. It waddles down a ramp with a motion reminiscent of a penguin walking. The core idea inspired by this toy is passive-dynamic locomotion. Mechanisms utilizing this principle move dynamically and are even dynamically stable, but they contain no controller subsystem. With the low cost of modern computation and the sophistication of modern control theory, one might question the utility of studying such purely mechanical devices. Nonetheless, the study of passive dynamics in walking has led to energy efficient robots [2]. And in control theory, understanding of the system (the "plant") is generally regarded as essential; one part of understanding how to control a system is to learn what the plant does, or can do with appropriate design, on its own, including the possibility of self-stabilization.

The subject of this paper, a device that hops down a ramp but is statically unstable, was inspired by Coleman and Ruina's Tinkertoy walker [3] (Appendix C of this paper). The Tinkertoy walker demonstrated something unique among passive devices, and something seemingly impossible; that a passive device (e.g., no motors, sensors, or controllers), without spinning parts could possess dynamic stability even though it could not stand still stably. My goal was to make an analogous hopping device, one that could hop stably without any apparent contribution from the existence of a statically stable configuration, depending only on a special mass distribution and passive mechanical elements like springs and dampers.

2 Background

Single leg forward hopping that relies on sensing and feedback has been widely studied and modeled [4], most famously by Raibert et al. [5]. Fewer have investigated embodiments that eliminate the need for sensing and instead involve periodic "open loop" actuation schemes [6]. Purely passive forward hopping and running has mostly been studied with mathematical models and numerical simulation. One theoretic, statically unstable design, a single leg offset mass hopper (SLOM) assumes a massless leg which was optimized for passive forward hopping. This device was capable of approximately 16 hops in simulation before falling [7]. Similarly, a statically unstable passive hopper based on the configuration of a kangaroo with tail was capable of several hops in simulation [8]. Neither of the models in these later two studies was tested physically. Nor does it seem that these designs are truly stable, even in their mathematical forms. Prior research into widefoot (statically stable) open loop hopping has involved actuation [9]. Iida et al. have described a related passive mechanism involving a large flat base [10].

More related to our present work, in 2D numerical simulations McGeer [11] demonstrated the possibility of stable passive bipedal running for a device that cannot stand upright when the hip spring is in a neutral position. Such passive running has been, perhaps, partially realized in a physical model that can run several consecutive strides [12].

While running and hopping (gaits with flight phase) do not depend on a springing bounce [13], both hopping and running are typically viewed as springy gaits. The energy preserving nature of springs account for the evolutionary advantages of these gaits in nature [14,15]. Or, conversely, the advantages of springs in running and hopping may explain the evolutionary advantages of tendons in devices that would run even without them [13].

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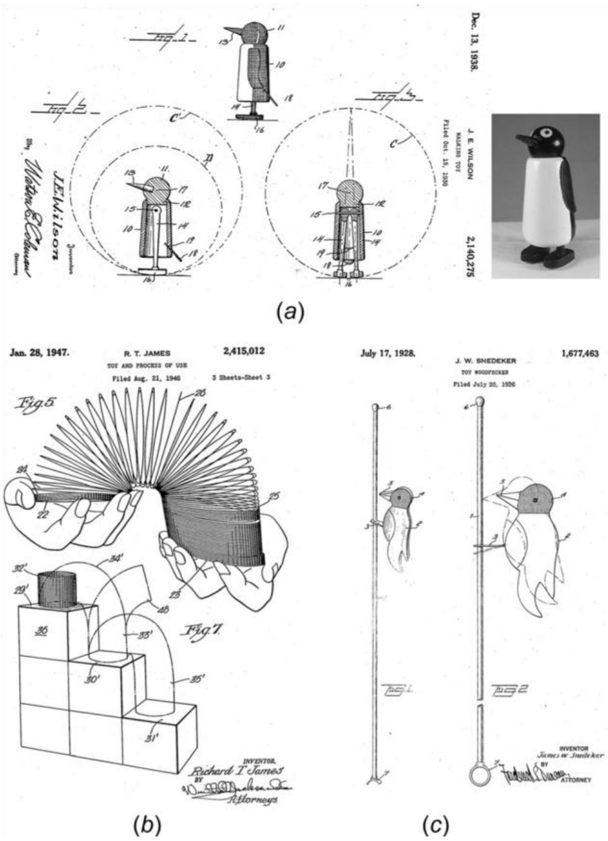

Fig. 1 U.S. Patent drawings of passive dynamic toys. (a) The passive dynamic ramp toy known as the Wilson Walkie, as well as a photograph of the toy. (b) The Slinky, another passive dynamic toy capable of walking down stairs. (c) A woodpecker toy that descends a pole using in a piecewise pecking motion.

The first mechanical analogs to natural locomotion, passive dynamic toys, like the Wilson Walkie, are gravity-powered devices that have periodic movement and that utilize gravitational potential energy and dissipation by collisions and friction. These passive-dynamic toys include bipedal and planar walking toys, the Slinky, the woodpecker-on-a-pole that pecks and slides its way down (Fig. 1 and Appendix C of this paper), and brachiation (ape swinging) toys [16] (and Appendix C of this paper). Unlike the device we are going to describe, none of these toys appear to have a free flight phase (a period of time with no ground contact).

3 Purpose

My initial goal was to develop a passive, intermittent-contact "toy" that could steadily hop down a ramp without, as for all passive dynamic toys, conventional feedback control, actuation, or even any spinning parts. This led to an investigation of whether a similarly constrained device, but one that lacked a statically stable upright position, might also be possible (the closest analog was the statically unstable Tinkertoy). The core question was whether sustained forward hopping, without the static stability of a broad foot, could be realized in a minimal passive mechanism.

4 Methods

Prior to the passive hopping experiments which are most central in this paper, a series of exploratory ramp descending models were developed (multimedia extension 1 video). These were variations of classic passive-dynamic locomotion toys described above. Initial experiments were performed with models composed of springs, dowels, masking tape, masses of clay, and other common materials. These were tested on a sloped and flat wooden ramp.

Development of the hopping devices proceeded by empirical means, with the early mechanisms seemingly far removed from what might be considered a legitimate hopper precursor. However, these simple mechanisms and sequential modifications provided a postereriori knowledge utilized in construction of the final hopper form. The management of multiple mass interactions, and optimization for shallow ramp descent, were among the skills acquired and applied to subsequent models.

A rubber ball bouncing down a ramp illustrates the primary challenge of passive upright hopping; acceleration occurs as the ball travels down the ramp and rotates with each bounce. Converting such motion to regular forward hopping requires regulation of both body attitude and speed.

The first device to give some indication of this possibility consisted of a toy car, a thin spring wire, and a rubber ball (Fig. 2) (multimedia extension 2 video). The ball was rigidly fixed to a curved wire spring which, in turn, was connected to the top of the toy car with a masking tape hinge that allowed free revolute movement. When set in motion at the top of the ramp, the ball, trailing behind, would bounce repetitively. The trajectory of the ball, as constrained by the hinged spring wire, seemed to mimic forward hopping by a human on a pogo stick. As with a spring powered clock, where the release of energy is regulated by a pendulum and escapement, the hopping ball was propelled by gravity while regulating the speed of forward motion. Rotation of the ball was limited by the curved wire so that equal forward and reverse pitching occurred during one hopping cycle. The angle of the slope, the gauge of the wire, its length, and its curve, were optimized by trial and error until the stalling or runaway behavior was minimized.

Various previous powered hopping models have involved a kind of tethering constraint relative to ground plane [17]. The toy car here was analogous to a tether, providing constraint and reference to the ground. However, no actuation, besides gravity on a slope, was involved. More complex than simply periodic “period 1” behavior was also observed, depending on the drop height of the ball at launch. Perhaps the dynamics of the ball-and-car are somewhat analogous to an apparently related toy, the woodpecker descending a pole (Fig. 1(c)). A mathematical model of the woodpecker toy revealed both period-1 and period-2 motions [18] (and Appendix C).

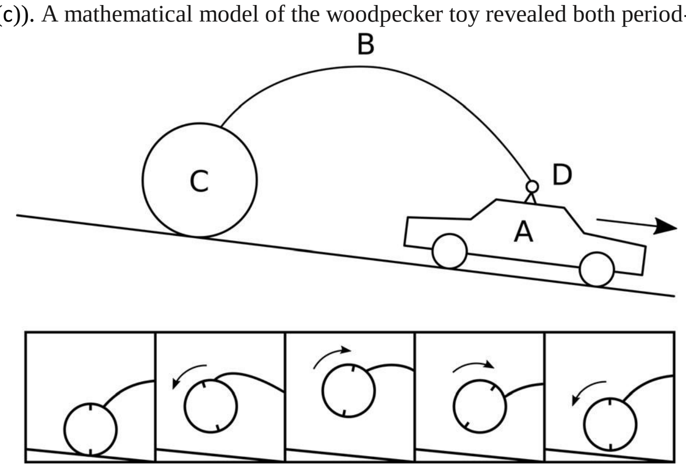

Fig. 2 Ball and car toy. A toy car A is attached by a curved wire spring B to highly elastic rubber ball C. One end of the wire spring is embedded in the ball. The opposing end of the wire spring is attached by hinge D to the top of the toy car. The lower panel shows the approximate orientation of ball during one hop cycle, as the car leads the ball down the hill. Arrows indicate the alternating direction of ball pitching between collisions (multimedia extension 2 video).

Forward hopping is an intermittent motion involving a stance phase and a flight phase. In order to eliminate a ground reference system, of the type provided by the toy car, control of body attitude in passive forward hopping must rely on forces transmitted during stance. This notion of such stabilizing torque, from ground contact, is explicitly used in powered and controlled runners and hoppers, e.g., the hopping robots of Raibert et al. [5]. In subsequent experiments, I explored the possibility that body pitch could be regulated by torques from the passively generated (uncontrolled) ground forces.

All of the subsequent untethered hopping models had three general components: a body with non-negligible mass, at least one spring, and at least one leg. First, such related device comprised a coil spring with a mass attached to the top. This could be made to vibrate and proceed down a slope at an approximately constant speed. The flat base of the spring acted as a foot intermittently contacting the ramp. It was unclear, however, whether friction was primary factor in regulating the acceleration of such a flatfooted device [19]. Subsequently, a thin piece of plastic sheet (52 mm circular diameter, 1 mm thick) was heat-formed into a shallow (152 mm radius) hemispheric foot and fixed to the bottom of the previous spring/mass device. With this configuration, the center-of-mass of the device was well below the center of radius of the foot, giving the device an undesirable (for our eventual goals) static stability (Fig. 3).

Sustained hopping was more easily obtained when the foot mass was much smaller than the upper mass. This wide-footed hopper displayed periodic hopping as well as more chaotic and multiperiodic modes (multimedia extension 3 video). During spring compression phase, the body mass underwent a forward pitch rotation. This was the result of two combined motions: rolling of the entire mechanism on the wide-curved foot during compression, and forward pitching of the body mass relative to the foot. In the prior ball and car toy, the curved spring allowed reorientation of the ball during flight, but it relied on the constrained rolling mass of the car. Here, the greater inertial mass was the body of the hopper, and a free spring facilitated reorientation of the entire mechanism during flight. For this device, it appears that most of the gravitational energy is lost through collisions, although there may be some energy lost to frictional slip. The unconstrained coil spring in this hopper allowed more than one-

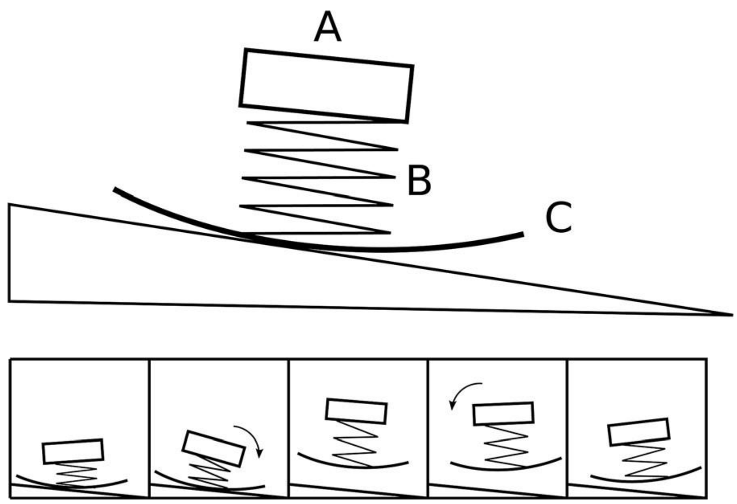

Fig. 3 Curved foot hopping toy. A ramp-descending toy composed of body mass A, coil spring B, and curved foot C is shown in cross section. The mass of A was much greater than that of the coil spring B, or foot C. The curved foot was formed from a thin stiff plastic sheet into a shallow hemisphere. The lower panel shows a hopping sequence from left to right, beginning with collision. Following collision, the spring is compressed while the mass rotates forward, compressing the spring to a greater degree on the forward edge. The foot rolls forward. During flight, this asymmetric spring force is released, causing slight counter-rotation of the entire device before landing at a forward position. This can be observed in the slow motion portion of the footage (multimedia extension 3 video).

degree-of-freedom motion of the upper mass relative to the foot. Some amount of three relative 2D motions are possible: pitching, compression, and some amount of shear movement. Multidegree-of-freedom hip springs were a feature used in all the subsequent hopper embodiments. Sliding and rotary joints were avoided to minimize frictional losses; only springs connecting the leg to the body were employed. The spring ends were rigidly fixed to these parts.

The curved footed hopper demonstrated that reorientation of an untethered body/leg structure could be achieved by passive ground forces within a hopping cycle. It also revealed that it was desirable for the spring to do more than simply allow effective leg compression. If the spring is considered a leg, then, a small amount of leg swing was apparent during the ground contact phase of the hopping cycle.

With inspiration from the static instability of the Tinkertoy walker [3], and the suggestion by its inventor (Coleman, private communication), the next step in the evolution of my devices was an attempt to reduce the size of the hopper foot. The goal was to maintain the repetitive downhill hopping but achieve it with a device lacking a statically stable position. A successful passive-dynamic hopping device would be the first of its kind in this category. I pursued designs of a more planar form that might tend to more planar (longitudinal) motions, simplifying both construction and possible future analysis.

The prior study by Seth et al. [7] involved a hypothetical (in simulation only) device with no leg swing relative to the body mass. It relied on a single degree-of-freedom leg spring and reorientation of the entire device during flight phase. Given the minimal number of hops achieved by this configuration, and the lack of any comparable physical device, a variation on the multidegree-of-freedom hip spring of the curved foot hopper was pursued.

To reduce the reliance of rolling on a wide foot during stance phase, it was necessary to both reduce the width of the foot and increase leg swing relative to body. To facilitate this, another form of spring was used: rubber bands. These were connected between a u-shaped plywood body frame that provided clearance for the leg movement as well as control over center of mass of the entire device (Fig. 4). Mass weights were attached to the ends of the u-shaped frame and could be altered to bias the hopping motion in a forward direction.

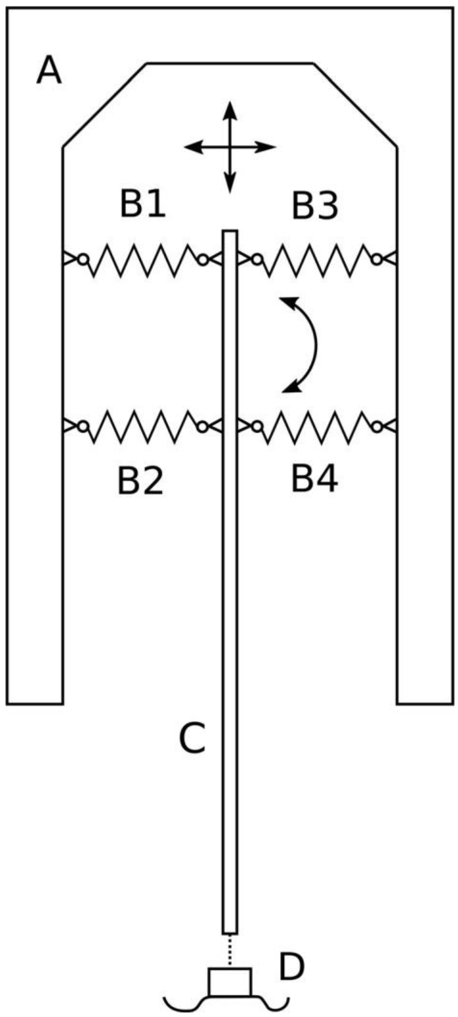

Fig. 4 Initial rubber band sprung hopper. Body A (plywood frame) was attached by rubber bands B1–B4 to leg C. The frame had much more mass than the leg. This suspension allowed three (in the plane) degrees-of-freedom between leg and body: vertical bounce, fore-aft leg swing and some, but relatively less, horizontal displacement. If B15B3 and B25B4, vertical compression did not cause rotation. If, e.g., B1>B3 and B4>B2, and have equal spring rest lengths, then large vertical compression caused a counterclockwise rotation, thus making compression and rotation coupled, at least for large motions. Initial experiments used a narrow foot. A subsequent version had a springy brass foot D, which helped sustain hopping motion, but also resulted in (undesirable) static stability (multimedia extension 4 video).

This rubber band suspension, when made unsymmetrical with B1>B3 and B4>B2, allowed large displacements of the leg relative to the body, which allowed and caused passive body pitching. As an evolution of the initial curved foot hopper, the rubber band suspension decreased the reliance on body pitching and increased leg swing during flight phase. A version of this suspension was applied to a hopper with a small foot (multimedia extension 4 video). The foot was a piece of brass shim stock bent to form a springy toe, arch, and heel. The brass spring foot seemed to help sustain hopping motion. This device appeared to hop stably, but, unfortunately, could also stand in a stable, static position. Without the brass foot this device could hop for a few steps down the ramp, but quickly accelerated and toppled. The rubber band suspension revealed that a relatively planar spring form could provide both substantial leg swing and body pitching. However, this suspension ultimately proved problematic for fine tuning leg and body motion, and was subsequently abandoned.

A more robust and adjustable suspension was required. In the next embodiment of the hopper, elastic connection of the body to the leg was accomplished with a flat leaf spring. A flat leaf spring allows rotation as well as motion orthogonal to the leaf. First, tests of this spring were made in a configuration with a single narrow leg (first portion of multimedia extension 5 video). This proof-ofconcept model with a single leaf spring suspension displayed substantial leg swing and vertical hopping ability. The hopping cycle was rapid compared to that achieved with the rubber band suspension. Some repetitive hopping was observed, but the device accelerated, pitched, and crashed quickly. It also lacked any directional (steering) stability, due possible to twisting of the leaf spring and the single-point contact of the foot.

In a subsequent iteration, a pair of steel leaf springs were attached to the top of a leg assembly and to the inside front edge of an inverted “u” shaped body frame (Fig. 5 and last portion of multimedia extension 5 video). Two wooden legs were used. These were separated by a masking tape and oriented normal to the plane of motion, hence acting as a linear, instead of point foot, contact. As carried over to the final hopper, this arrangement was intended to provide lateral stability, while keeping the overall device construction in the planar realm.

In the prior nonleaf spring hoppers, body pitching during ground contact seemed necessary for forward movement. In the statically unstable leaf spring hoppers, body pitching was more problematic. It was unclear what amount, if any, body pitch was needed. Consequently, a tail was added as a means of slowing the pitch motion of the device. The long tail increased the inertia of the hopper body without adding substantial mass. This hopper was capable of several repeated steps, and the two-foot configuration provided an unexplained steering effect that helped maintain heading down the ramp. This hopper was not statically stable; it could not maintain a standing posture. However, it was not particularly dynamically stable either. It would always topple within a short distance of motion down the ramp. In most of the trials, there was an apparent tottering in the fore-aft plane over several hop cycles. This was also apparent in the pattern of emitted sounds. Although heading was maintained, it appeared likely that some greater foot profile radius was needed.

In the “U” shaped hopper, the center of mass was controlled by adjusting clay, metal weights, and clamps attached to the frame of the hopper. The correct body mass allowed the upright bouncing mode to synchronize with the period of leg swing. With too much mass restricting the bounce height, the leg would not swing adequately back into place and the device would fall backward. If body mass was too low, the opposite would occur and the leg would swing past the desired point. The fore/aft position of the mass also effected stalling or accelerating behavior. The initial conditions of drop height, rotational angle of body, and duration of grasp before release—all intuitively learned techniques—factored into to the success or failure of trial runs.

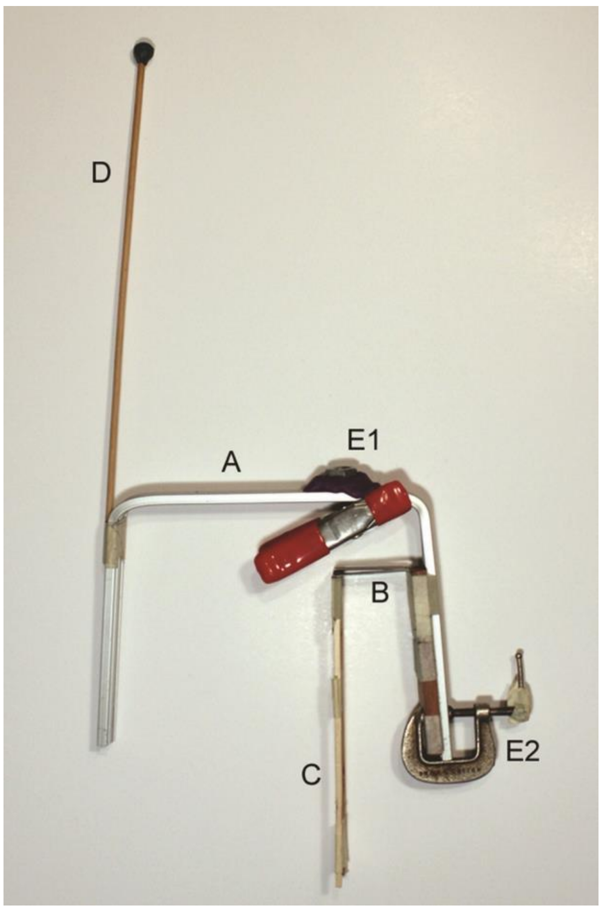

Fig. 5 Early use of leaf spring in hopper. The body frame A was a U shaped bent aluminum extrusion. Leaf spring pair B connected dual wooden legs (chopsticks) C to the frame. The tail mast D was a wooden dowel with clay adhered at top. Clamps and clay were used as adjustable masses E1–E2 (multimedia extension 5 video).

The final hopper (Fig. 6) was more robust in construction, but was otherwise similar to the previous configuration. The body frame was U-shaped welded aluminum channel. The legs and tailmast were made of carbon fiber tubing. Attempts to reduce the foot size continued, but there appeared to be a minimal radius for reasonable hopping (see Appendix B for details). The deformation of the foot during collision may have some effect on the performance. As noted, the leaf spring allowed both vertical bounce and hip rotation. Observations were made by reviewing high speed video. Following collision, and during initial compression, the spring is bent into a slight “S” shape. This deformation precedes bending of leaf spring in the more dominant flexure mode, drawing the top of the leg closer to the body during full compression. A slight forward pitch is induced in the body of the hopper during each bending of the leaf spring. The body pitching was subtle and somewhat counteracted by the resistance of the flexible tail rod.

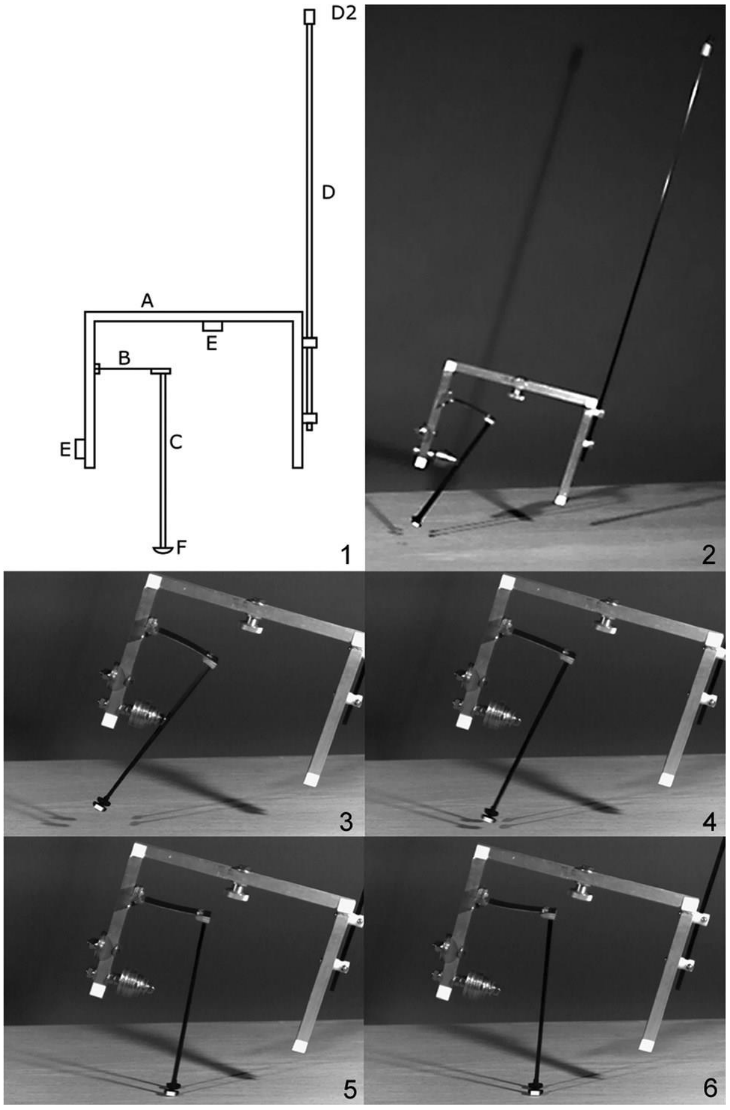

Fig. 6 Final form of the statically unstable hopper. Schematic (1) shows body frame A, leaf spring B, and leg C. Leg and frame were joined by leaf spring B. The tail mast D with small terminal mass D2 were adjustably attached to the frame. Mass E was adjustable and could also be secured in various positions as a means of positioning the net center of mass. Foot F was hemispheric rubber. Complete specifications are in Appendix B. Photographs were extracted from high speed video footage and show the full device (traveling from right to left) in flight (2), and in close-ups of the leg movement during a hop cycle (3–6). Flexure modes of the leaf spring are shown.

5 Results

The final statically unstable hopper could hop unassisted down a 16 foot plywood ramp for more than 70 steps (multimedia extension 6 video). High speed video shows the details of the leg motion (multimedia extension 7 video). It was found that the manual launching technique did not require significant skill.

Analysis of the audio track from one of the long hopping trials reveals a frequency of about 10.5 hops per second. A comparison between the first ten hops after release, and the last ten hops near the end of the ramp revealed a slight decrease in hop frequency (from ten hops per second to 9.65 hops per second) during the length of the run.

The hopper pitched slightly forward and backward during an interval of several hops. In the final hopper, the cycle repeated after approximately ten hops, while in the preceding lighter-mass model the cycle repeated approximately every five hops. A, perhaps related, "totter mode" was predicted by McGeer in the first paper on passive dynamic running [11].

Retrospective evaluation of the hopper models, in order of construction sequence, showed a decrease in the leg mass (including spring) relative to the body mass. The wide foot hopper had a leg-to-body mass ratio of 0.23, while the subsequent devices had leg-to-body mass ratios of 0.11, 0.06, and 0.05, for the rubber band suspension hopper, the U-shaped aluminum frame hopper, and the final form hopper, respectively.

Light leg mass relative to body mass is common in the animal kingdom. Rear leg muscles in hopping animals, such as wallabies, move very little during a hopping cycle. Low mass tendons provide a significant portion of the animal's elastic energy storage [20]. The final version hopper, with its very light leg could be comparatively viewed as functioning with tendons alone.

A "duty factor," or time of stance phase compared to flight phase, of 0.36 was found from high-speed video of the final statically unstable hopper. Coincidentally, or not, this approximately matches the duty factor of a kangaroo rat [21] (see high-speed video therein).

A measure of energy effectiveness in locomotion is the specific cost of transport CoT [22]. For passive models, which rely exclusively on gravitational potential energy to replace energy dissipated as they go down a ramp, the specific cost of transport is the sine of the ramp slope [23] (a smaller slope corresponds to smaller CoT and greater effectiveness). For this passive hopper slope=CoT=0.079, placing it between 0.05 for the passive biped with knees [24] and CoT=0.22 for a passive running biped [12].

Following the development of the statically unstable hopper, I was aided by others who provided independent mathematical analysis [25].

6 Discussion

We have designed, built, and tested a physical passive mechanical device with somewhat animal-like hopping characteristics. In that these robots have no electronic control, their stability relies on mechanics alone; i.e., they can only achieve stable hopping due to the interaction of forward inertia, material behavior, kinematic constraints, and mass distribution. The apparent stability of the final two models is remarkable, because it is essentially dynamic; when not hopping, they cannot stand up.

While one could imagine that the design process used here might have been replaced by numerical optimizations of simulations, it is also possible that human intuition was (at this point in the development of automatic design) more effective.

It is my hope that the mechanisms presented will inspire future toy makers and investigators of passive dynamics generally. Certainly, this project expanded my appreciation of the mechanics underlying nature. It also opened my eyes to the wide range of potential passive dynamic locomotion devices. For instance, I found that a simple block of Styrofoam was an almost stable hopper (multimedia extension video 8).

Acknowledgment

Thank you to Andy Ruina for his encouragement, years of correspondence, and editorial help; to Tim van Oijen and Gregg Stiesberg for editorial help; and to Michael Coleman for his inspiration, encouragement, and editorial help. High speed video footage was graciously provided by Tim Richardson of Fast Video NW, Vancouver, WA.

Appendix A

The multimedia extensions to this article can be found in our Supplemental Material. The table below provides an index to the multimedia extensions.

Extension	Media type	Description	Year of footage
1	Video	Passive toy compilation	2006–2009
2	Video	Car-and-ball toy	Remake of 2006 experiment
3	Video	Small hopping toys	2006–2009
4	Video	First upright hopper	2006
5	Video	First statically unstable hoppers	2007
6	Video	Final statically unstable hopper	2008
7	Video	Slow motion footage of final hopper	2008
8	Video	Styrofoam beam hopper	2009

Summary notes on videos

(1) Passive toy compilation: Various ramp-descending models were produced to explore the range of body configuration and locomotion modes possible with ramp powered toys. Devices displaying periodic forward movement on curved feet are shown, along with conceptual drawings and paper models.
(2) Car-and-ball toy: A mechanism consisting of a toy car, curved wire spring, and rubber ball display rudimentary hopping motion. Slow motion footage shows ball trajectory during hopping cycle.
(3) Small hopping toys: Toys with body mass, coil spring, and lightweight curved foot display repetitive hopping. A metal ball concealed within clay comprises the body mass of the first device shown. Slow motion footage shows body pitching during stance phase.
(4) First upright hopper: A spring suspension is shown in simulation and is applied to a statically stable upright model with springy foot and swinging leg. Repetitive hopping is demonstrated.
(5) First statically unstable hoppers: Initial trial of statically unstable hoppers displays short distance hopping with a fore/aft pitching oscillation mode. Two devices are shown. 00:06.07 First statically unstable device with single leg displays repetitive hopping motions, but accelerates and topples quickly.
00:22.68 Statically unstable hopper with aluminum frame and two legs fixed in a linear configuration shows increased hopping distance, but still accelerates.
(6) Final statically unstable hopper: Statically unstable hopper with small curved feet performs runs of more than 70 unassisted hops.
(7) Slow motion footage of final hopper: Statically unstable hopper is shown in slow motion and close-up to reveal gait and spring movement characteristics.
(8) Styrofoam beam hopper: Subsequent to the final hopper, a beam-shaped piece of Styrofoam, previously packaging material, is found to flex and hop in a nearly stable way. The concept was unrefined, but interesting in its simplicity.

Appendix B: Hopper Technical Details

Material list
Body material: $^{1}=_{2}$ in./12.7 mm U-shaped aluminum extrusion with welded corners
Tail mast material: Carbon fiber kite tubing. 1880 in./4.8 mm diameter
Leg material: Carbon fiber kite tubing. 1570 in./4.0 mm diameter (legs were epoxy glued into thin plywood mount).
Leaf spring: Spring steel 0.045 mm thick, 16 mm wide. Two pieces totaling 10 g (mass of spring exceeds mass of leg assembly). The mass could be considered in numeric modeling. Note free spring distance and angle relative to body in Fig. 7(b).
Total device weight, including tail mast: 412 g
Total tail mast length including top mass: 67.9 cm
Tail top mass: 12 g
W1 washer assembly mass: 52 g
W2 washer assembly mass: 152 g
W3 adjustable mass: 12 g
Leg assembly weight, including wood, hardware, and feet: 9.5 g
Center of balance of leg assembly: approximately 79 mm from top
Slope of ramp used in long hopper run videos: 0.07923 rad Free oscillation rate of leg assembly as measured with strobe: 14.83 cycles per second
Total angular rotation of leg during hopping and stance: 31 deg
Foot components: vinyl end cap from a kite supply company and curved rubber foot made from a 5/8 in. diameter conical household faucet washer cut and profiled into an approximate 0.50 in. radius.
Leg oscillation rate seemed critical to operation of the hopper. Future design considerations could include a sliding adjustable means for altering leg mass distribution.

Appendix C: Various Historical Passive Dynamic Toys

(1) Page devoted to the Tinkertoy brachiator, including videos and publications (with some construction details and patent references for old ape-swinging toys): http://www.cem.uvm.edu/~mcoleman/tinkertoy_brachiator_images.html
(2) Page devoted to the Tinkertoy walker, including publications, video, and construction details: http://ruina.tam.cornell.edu/research/topics/locomotion_and_robotics/tinkertoy_walker/
(1) Ramp walker of the Wilson Walkie type design: https://www.youtube.com/watch?v=gSaRQ3yR_b4
(2) Shows variations on passive tumbling toy: https://www.youtube.com/watch?v=CfCqWEdos2A
(3) Several planar wood walking toys: https://www.youtube.com/watch?v=qQjqNnOZbf8
(4) Demonstration of a McGeer-type passive walker on treadmill: https://www.youtube.com/watch?v=CK8IFEGmiKY

(5) Traditional Japanese tumbling toy: https://www.youtube.com/watch?v=Gmh5jeAWSGU
(6) Woodpecker toy, with slow motion video: https://www.youtube.com/watch?v=s3YSnNAIHDg
(7) Slinky shown on treadmill: https://www.youtube.com/watch?v=711bZ_pLusQ
(8) Clock that operates by slowly descending a ramp: https://www.youtube.com/watch?v=6BdpT8yjRm8

References

[1] Wilson, J. E., 1938, "Walking Toy," U.S. Patent No. 2,140,275.
[2] Collins, S., Ruina, A., Tedrake, R., and Wisse, M., 2005, "Efficient Bipedal Robots Based on Passive-Dynamic Walkers," Science, 307(5712), pp. 1082–1085.
[3] Coleman, M. J., and Ruina, A., 1998, "An Uncontrolled Walking Toy That Cannot Stand Still," Phys. Rev. Lett., 80(16), p. 3658.
[4] Matsuoka, K., 1979, "A Model of Repetitive Hopping Movements in Man," Fifth World Congress on Theory of Machines and Mechanisms, Montreal, Canada, July 8–13, pp. 1168–1171.
[5] Raibert, M. H., Brown, H. B., and Chepponis, M., 1984, "Experiments in Balance With a 3D One-Legged Hopping Machine," Int. J. Rob. Res., 3(2), pp. 75–92.
[6] Mombaur, K. D., Longman, R. W., Bock, H. G., and Schl€oder, J. P., 2005, "Open-Loop Stable Running," Robotica, 23(1), pp. 21–33.
[7] Seth, B., Seshu, P., Shanmuganathan, P. V., Vichare, V. V., and Raj, P., 2007, "Search for Initial Conditions for Sustained Hopping of Passive Springy-Leg Offset-Mass Hopping Robot," ASME J. Dyn. Syst., Meas., Control, 129(4), pp. 522–526.
[8] Seshu, P., Seth, B., and Shanmuganathan, V., 2006, "Passive Dynamics of Kangaroo-Like Hopping Robot," J. Inst. Eng. (India), 87(3), pp. 39–45.
[9] Ringrose, R., 1997, "Self-Stabilizing Running," IEEE International Conference on Robotics and Automation (ICRA-97), Albuquerque, NM, Apr. 20–25, Vol. 1, pp. 487–493.
[10] Iida, F., Reis, M., Maheshwari, N., Yu, X., and Jafari, A., 2012, "Toward Efficient, Fast, and Versatile Running Robots Based on Free Vibration," 7th Annual Dynamic Walking Conference (DWC2012), Pensacola Beach, FL, May 21–24, pp. 162–163.
[11] McGeer, T., 1990, "Passive Bipedal Running," Proc. R. Soc. London, Ser. B, 240(1297), pp. 107–134.
[12] Owaki, D., Koyama, M., Yamaguchi, S., Kubo, S., and Ishiguro, A., 2011, "A 2-D Passive-Dynamic-Running Biped With Elastic Elements," IEEE Trans. Rob., 27(1), pp. 156–162.
[13] Srinivasan, M., and Ruina, A., 2006, "Computer Optimization of a Minimal Biped Model Discovers Walking and Running," Nature, 439(7072), pp. 72–75.
[14] Alexander, R. M., 1990, "Three Uses for Springs in Legged Locomotion," Int. J. Rob. Res., 9(2), pp. 53–61.
[15] Cavagna, G. A., 1970, "Elastic Bounce of the Body," J. Appl. Physiol., 29(3), pp. 279–282.
[16] Coleman, M. J., 2006, "Upside Down Walking: A Passive-Dynamic Brachiating Robot Toy," Dynamic Walking 2006: Mechanics and Control of Human and Robot Locomotion, Ann Arbor, MI, May 4–8.
[17] Zeglin, G., and Brown, B., 1998, "Control of a Bow Leg Hopping Robot," IEEE International Conference on Robotics and Automation (ICRA-98), Leuven, Belgium, May 16–20, Vol. 1, pp. 793–798.
[18] Leine, R. I., Van Campen, D. H., and Glocker, C. H., 2003, "Nonlinear Dynamics and Modeling of Various Wooden Toys With Impact and Friction," J. Vib. Control, 9(1–2), pp. 25–78.
[19] Steinkamp, P., 2007, "Homemade Passive Dynamic Toys," Dynamic Walking III Conference, Mariehamn, Åland, Finland, June 24–30.
[20] Biewener, A. A., Konieczynski, D. D., and Baudinette, R. V., 1998, "In Vivo Muscle Force-Length Behavior During Steady-Speed Hopping in Tammar Wallabies," J. Exp. Biol., 201(11), pp. 1681–1694.
[21] Gutmann, A. K., Lee, D. V., and McGowan, C. P., 2013, "Collision-Based Mechanics of Bipedal Hopping," Biol. Lett., 9(4), p. 20130418.
[22] Tucker, V. A., 1970, "Energetic Cost of Locomotion in Animals," Comp. Biochem. Physiol., 34(4), pp. 841–846.
[23] Garcia, M., Chatterjee, A., and Ruina, A., 2000, "Efficiency, Speed, and Scaling of Two-Dimensional Passive-Dynamic Walking," Dyn. Stab. Syst., 15(2), pp. 75–99.
[24] Collins, S. H., Wisse, M., and Ruina, A., 2001, "A Three-Dimensional PassiveDynamic Walking Robot With Two Legs and Knees," Int. J. Rob. Res., 20(7), pp. 607–615.
[25] Stiesberg, G., van Oijen, T., and Ruina, A., 2016, "Steinkamp's Toy Can Hop 100 Times But It Can't Stand Up," ASME J. Mech. Rob., 9(1), p. 011013.