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A QUASISTATIC ANALYSIS OF SOME DRY FRICTIONAL BEHAVIOR

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November 1980

This manuscript is the text of a thesis submitted by the author in partial fulfillment of requirements for a Ph.D. in Engineering at Brown University.

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November 1980

ABSTRACT

Failures of traditional friction laws motivate an internal variable representation for a memory dependent friction law. Conditions are stated for the validity of a description with one internal variable and found to be approximated by, but technically in disagreement with limited accurate experiments which require two internal variables for accurate description. The internal variable friction laws predict the following results: Stability of massless elastic systems in contact with a boundary governed by a memory dependent law depends largely on $v(d\mu^*(v)/dv)/d_c$ where v is slip velocity, $\mu^*(v)$ is the steady state dependence of the coefficient of friction μ on v and d_c is a characteristic displacement of the memory. Stability also depends on the ratio of elastic compliance to normal stress. At neutral stability a spring and massless block system may oscillate steadily. A simple one dimensional fault allows propagating creep waves. A limited class of internal variable friction laws yield a scaling rule that predicts, for example, that in some experiments 'time dependence' of static friction must be scaled by previous and successive slip velocity. An extension of a one internal variable friction law to continuous deformation predicts localization of deformation before any elastic unloading instabilities.

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Acknowledgement

All of the ideas here have been inspired and developed by discussion with Jim Dieterich, Jim Rice, Don Simons and others. The author has been supported in part by a National Science Foundation Energy Traineeship and by the U.S. Geological Survey, and the work has been supported by grants from the NSF Geophysics Program and U.S.G.S. Earthquake Hazards Reduction Program to Brown University.

I am grateful to the following secretaries at the Division of Engineering of Brown University for assistance with the preparation of the final manuscript: Ms. Sara Mancino, Ms. Patricia Orzechowski, and Ms. Linda Trevitt.

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Introduction

The motion occurring when two objects in frictional contact slide results from the interaction of the frictional surface with the surrounding mechanical system (i.e., the adjoining bodies and loading machinery). Unstable motion ('stick-slip' for example) is a property neither of a material surface nor of the surrounding machinery, but rather of the combined system. Systems exhibiting frictional instabilities include string instruments, squeaky machinery, and the earth's crust. It is not clear, however, what the phenomena associated with these systems have in common, either in the frictional or the "machine" properties. From what we know of earthquakes, for example, they may only be related to friction, or what is commonly called friction, by the observation that they involve concentrated shear deformation.

Various models can and have been used to discuss the motion associated with 'stick-slip' and related instabilities in the context of many mechanical systems. Of particular interest in this paper are the conditions under which instabilities can be expected, with emphasis on friction laws rather than complex mechanical systems. In particular, several commonly used friction laws will be briefly reviewed along with some stability implications, mostly in the context of the spring-block model of Figure 1. This discussion will make it clear that for proper modeling of instabilities the slip (or memory) and slip rate dependence of friction must be understood. A new friction law from work of Dieterich (1978), related to concepts introduced earlier by

Rabinowicz (1958) will be discussed which will be followed by a discussion putting this law in the more general context of an internal variable law. Recent relevant experiments will then be briefly discussed. Various stability implications are found for this type of law. First, stability criteria for constant force loading and steady sliding in the spring-block model of Figure 1 are determined. Quasi-static (i.e., intertia neglected) oscillations will be found to be an almost necessary consequence of the Dieterich-like friction laws. Experimental and numerical verification of these oscillations will be mentioned. Next, a simple one-dimensional model will be used to consider a continuum described by a Dieterich-type law (for shear strain rather than slip) and localization is found to be possible even while 'strengthening' and is at least initiated before elastic unloading instabilities are possible. Lastly, some applications of the frictional slip laws are made to fault mechanics and the possibility of creep waves is discussed by analysis of the stability of steady slip in an elastic slab.

Spring-Block Model

The simplest mechanical system in which to examine stability of sliding is the spring-block of Figure 1. A rigid block is held in frictional contact with another rigid surface by a normal force N that may, in general, depend on the sliding displacement δ or the load point displacement u. The load point is connected to the block by a spring of stiffness $\hat{k}$ which transmits a force F in the direction of sliding. In this model all the inelastic shear deformation at a frictional surface region is modeled by δ, the slip displacement. The load point displacement u represents the displacement at the point at which the loading machine is controlled. The displacement u, no matter what is represents (hydraulic oil pumped, gears turned, or the shift of the base of a continental plate, for example) is measured in such a way that $u = \delta$ for constant force loading. All elastic compliance, e.g., elastic sample deformation, is incorporated in the spring constant $\hat{k}$. The model does not usually include inelastic deformation away from the frictional interface or non-linear elastic response, but these effects can be added. If the frictional phenomena to be discussed scale with normal force, as is commonly assumed, the stiffness $\hat{k}$ and friction force F can be normalized by the possibly varying normal force N. Then F is replaced by $\mu \equiv F/N$ and $\hat{k}$ by $k \equiv -d(F/N)/d\delta$ (fixed u). There is little explicit evidence that all terms in the laws to be discussed do in fact scale well with normal force although that is perhaps the best naive guess and is at least not inconsistent with observations.

For quasi-static (inertia neglected) analysis, the spring-block model is not much of an idealization so long as slip in the modeled system is effectively homogeneous (not varying) on the slip surface. For dynamic analysis the block is endowed with a mass m. This is only an accurate model if the modeled system is dominated by a single mode of vibration. Furthermore, care must be exercised in applying the dynamical model if the normal force is not constant; in such circumstances the classical stick-slip calculations do not apply. Most frictional instability analyses use the dynamic (lumped mass) spring-block model. Our attention will focus on quasi-static motion and the onset of instabilities and so the block mass will most often be neglected and there will be no discussion of inertia governed slip times, stress drop overshoots, or other factors which depend on inertia or kinetic energy.

A further feature of the spring-block model is that specification of the load point motion $u(t)$ approaches displacement control or force control in the high and low stiffness limits so that the constitutive description is a limiting form of the model response.

Some Friction Laws

The friction force, for a given material, is dependent on the history of the sliding surface. Here we take the term "friction law" to mean the dependence of the friction force on the sliding history. Thus the same material may have different friction laws depending on the surface preparation, the chemical environment, the surrounding temperature, etc. The coefficient of friction μ might also depend on the history of the normal force N but this dependence will be neglected. Either the normal force will be assumed to be constant or all properties of μ will be assumed to be independent of any variations in N, depending on context.

The simplest friction law is that $\mu \equiv F/N$ is a constant during slip and that μ is not exceeded if the slip rate is zero. If the normal force N is constant, this law is stable in a quasi-static analysis of the spring-block model. For any forward motion of the load point $u(t)$ either (a) the force imposed by u via k is not enough to cause sliding, or (b) the equilibrium solution, $\delta = u-\mu/k$, is self-correcting (any imagined deviation of δ from this solution would cause a force on the block in the opposite direction from the deviation). If inertia is included, this model is neutrally stable, in that for constant $\dot{u}$, $\dot{\delta}$ can oscillate indefinitely about the equilibrium solution $\dot{\delta} = \dot{u}$ (the superposed dot denotes differentiation with respect to time).

A constant μ friction law does not necessarily lead to this moderately stable behavior in all systems, however. For example, if the normal force N is dependent on δ , unstable motion results if the impoed F/N increases with slip displacement δ for fixed u . This is the case for reverse slip in an elastic self-locking brake and can be the case for forward slip in saw cut specimens in the standard 'triaxial' machine of geomechanics (if the machine is sufficiently compliant in the axial direction and if overhanging slip surfaces are in communication with the confining fluid). In addition, a strongly non-linear spring, exhibiting a negative incremental spring modulus, in the spring-block model can lead to instabilities even with μ constant. We shall neglect these effects for the rest of this paper and shall assume that the spring is linear and that the normal force corrected stiffness $k \equiv -d(F/N)/d\delta$ (at fixed u) is positive.

A more sophisticated friction law is that the friction force is velocity dependent. That is, $\mu = \mu(\dot{\delta})$. An analysis of this law in the spring-block model shows that motion at a constant speed v by the load point cannot be followed stably by the block if $d\mu/d\dot{\delta} < 0$. That is, both dynamic and static models yield solutions near the equilibrium solution that grow in time for any finite k . If μ increases with slip speed $\dot{\delta}$, then sliding at constant speed is stable for all k . Thus, if friction is a decreasing function of slip velocity, and if a testing machine can be modeled as in Figure 1, the friction law cannot be measured in steady sliding experiments unless sufficient damping parallel to the spring is introduced (Vatta 1979). The apparent paradox that steady sliding is, in fact, observed in cases where friction is a decreasing function of velocity can be explained without external damping by use of the more complete frictional constitutive description discussed later.

Most often frictional instabilities are discussed in terms of a friction law that has a "static" coefficient of friction μ_{stat} that exceeds the sliding friction μ_k, where μ_k may or may not depend on slip velocity $\dot{\delta}$. Steady following of constant speed load point by the block is or is not possible depending on the sign of $d\mu_k/d\dot{\delta}$ as just discussed. This friction law does, however, allow episodic slip, even if μ_k is independent of $\dot{\delta}$, as described in most 'classical' discussions of "stick-slip." The load point first moves at a constant velocity while the block does not slide and the force transmitted by the spring increases. When the applied force reaches $N\mu_{stat}$, sliding begins and the friction force instantaneously drops to $N\mu_k$ causing an inbalance of applied forces and acceleration limited only by inertia of the block. Sliding proceeds as simple harmonic motion (if μ_k is independent of $\dot{\delta}$) until the slip velocity $\dot{\delta}$ returns to zero and the strength returns to the static level and the cycle repeats. This type of motion is possible at all load point speeds and spring stiffnesses no matter how μ_{stat} depends on time of contact or tangential load history so long as μ drops a finite amount to μ_k once any sliding occurs. The static coefficient μ_{stat} is commonly thought to be an increasing function of time of contact (Dokos 1946; Dieterich 1972). More recently it has been claimed (Johannes et al. 1973) that μ_{stat} actually depends on the rate of tangential load application $\dot{F}/N$. In either the case of contact time or load rate dependence, the diminished amplitude of stick-slip oscillations with increased sliding velocity can thereby be explained, but not the result that the amplitude of stick slip goes firmly to zero at a critical speed (Rabinowicz 1958).

Displacement Effects

Rabinowicz proposed that displacement memory effects (Heyman et al. 1955; Rabinowicz 1956, 1958) could explain inhibition of stick-slip at high load point velocities $\dot{u}$ and stiffnesses k. Two displacement effects (memory effects) were noticed by Rabinowicz and later with rocks by Dieterich (1978) and used to obtain stick-slip criteria. First, as sliding starts after a "stick," the friction force drops gradually to μ_k over a characteristic distance. Second, if the surface shows a velocity dependence and the slip velocity is suddenly changed from one velocity to another, the friction force changes its value over a characteristic distance denoted by d_c by Dieterich (1978). Rabinowicz (1956) further correlated this distance with a characteristic distance of force fluctuations during steady sliding. Rabinowicz argued that d_c represents a lower limit on the slip distance during a stick-slip event. If the slip distance that would be expected for a given μ_{stat} and μ_k (with instantaneous displacement weakening) is less than d_c, no stick-slip will occur. Since the distance slid in the simple dynamic analysis is $2(\mu_{stat}-\mu_k)/k$ this gives the instability condition

$$2(\mu_{static}-\mu_{kinetic})/k > d_c \rightarrow \text{instability} . \quad (1)$$

Dieterich (1978) observes that instability of a quasi-static slip motion will occur if the frictional force the surface can support drops more than the spring force during an increment of displacement. Hence, for a friction law that can be approximated by a linear displacement dependent drop from the static to the kinetic friction force, this yields

$$(\mu_{static} - \mu_{kinetic})/k > d_c \rightarrow \text{instability} \qquad (2)$$

which, although argued differently, is essentially identical to (1) since d_c has not been defined carefully.

Using the concept of a critical distance, both Rabinowicz (1958) and Dieterich (1978) attempted to explain the relationship between the dynamic coefficient of friction as a function of velocity $\mu_k(\dot{\delta})$ and the static coefficient of friction as a function of time of contact $\mu_{stat}(t)$. Rabinowicz calls d_c a lower limit on the 'resolving power of the friction process' implying the equivalence of steady sliding at speed v and the intermittent sliding (at an unspecified speed) of distances d_c with stationary times of d_c/v. Thus,

$$\mu_k(v) = \mu_{stat}(d_c/v) \quad , \qquad (3)$$

where v is a steady state slip velocity and d_c/v is an equivalent stationary contact time that causes the same friction force. Similarly, Dieterich claimed that friction is fundamentally a function of the average time of asperity contact. If d_c is the slip displacement required to change from one 'population' of asperity contacts to a new population then the average time of asperity contact is given by d_c/v. This again leads to Eq. (3). Both Rabinowicz and Dieterich back their claims with experimental evidence showing that the roughly logarithmic increase in the coefficient of 'static' friction with time can be correlated with the roughly logarithmic decrease of kinetic friction with slip velocity by introduction of a characteristic distance d_c. For sliding with varying velocity where the steady state results do not apply, Rabinowicz suggested that friction should fundamentally be taken as a function of some kind of weighted average of the slip velocity over the distance d_c.

More recently Dieterich (1979a) recognized from his experimental results with rocks that his average time of contact variable τ was not sufficient to describe the frictional behavior. He observed the following: (1) When the slip velocity was suddenly increased from one constant value to another the relaxation to a lower friction force over the characteristic distance d_c was preceded by a transient increase in the friction force. (2) After nominally stationary contact which followed steady sliding the force achieved when steady sliding was resumed was greater for large resumed starting speeds (note that this result is the opposite of that of Johannes et al., (1973), although differences in the "static" friction experiment may account for this (Ruina 1980)). Dieterich accounted for these observations by giving the friction coefficient an explicit dependence on the slip velocity as well as on his average time of contact τ. He also gave a rule for how the variable τ should change when the slip velocity is suddenly changed from one constant value to another. He then viewed a continuously varying slip velocity as a sequence of short constant velocity steps. The full law Dieterich (1979) used is

$$\mu = C(\tau)F(\dot{\delta}) \tag{4a}$$

$$C(\tau) = c_1 + c_2\log_{10}(c_3\tau+1) \ , \ F = f_1 + [f_2\log_{10}(f_3/\dot{\delta}+10)]^{-1} \tag{4b,c}$$

$$\tau = (d_c/\dot{\delta})(\dot{\delta}\tau_o/d_c)^{\exp(-(\delta-\delta_o)/d_c)} \tag{4d}$$

where c_1 , c_2 , c_3 , f_1 , f_2 , f_3 and d_c are material constants. Equation (4d) describes the evolution of τ when the slip speed is the constant $\dot{\delta}$ beginning at δ_o , at which point τ has the value τ_o. During constant rate sliding new τ_o , δ_o pairs may be assigned from the current values of τ, δ without altering the evolution of τ.

Equations (4) were constructed to have the following properties: (1) Experimental results of Dieterich (1972) for time dependent static friction are reproduced if τ is replaced by time of stationary contact in (4a,b) and one slip speed is used for all experiments; (2) During slip at constant rate $\dot{\delta}$, the "average contact time" τ approaches $d_c/\dot{\delta}$ in the evolution law (4d) so as to reproduce the idea expressed in the discussion after Eq. (3); (3) A sudden change in slip velocity causes a sudden change in friction force of the same sign as expressed by the continuity of τ with $\dot{\delta}$ in (4d) and the direct slip rate dependence in (4a,c); (4) After sudden changes in slip velocity, the friction force approaches a steady state level over a characteristic distance d_c ; (5) The direct dependence of μ on slip velocity in (4c) is roughly logarithmic for an intermediate range of velocities but finite for all slip velocities.

One may note that the dependence of μ on slip velocity makes the concept of static friction somewhat ambiguous, however. Furthermore, Eq. (4c) is not compatible with time dependence of static friction, as will be discussed, leaving only a subtle connection of Eqs. (4) to static friction experiments. The association made between $\mu_k(\dot{\delta})$ and $\mu_{stat}(\tau)$ in (3) is also effectively lost because of the direct slip velocity dependence in (4a,c).

Familiarity with the Dieterich law (4) can be obtained from studying Dieterich (1979a) and especially Figure (6) therein.

Due to the complexity of Eqs. (4), analytic results are difficult to obtain. Dieterich (1978, 1979a) approximates the law as a displacement weakening law to get stability results similar to Eq. (2). He also has done a finite element analysis of a deformable block with an interface law described by Eqs. (4) (1979b), and some numerical spring-block models using Eqs. (4) or variations thereof (1980). Kosloff and Liu (1980) have used Eqs. (4) to numerically solve for displacement vs. time in constant force loading.

Although the initial attention of both Dieterich and Rabinowicz on friction laws was focused on time dependence of static friction, later emphasis was on transients in the friction force during sliding. In fact, in the light of the friction law just described, it is not clear what is meant by static coefficient of friction μ_{stat}.

In both experiments, and in this theory, visible sliding often precedes the peak friction force. Furthermore, there is strong evidence that the peak friction force depends on the sliding history preceding the stationary time, the imposed force history during nominally stationary contact, and the rate at which displacement or force rate is resumed afterwards (Ruina 1980).

In the next section we introduce a Dieterich-like friction law without giving "static" friction such a central role. The law is deduced on the basis of assumptions that have been or could be tested experimentally.

State and Steady State

A constitutive law for friction describes the properties of a 'point,' or representative small section on what is loosely called the sliding surface. By "sliding surface" is meant a layer near the surface thick enough to include all the inelastic effects that are not in the surrounding bulk material, yet thin enough to be considered mathematically a surface in terms of the bulk material. Further, for this paper we want the full interaction between the "surface" and the bulk material to be adequately described by the traction vector across their common boundary. Thus, if the friction force is affected by temperature alterations due to slip, these alterations must be confined to a boundary layer small compared to the bulk sample size or else either of the extremes of an isothermal or adiabatic process must be assumed (the steady state, to be discussed, is not, however, compatible with an adiabatic process). We leave as unspecified which of these three alternatives is most appropriate in explaining experimental observations and do not refer to temperature explicitly in the constitutive description, despite the fact that full understanding of the physics of the frictional process is thus being precluded. If one does want to account for thermal interaction between the 'surface' and bulk material, assumptions must be made about the surface constitutive behavior with respect to temperature variations. Similar considerations apply for diffusing reactive chemicals and pore fluid where something like the Terzaghi effective stress law may hold (Jaeger and Cook 1976). These interactions may well be important in many situations; however, their explicit consideration does not seem to be necessary for the observed slow slip rate experiments that motivated this paper. Note that neglect of thermal and chemical interactions does not imply that temperature and chemical environment are not extremely important--only that the effect of variations of these quantities due to the frictional process itself may be neglected or is tacitly incorporated in the friction law. Neglect of the normal traction can be rationalized, as discussed earlier, by assuming it is constant or not relevant to properties of the friction coefficient μ .

At any given instant a point on the surface is fully characterized by the details of the microscopic asperity geometry and composition as influenced by the external load history. For the following discussion the term 'state' means a sufficient amount of this information about the surface in order to fully predict the instantaneous response to any controlled macroscopic variables. The term 'response' denotes the values of any uncontrolled macroscopic variables of interest as well as the rate of change of the state. In this paper either the slip rate $\dot{\delta}$ or the imposed coefficient of friction F/N will be considered as the controlled macroscopic variable and the other as part of the response. From knowledge of the state at any instant, μ as a function of the instantaneous value of $\dot{\delta}$, may be thought of as a property of the state. This particular separation into state and property is motivated by the observation that the coefficient of friction appears to respond instantly to changes in slip rate $\dot{\delta}$ in a way that changes continuously depending on $\dot{\delta}$ (i.e., $d\mu/d\dot{\delta}$ evaluated at a fixed state is finite and generally non-zero). On the other hand, even in a very stiff machine (Ruina 1980), step changes in $\dot{\delta}$ only cause finite changes in μ so there is apparently no explicit dependence of μ on $\ddot{\delta}$ or higher derivatives at a fixed state. In some special cases an explicit dependence on $\ddot{\delta}$ can provide a course scale approximation to a constitutive law of the form being developed.

In order for the concept of state to be useful it must be describable with very few numbers θ_i or collectively $\underline{\theta}$. The discussion in the past few paragraphs can then be summarized by

$$\mu = \mu(\theta,\dot{\delta}) \tag{5a}$$
$$\dot{\theta}_i = \dot{\theta}_i(\underline{\theta},\dot{\delta}) \tag{5b}$$

where the functions $\dot{\theta}_i(\underline{\theta},\dot{\delta})$ are finite for finite $\dot{\delta}$. We shall maintain the somewhat ambiguous notation that μ and $\dot{\theta}$ are both the current value of the quantities they describe and functions of their respective arguments (which may depend on context).

The variables θ_i represent some kind of averages of the undoubtedly complicated state on the surface when examined microscopically. For example, Dieterich (1978, 1979a) suggests a single variable representing average asperity 'contact time' and more macroscopically, Rabinowicz (1958) suggests average recent slip velocity. We leave for speculation or future investigation what, physically, the variables θ_i represent. Whether or not there are microscopic models that approximately or rigorously yield such macroscopic variables as θ_i is unknown. Their usefulness, like temperature and entropy in thermodynamics, does not, however, rest on microscopic justification, although such justification would add tremendously to the credence and usefulness of the theory.

A further feature that friction laws seem to have is a fading memory. This attribute was implicit throughout the Introduction. More specifically, if a slip rate history $\dot{\delta}(t)$ is imposed on a surface and the response $\dot{\delta}(t)$ is observed then if subsequent to arbitrary sliding the slip history $\dot{\delta}(t)$ is repeated (time origin offset) then the response $\mu_2(t)$ will approach $\mu_1(t)$ once sufficient time or displacement has elapsed, depending on the nature of the fading memory. In other words, reproducible results may be obtained with a single surface merely by offsetting the time and displacement origin. A fading memory implies the existence of a steady state corresponding to constant velocity sliding since then the same slip history is being reapplied continuously and continuously repeatable results must be obtained. In terms of the friction law (5) the existence of a steady state is interpreted to mean that for any value of $\dot{\delta}$ there are corresponding values of θ_i and μ denoted $\theta_i^*(\dot{\delta})$ and $\mu^*(\dot{\delta})$, to which θ and μ must approach closely after sufficient time or displacement a a constant slip rate. Since experimental results of Ruina (1980) fit well in such a framework we shall for simplicity assume that the evolution of each state variable depends only on its own value, $\dot{\theta}_i(\underline{\theta},\dot{\delta}) = \dot{\theta}_i(\theta_i,\dot{\delta})$. Note that this precludes oscillatory approach of θ_i to steady state for fixed $\dot{\delta}$. So we now have

$$\mu = \mu(\underline{\theta}, \dot{\delta}) \tag{6a}$$

$$\dot{\theta}_i = \dot{\theta}_i(\theta_i, \dot{\delta}) \tag{6b}$$

where $$\infty > \dot{\theta}_i(\theta_i, \dot{\delta})/(\theta_i^*(\dot{\delta}) - \theta_i) > 0 \tag{6c}$$

and $\theta_i^*(\dot{\delta})$ (the solution of $\dot{\theta}(\theta, \dot{\delta}) = 0$ for θ in terms of $\dot{\delta}$) is, again, the steady state value of θ_i corresponding to $\dot{\delta}$. The inequality of (6c) implies exponential approach to a unique steady state value.

Before proceeding further it should be pointed out that many friction experiments with rocks do not show the assumed fading memory and steady state behavior. A typical triaxial test shows a long term transient in the level of the coefficient of friction for the duration of the experiment (e.g. Summers and Byerlee, 1967). These tests are, however, limited in the amount of slip available and are also possibly affected by the constantly changing normal stress. Further, experiments by Dieterich (1980) and Ruina (1980), aimed to look at effects like those described by eqs. (4) or (6) found long term transients in the short term behavior. Dieterich (1980) suggests that both of these long term transients are due to the increase in a number of wear particles (fault gouge) as sliding proceeds and Ruina (1980) notes that progressive chemical contamination may also contribute. The long term transients of Dieterich and Ruina changed very slowly after sufficient displacement (1-4 cm., which is much larger than the .001 to .1 mm. of slip in which the features we study here are exemplified) implying the eventual validity of the steady state assumption. Even during the transient behavior, a description like eq. (4) or (6) seemed to apply during any small distance of sliding (small compared to total experiment duration but large with respect to characteristic distances implicit in 6b). In the presence of gouge, large drops in the imposed coefficient of friction, larger than would occur naturally from instability, were found to cause increases in μ that did not go away over the distances associated with the fading memory (Dieterich (1980), Ruina (1980), Summers and Byerlee (1977)). This effect, whatever its mechanism, is beyond what we claim is explicable in terms of eqs. (6), (as they are now understood).

Characteristic Distances

As noted, equations (6) have, as one solution, the steady state solution, when $\dot{\delta} = v = \text{constant}$:

$$\mu = \mu(\underline{\theta}^*(v), v) = \mu^*(v) \tag{7a}$$

$$\dot{\theta}_i = 0 \tag{7b}$$

Linearizing eqs. (6) with regard to θ near this steady state we obtain

$$\mu = \mu^*(v) + \sum_{\text{all } i} \mu_{\theta_i} \hat{\theta}_i \tag{8a}$$

$$\dot{\theta}_i = \dot{\hat{\theta}}_i = (\partial\dot{\theta}_i/\partial\theta_i)\hat{\theta}_i \tag{8b}$$

where $\mu_{\theta_i} = \partial\mu/\partial\theta_i$ and $\hat{\theta}_i = \theta_i - \theta_i^*(v)$. Equations (8) describe the friction force near the steady state for an unvarying $\dot{\delta}$ and have the full solution:

$$\mu = \mu^*(v) + \sum_{\text{all } i} c_i \exp((\partial\dot{\theta}_i/\partial\theta_i)\delta/v) \tag{9}$$

where t has been replaced by δ/v and c_i are arbitrary constants. Since $(\partial\dot{\theta}_i/\partial\theta_i)$ is negative the final approach to steady state is thus a sum of exponentials with characteristic distances equal to $-v/(\partial\dot{\theta}_i/\partial\theta_i)$. Experiments indicate that these distances are constants independent of v (Dieterich (1978, 79a, 80), Ruina (1980)). For sudden small changes in slip velocity the number of exponentials needed to well approximate the approach to steady state is not greater than the number of internal variables needed to describe the friction law and is equal if

$$\partial\dot{\theta}_i/\partial\theta_i < 0 \quad , \quad |\mu_{\theta_i}| \,.\, 0$$

and

$$\partial\dot{\theta}_i/\partial\theta_i \neq \partial\dot{\theta}_j/\partial\theta_j \quad \text{for} \quad i \neq j \ . \tag{10}$$

For slip histories with characteristic wavelengths in the variation of slip velocity much greater than the characteristic distance ($d_i \equiv -v/(\partial\dot{\theta}_i/\partial\theta_i)$) associated with a given internal variable θ_i, that variable may be approximately described as always having its steady state value $\overset{*}{\theta}_i(\dot{\delta})$ corresponding to the instantaneous value of $\dot{\delta}$. For slip histories with characteristic wavelengths much shorter than the characteristic distance associated with a given θ_i that variable may be regarded as constant in some circumstances (e.g. if the velocity fluctuations are not so large that nonlinearities in $\dot{\theta}_i(\theta,\dot{\delta})$ cause an effective reduction in d_i). Thus, even though a full description of the friction may require several internal variables, only those with characteristic distances on the same scale as the slip histories of interest may need to be treated carefully.

One Internal Variable

Here we focus our attention on a friction law adequately described by only one internal variable, e.g.:

$$\mu = \mu(\theta,\dot{\delta}) \quad , \quad \dot{\theta} = \dot{\theta}(\theta,\dot{\delta}) \qquad \text{(11a,b)}$$

where eq. (6c) is still a restriction on (11b). First it should be noted that the expression (11) is not unique for a given material. The variable θ could be replaced by $\alpha(\theta)$ where α is any monotonic function. New functions $\tilde{\mu}(\alpha,\dot{\delta})$ and $\alpha(\alpha,\dot{\delta})$ can then be constructed from (11) that represent the same fiction law.

If a representation such as (11) exists it can be found experimentally as follows: Assign to θ the value of a paremeter describing a single parameter family of slip histories, each of which is long enough to uniquely determine the state. For example, if the single parameter family of histories is slip at constant velocity (for sufficient distance to clear the memory), θ may be taken as some point function of the speed of slip. Or, <u>if</u> stationary contact time could be well defined in terms of a single parameter it could be used for θ . (There is some doubt that this is reasonable, however (Ruina 1980).) Eq. (11) is then deduced by imposing a range of slip histories $\delta(t)$, each of which is followed by a range of slip velocities $\dot{\delta}$. The function $\mu(\theta,\dot{\delta})$ of (11a) is just the value of μ immediately following initiation of slip at speed $\dot{\delta}$. The function $\dot{\theta}(\theta,\dot{\delta})$ is found by solving (11a) for $\theta(\mu,\dot{\delta})$ and the time derivative of (11a) for $\dot{\theta}(\theta,\dot{\mu},\dot{\delta},\ddot{\delta})$ so that (11b) can be determined from measurement of $(\mu,\dot{\mu},\dot{\delta},\ddot{\delta})$ at a number of points for various slip histories. On the other hand, the sets $(\mu,\dot{\mu},\dot{\delta},\ddot{\delta})$ are overdetermined if (11b) exists since then one could solve, say, for $\dot{\mu}(\mu,\dot{\delta},\ddot{\delta})$. So, if for <u>given</u> $\mu,\dot{\delta},\ddot{\delta}$ the variable $\dot{\mu}$ <u>also</u> depends on previous slip history then <u>no</u> representation of the form (11) exists.

We now turn to the question of how experimental data (i.e. pairs of functions $\mu(t)$ and $\delta(t)$) could be used to suggest that a formulation like (11) applies. Experimental results consistent with (11) need to satisfy the following two conditions that cannot be proved by data but could be strongly suggested by lack of discordant evidence: 1) The value of $\mu(t_1)$ is uniquely determined by $\mu(0)$ and $\dot{\delta}(t)$ for $0 \leq t \leq t_1$. That is, all slip velocity histories equal to $\dot{\delta}(t)$ beginning at an arbitrary time 0 and causing a given value of μ at time = 0, cause the same values for $\mu(t)$ for all subsequent time. In other words, the state is uniquely determined by the instantaneous values of μ and $\dot{\delta}$; 2). For given initial conditions, $(\mu_o,\dot{\delta}_o)$, very fast changes (corresponding to changes at a fixed state) must lead to a unique monotonic relationship between μ and $\dot{\delta}$ independent of the nature of the fast change.

Note that condition (1) above is much stronger than the similar sounding condition for the existence of a steady state.

Construction of the form (11) from the above results proceeds as follows. The condition (2) leads to a family of curves on the $\mu-\dot{\delta}$ plane that, by condition (1), cannot intersect. Numbers assigned to these curves in an arbitrary but monotonic manner can be identified as values of θ. Thus $\theta = \theta(\mu,\dot{\delta})$, which inverted is (11a). For fast changes in $\dot{\delta}$ the state must be constant thus $\dot{\theta}$ can only depend on $\ddot{\delta}$

and higher derivatives in a manner that gives no singularities in $\dot{\theta}$ for singular $\ddot{\delta},\dddot{\delta}$ etc. Assuming, then, that $\dot{\theta}$ does not depend at all on $\ddot{\delta},\dddot{\delta}$ etc. we have that $\dot{\theta}$ can only depend on μ and $\dot{\delta}$. Using then $\theta = \theta(\mu,\dot{\delta})$ we obtain $\dot{\theta} = \dot{\theta}(\theta,\dot{\delta})$.

Experiments tending to confirm conditions (1) and (2) above thus tend to confirm the validity of (11a,b). Experiments that violate these conditions, like experiments that violate any predictions of (11a,b), demonstrate that one internal variable is not sufficient to describe the friction law. However the form (11a,b) may still be a useful approximation. If, for example, several internal variables with disparate characteristic distances are required for a full constitutive description, only one may dominate effects due to slip with the appropriate characteristic wavelength of variation, as explained before. Also, a description with only one internal variable may be used as an approximation if for example the conditions (1) and (2) are not too strongly violated.

Sample Laws With One Internal Variable

The Dieterich friction law, eq. (4), can be expressed in the form (11) (Kosloff and Liu, 1980). Differentiating (4d) with respect to time (not 'contact time') remembering that (4d) applies with $\dot{\delta}$ fixed, gives

$$\dot{\theta} = -(\theta/\theta^*(\dot{\delta}))\ell n(\theta/\theta^*(\dot{\delta})) \qquad (12)$$

where τ has been replaced by θ and $\theta^*(\dot{\delta}) = d_c/\dot{\delta}$. Eq. 12 is equivalent to (4c) as used by Dieterich (1979a,b) for non-constant slip velocities. When the slip speed goes to zero, for any given value of θ, eq. (12) predicts that $\dot{\theta}$ goes to zero as well. Thus θ does not evolve with time for stationary contact and cannot be interpreted as 'average contact time' as stated by Dieterich (1978,79a). This does not detract from the usefulness of the law in either its general or specific form, however.

For the purpose of obtaining a few simple analytical results and in order to highlight the essential features of the Dieterich law a few simplifications can be made. Equation (4a) may be written as the sum of a memory term and direct velocity term, rather than as a product, without much discrepancy, since the total variations in μ that are observed in friction experiments are small fractions of μ. Since we have lost the 'contact time' interpretation of θ the '1' in the first of (4b) need not be included. Using Dieterich's (1979b) value of $c_3 = .5$, this simplification only shows up at slip speeds approaching $\dot{\delta} = .5\ d_c$ and greater (where time is measured in seconds). Dieterich has chosen the second of (4b) to be finite for high and low speed limits but to be roughly logarithmic in between. Replacing Dieterich's F with $F(\dot{\delta}) = B\ell n(\dot{\delta})$ seems to cause little problem in practice, however, since extremely large and small values of $\dot{\delta}$ are required before any paradoxes are reached, for representative values of B. The simplified Dieterich law is then, with (12):

$$\mu = A + C\ell n\theta + B\ell n\dot{\delta} \tag{13}$$

where $\log_{10}$ has been replaced with ℓn for mathematical convenience and the constant A will depend partially on the units used to measure θ and $\dot{\delta}$. This simplified Dieterich law can perhaps be made more transparent by replacing the memory term $C\ell n\theta$ by $\mu_m + C\ell n d_c$ and solving for $\dot{\mu}_m$ from (12) to give

$$\mu = A + \mu_m + B \tag{14a}$$

$$d\mu_m/d\delta = (\mu_m^*(\dot{\delta}) - \mu_m)/d_c \tag{14b}$$

$$\mu_m^*(\dot{\delta}) = -C\ell n\dot{\delta} \tag{14c}$$

where A in (14a) is greater than A in (13) by $C\ell n(d_c)$. An expression similar to (14b) is given in Kosloff and Liu (1980). Thus the memory term μ_m is described by the fact that it always tends towards the steady state value corresponding to the instantaneous slip rate $\mu_m^*(\dot{\delta})$. It approaches the steady state value at a displacement rate proportional to its difference from that value. In this case the steady state value is $-C\ell n\dot{\delta}$. Solution of (14) for constant $\dot{\delta}$ shows that, with this law, approach to steady state is exponential in displacement even for large deviations from steady state.

Two examples of the simplified Dieterich friction law, eqs. (14) are plotted in figs. (2). Lines of constant state, μ_m, are light solid lines and show the instantaneous positive dependence of μ on slip velocity $\dot{\delta}$. The heavy line is the steady state friction law and is either a decreasing (2a) or increasing (2b) function of slip velocity. Above the steady state line μ_m decreases, below it μ_m increases, as indicated by the arrows. The dotted lines are lines of constant $\dot{\mu}_m$. Any slip motion corresponds to a curve on a plot of fig. 2 and is the simultaneous solution of the friction law and any constraints imposed by the loading mechanism. The imposition of constant slip rate, for example, constrains the motion to a vertical line on either fig. 2 and μ approaches the steady state value (solid line) for the given $\dot{\delta}$, as dictated by the arrows. Constant force loading (imposed constant F/N) corresponds to a set of points on a horizontal line and will be discussed later.

Further insight into the simplified Dieterich Friction law can be obtained from integration of (14b):

$$\mu_m(\delta) = (1/d_c)\int_{-\infty}^{\delta} e^{-(\delta-\delta')/d_c}\, \mu_m^*(\dot{\delta}(\delta'))d\delta' . \qquad (15)$$

Equation (15) says that the memory term μ_m is the weighted average of $\mu_m^*(\dot{\delta})$ (the steady state value of μ_m corresponding to the instantaneous value of $\dot{\delta}$) over the recent sliding past. For the simplified Dieterich law, where (14c) applies, μ_m is the exponentially weighted average of $\mu_m^* = -C\ell n\dot{\delta}$. The Rabinowicz (1958) idea, that the governing variable should be some kind of average of the recent slip velocity is thus incorporated in (14b) so long as exponential decay is an adequate 'distance function' for weighted memory and other functions besides $\ell n\dot{\delta}$ are allowed for $\mu^*(\dot{\delta})$. More generally we should replce μ_m by θ in (14b) and (15) for use in (4b), (11a) or (13) so as not to restrict dependence on the internal variable θ to a linear one (as is the case with μ_m in (14)).

Other internal variable friction laws can be generated from the exponentially fading weighted average of some function of the slip velocity. For example if the reciprocal of the exponentially weighted slip velocity is used as the governing variable θ we have

$$\theta = [(1/d_c)\int_{-\infty}^{\delta} e^{-(\delta-\delta')/d_c}(\dot{\delta}(\delta')/d_c)d\delta']^{-1} \tag{16a}$$

$$\dot{\theta} = (\theta/\theta^*(\dot{\delta})) - (\theta/\theta^*(\dot{\delta}))^2 \quad , \quad \theta^*(\dot{\delta}) = d_c/\dot{\delta} \tag{16b,c}$$

where (16b,c) follow from differentiation of (16a).

This law, like the Dieterich law (12) gives no change of state for zero slip velocities. On the other hand if the governing variable is taken to be the exponentially weighted average of the 'slowness' = $1/\dot{\delta}$ then (15) and (14b) become

$$\theta(t) = (1/d_c)\int_{-\infty}^{t} e^{-(\delta(t)-\delta(t'))/d_c}dt' \tag{17a}$$

$$\dot{\theta} = 1 - \theta/\theta^*(\dot{\delta}) \quad , \quad \theta^*(\dot{\delta}) = d_c/\dot{\delta} \tag{17b,c}$$

If this definition of θ is used in (13) or (4b) one of the original aims of Dieterich (1979a) is met that is missed by (12) or (14b,c). Equations (17b,c) are the simplest definition of θ that give $\dot{\theta} = 1$ for $\dot{\delta} = 0$, and thus the interpretation of θ as effective 'contact time.' This law (17b,c) also gives the steady state solution $\dot{\theta} = 0$ when $\theta = d_c/\dot{\delta}$. Eqs. (17) have however what appears to be a drawback in that at constant slip velocity, far above the steady state velocity $\dot{\delta}^*(\theta)$, the steady state value of μ is approached linearly with respect to δ. This can be extracted from (13) and (17b,c) or from eq. (6.24) of Rice (1980). Recent experiments (Ruina (1980), Dieterich (1980)) in a stiff machine do not seem to confirm this linear decay.

Depending on the constants or functions used in (11), μ can either increase or decrease with steady state velocity. The general property that all of the laws just described have is that μ is an increasing function of the instantaneous slip velocity that is partially, fully, or more than fully countered by the 'memory' after a sliding distance greater than d_c. The variations between the laws are in terms of the following; 1) The shape of the μ vs. δ curve for fixed $\dot{\delta}$, 2) Whether or not the evolution of state is connected fundamentally with time (i.e. whether state changes in completely stationary contact), 3) How the effects in the law vary with $\dot{\delta}$ or θ (for example in the Dieterich law (4b) the direct velocity dependence becomes negligible at high and low slip speeds).

Even in their simplest forms, the properties of these laws are somewhat more subtle than in classical velocity, 'time' or displacement dependent laws. Loosely one may think of μ as a function of quality of contact and slip velocity. Quality of contact may or may not correspond to average asperity contacts, etc.. The friction force increases with both slip velocity and 'quality.' The quality of contact always tends towards its corresponding steady state value, which is a decreasing function of slip velocity.

Experimental Evidence

Indirect affirmation of the truth of the general one variable law (11) or a law like the Dieterich law (4) or (14) comes from the predictions these laws make (mentioned in later sections and by Dieterich 1978, 79a, 79b, 80). Successful predictions do not necessarily come from a unique theory, however (and definitely do not validate it like an unobserved necessary consequence invalidates it). An incorrect model can usefully make predictions if the predictions do not depend on aspects of the model that are incorrect. This, at best, is the situation with laws of the form (11). The usefulness of laws the form of (11) or more specifically (14) come from their incorporation in a simple mathematical form having the following essential features:

1) Fading memory and steady state
2) Positive direct velocity dependence competing with
3) Negative velocity dependence of non-instantaneous (but fading) memory

and their ability to include the possibly essential features of

4) Fading memory with a characteristic distance of decay independent of velocity
5) Approximately logarithmic direct velocity dependence
6) Approximately logarithmic non-instantaneous dependence on velocity.

Time dependence or load rate dependence of 'static' friction may be viewed as a complex low velocity limit of a law that includes the above features (Ruina, 1980). In any case, the results obtained thus far by Kosloff and Liu (1980), Dieterich and in this paper do not utilize the concept of a distinct 'static' friction separate from the above features, and thus 'static' friction does not appear to be an essential basic property of these friction laws.

The fading memory and steady state feature is implicitly assumed in almost all experiments or discussion of metallic friction. Few rock friction experiments provide solid verification of this idea since they are apparently dominated by transients associated with first sliding. However, many of the experiments of, for example, Summers and Byerlee (1977) show a trend towards leveling of 'σ' vs. 'ε' or repeatable stick-slip events, both of which are indicative of fading memory. A superposition of a long term displacement dependence on a fading memory law (with possibly slowly changing parameters) is a possible correction in the cases where steady state is not observed. This correction is not adaquate for samples where the whole 'σ-ε' curve is often offset by 'static' holds. Thus, although certain features such as occasionally observed oscillations and slip preceding instability in the tests by Summers and Byerlee may seem indicative of the laws we propose here these experiments do not offer much support.

Recent experiments of Johnson (1980) in a servo controlled triaxial testing machine show evidence of a steady state, both in the leveling of 'σ vs. ε' curves and repeatability of experiments. Additionally his experiments substantiate the idea of a direct velocity dependence competing with a memory to the same extent as the earlier work of Dieterich (1979a). Besides these directly supportive aspects of Johnson's work he also has further results that are consistent with approximations to the Dieterich law (13) or (14) such as a roughly logarithmic drop in μ with time during static holds followed by, with resumed sliding, a peak in μ that is roughly logarithmically dependent on the product of 'static' time and resumed load point velocity as will be discussed with regard to the rate scaling rule. Servo-controlled experiments by Dieterich (1980) with fault gouge also give results in support of his earlier work. Because of the artificial stiffness that servo control provides, this later work by Dieterich shows clearly the distinctness of the essentially instantaneous direct velocity dependence from the memory dependence. Some of his results (unpublished) show a complex relaxation to steady state curve, however, that does not seem well suited to modelling by the likes of (4) or (14) even when the compliance in the testing machine is taken into account.

Recent experiments (Ruina 1980) in the 'sandwich' shear apparatus of Dieterich using servo-control on displacement, measured close to the slip surface, show some features consistent with the recent discussion and some new features. Typical results are shown in figure 3a. The effective machine stiffness is high enough (dotted line) so that the curves may be viewed as μ vs. δ. For experiments conducted in one day, without much sliding in between, repeatability was excellent as was the existence of a steady state independent of recent history. In figure (3a) two experiments are shown for step changes in load point velocity and in both cases μ approaches the same level despite the difference in previous velocities. A step change in $\dot{u} \approx \dot{\delta}$ leads to a step change in μ as expected. Often electrical noise would cause an unwanted servo 'correction' causing a sudden transient jump in μ and $\dot{\delta}$. These very short disturbances ($<$ .1 sec $\Rightarrow \Delta < .1\mu$) caused little or no memory effects thus supporting the idea of θ as continuous with δ

and our separation of state (θ) and property (μ vs. $\dot{\delta}$ for fixed θ). The new features are these: 1) The curve for step changes from $.1\mu$/sec to 1μ/sec does not retrace the curve for step change from $.01\mu$/sec to 1μ/sec. This implies that one internal variable is <u>not</u> sufficient for describing μ especially if the short term transient is to be described. 2) The curves retrace almost exactly an exponential decay after an initial transient decay that also is roughly exponential. Not visible in figure 3 but clearly observed (Ruina, 1980) was the independence of the characteristic distances of the exponential decays from slip velocity. Further, all the jumps in μ and ultimate relaxations were roughly proportional to the log of the ratio of the velocity after slip to the velocity before slip.

Assuming that an internal variable representation exists and that each of the exponential decays corresponds to one internal variable, the experiments are quite accurately described by a friction law of the form;

$$\mu = \mu_o + \mu_1 + \mu_2 + B\,\ell n\,\dot{\delta} \tag{18a}$$

$$\dot{\mu}_1 = -(\dot{\delta}/d_1)(\mu_1 + C_1\,\ell n\,\dot{\delta}) \tag{18b}$$

$$\dot{\mu}_2 = -(\dot{\delta}/d_2)(\mu_2 + C_2\,\ell n\,\dot{\delta}) \tag{18c}$$

two trajectories of which are ploted in fig. 3b as computed with the spring block model of fig. 1 with a very stiff spring. Eqs. (18) fit well for velocity drops as well as jumps. Since the memory terms μ_1 and μ_2 are each identical to the memory term in the simplified Dieterich law, they do not give 'time dependence' in the limit of zero slip velocity (e.g. $\dot{\mu}_1$ and $\dot{\mu}_2$ go to zero as $\dot{\delta}$ goes to zero).

In these particular experiments the short term relaxation given by μ_1 in (18b) almost exactly negates the direct velocity dependence $B\ell n\dot{\delta}$. If $C_1 = B$ exactly, eqs. (18) can be re-expressed by combining μ_1 and $B\ell n\dot{\delta}$ into a single short term response μ_s so that

$$\mu = \mu_o + \mu_s + \mu \tag{19a}$$

$$d\mu_s = -(\mu_s/d_1)d\delta + (B/\dot{\delta})d\dot{\delta} \tag{19b}$$

$$\mu_s = (1/d_1)\int_{-\infty}^{\delta} e^{-(\delta-\delta')/d_1}(-d_1B/\dot{\delta})d\delta' \tag{19c}$$

where $\mu_s = \mu_m + B\ell n\dot{\delta}$, μ_n still obeys (18c) and μ_o is still a constant. The superposed dots denote differentiation with respect to time. The term μ_s includes both the short term memory and direct velocity dependence. It exponentially approaches zero for constant slip velocity ($d\dot{\delta}=0$). If the fractional slip velocity change over the slip distance d_1 is small and the velocity is slowly changing (characteristic wavelengths $\gg d_1$) then (19) reduces to $\mu_s = d(-d_1B/\dot{\delta})/dt = (d_1B/\dot{\delta}^2)\ddot{\delta}$. That is, for motion with a distance scale large compared to d_1 the coefficient of friction appears to depend on slip acceleration. The surface can be thought of as having a velocity dependent "inertia."

In numerical spring-block models the two internal variable laws (18) or (19) yield results that differ in some details from use of the one variable law (14). However the differences are in the detail of behavior not the quality. For example one or the other of the two characteristic lengths seem to dominate depending on the simulated experiment. The calculations that follow (except the scaling law) are based on the use of one internal variable.

Constant Force Loading

The constitutive laws thus far have been written for μ in terms of slip history. However, assuming the necessary invertability they acan be re-expressed to solve for slip from the friction force. For example the simplified Dieterich law eqs. (14) can be expressed:

$$\dot{\delta} = \exp((\mu - A - \mu_m)/B) \tag{20a}$$

$$d\mu_m/d\delta = (1 - C/B)(\mu_m^*(\mu) - \mu_m)/d_c \tag{20b}$$

$$\mu_m^*(\mu) = (C/B)(\mu - A)/(1 - C/B) \quad . \tag{20c}$$

For constant μ loading this system can be solved for $\dot{\delta}$ in terms of δ as:

$$\dot{\delta} = \dot{\delta}^*(\mu)(\dot{\delta}_o/\dot{\delta}^*(\mu))^{\exp(((C/B)-1)\delta/d_c)} \tag{21}$$

where $\dot{\delta}^*(\mu)$ is the steady state value of $\dot{\delta}$ that corresponds to μ, $\dot{\delta}_o$ is the slip velocity at $\delta = 0$. For $C < B$ the solution decays to the steady state solution. For $C > B$ steady sliding is extremely unstable. If $\dot{\delta}_o > \dot{\delta}^*(\mu)$ infinite velocities are reached in finite time since in (19) $\dot{\delta}(\delta)$ is of greater order than δ (i.e. $\int dt = (1/\dot{\delta}(\delta))d\delta$ = finite). If $\dot{\delta}_o < \dot{\delta}^*(\mu)$ slip stops in finite time. The condition $C > B$ corresponds to a friction law that has μ decrease with steady state slip speed, e.g. $d\mu^*(\dot{\delta})/d\dot{\delta} < 0$.

Constant force loading constrains the motion to horizontal lines on figure 2. The arrows governing $\dot{\mu}_m$ indicate the subsequent velocities for given initial conditions. The stability conditions from the last paragraph are apparent with brief inspection.

The figures (2) can be used to show qualitative response for other loading by remembering that sudden changes in μ or $\dot{\delta}$ (or some combination) must be along lines of constant state. Slower changes must yield to the arrows.

As noted in Kosloff and Liu (1980) and Dieterich (1979a) sliding occurs, with this style law, before instabilities. In particular, unless sufficient force is applied to bring the slip speed up to the steady state value corresponding to that state, accelerating slip instabilities cannot occur. If the friction force is an increasing function of the steady state slip velocity, fig. 2b, slip velocities may grow quickly during and subsequent to quickly growing imposed load μ, but they remain finite and approach the steady state velocity corresponding to μ.

Unsurprisingly, stability of sliding at constant μ for any law of the form (11) is critically dependent on $d\mu^*/d\dot{\delta}$, as was the case in the last example. We examine the stability by looking for solutions near a steady state solution at slip speed $\dot{\delta}^*$, $\mu^*(\dot{\delta}^*)$, $\theta^*(\dot{\delta}^*)$ and $\dot{\theta} = 0$. Linearizing (11) near this solution one obtains:

$$\mu = \mu^*(\dot{\delta}^*) + \hat{\mu} = \mu^*(\dot{\delta}^*) + \mu_\theta \hat{\theta} + \mu_{\dot{\delta}} \hat{\dot{\delta}} \tag{22a}$$

$$\dot{\theta} = \hat{\dot{\theta}} = \dot{\theta}_\theta \hat{\theta} + \dot{\theta}_{\dot{\delta}} \hat{\dot{\delta}} \tag{22b}$$

where $\hat{\theta} = \theta - \theta^*$, $\hat{\mu} = \mu - \mu^*$, $\dot{\hat{\delta}} = \dot{\delta} - \dot{\delta}^*$ and subscripts denote partial differentiation.

For fixed μ loading, $\hat{\mu} = 0$. Solving for $\dot{\hat{\delta}}$ in (22a) and applying this to (22b) we have

$$\dot{\hat{\theta}} = (\dot{\theta}_\theta/\mu_{\dot{\delta}})[\mu_{\dot{\delta}} - (\dot{\theta}_{\dot{\delta}}/\dot{\theta}_\theta)\mu_\theta]\hat{\theta} \ . \tag{23}$$

The term in square brackets is $d\mu^*/d\dot{\delta}$. This can be seen by applying $d\theta^*/d\dot{\delta} = -\dot{\theta}_{\dot{\delta}}/\dot{\theta}_\theta$ (from implicit differentiation of $\dot{\theta}(\theta,\dot{\delta}) = 0$) to the 'total' derivative of (11a) with respect to $\dot{\delta}$ at steady state. The first term may be re-expressed by use of the association of $\dot{\theta}_\theta$ with $-v/d_c$ made in the discussion following eq. (9). So (23) reduces to

$$\dot{\hat{\theta}} = -(v/d_c\mu_{\dot{\delta}})[d\mu^*/d\dot{\delta}]\hat{\theta} \tag{24a}$$

$$\ddot{\hat{\delta}} = -(v/d_c\mu_{\dot{\delta}})[d\mu^*/d\dot{\delta}]\dot{\hat{\delta}} \tag{24b}$$

where (22a) has been employed to obtain (24b).

Assuming a positive direct velocity dependence $(\mu_{\dot{\delta}} > 0)$, eq. (24) has solutions that grow or die exponentially depending only on whether $d\mu^*/d\dot{\delta}$ is less or greater than zero.

So, for constant force loading the stability criterion with a memory and velocity dependent law is the same as with a strictly velocity dependent law. If force decreases with steady state velocity, steady slip is not stable. This similarity between the stability behaviors for the two laws breaks down, however, when a finite compliance is used in the loading.

Steady Sliding With a Spring-Block Model

Here we look at the nature of slip if the spring-block model (fig. 1) is used with a memory dependent law (eqs. 11) with the load point moving at a constant speed v. The friction force must equal the force transmitted through the spring.

$$\mu(\theta,\dot{\delta}) = k(u-\delta) \tag{25}$$

where for constant rate loading $u = vt$.

We are interested in the stability of steady-state sliding solution $\delta^* = vt - \mu^*(v)/k$, $\dot{\theta} = 0$. To examine the solutions near the steady state solution equations (25) and (11b) are linearized, as for constant force loading (eqs. 22), to give

$$\hat{\mu} = \mu_\theta \hat{\theta} + \mu_{\dot{\delta}} \dot{\hat{\delta}} = -k\hat{\delta} \tag{26}$$

where $\hat{\mu} = \mu - \mu^*(v)$, $\hat{\theta} = \theta - \theta^*(v)$, $\hat{\delta} = \delta - (vt-\mu^*(v)/k)$ and (22b) still applies for the linearization of $\dot{\theta}$. The coefficients of all the '^' variables are subsequently regarded as constants. The linear constant coefficient equations (22b) and (26) govern the motion of the block near steady state sliding with constant load point velocity.

The solutions of (22b) and (26) have the form:

$$\hat{\mu} = \mathrm{Re}[A_1 e^{st}] \quad , \quad \hat{\theta} = \mathrm{Re}[A_2 e^{st}] \tag{27}$$

where Re[ ] denotes the real part of [ ]. Application of the solution (27) to the linearized expression for $\dot{\theta}$ (22b) one can solve for A_2 in terms of A_1. Application of this result to (26) leads to the following quadratic equation and solutions for s

$$s^2 + (Tk/\mu_{\dot{\delta}})s + Dk^2/\mu_{\dot{\delta}}^2 = 0 \tag{28a}$$

$$s = (k/2\mu_{\dot{\delta}})(-T \pm (T^2-4D)^{1/2}) \tag{28b}$$

$$T = v(d\mu^*(\dot{\delta})/d\dot{\delta})/kd_c + 1 \,, \quad D = \mu_{\dot{\delta}} v/kd_c \tag{28c,d}$$

The identification of d_c as $-v/\dot{\theta}_\theta$ has been employed as well as the identity $d\mu^*/d\dot{\delta} = \mu_{\dot{\delta}} - \dot{\theta}_{\dot{\delta}}\mu_\theta/\dot{\theta}_\theta$. If, in either of the solutions of (28b) s has a positive real part then perturbations near the steady state will grow exponentially by (27). Since small perturbations can always be assumed to exist, steady following of the load point by the block is impossible if Re(s) > 0.

From the solution to the quadratic equation (28b) we have

$$T < 0 \text{ or } D < 0 \Rightarrow \text{instability} \,. \tag{29}$$

Since the direct velocity dependence $\mu_{\dot{\delta}}$ is expected to always be positive D is always greater than 0. So the stability criterion reduces to

$$-(d\mu^*/d\dot{\delta})/d_c > k/v \Rightarrow \text{instability} \tag{30a}$$

or

$$(C-B) > kd_c \Rightarrow \text{instability} \tag{30b}$$

Equation (30b) is (30a) applied to the simplified Dieterich law (14).

The dimensionless quantity T is negative when steady sliding is unstable and positive for possibly stable sliding and can serve as a measure for degree of stability. When s has an imaginary part the approach to, or growth away from the steady state solution will, from eqs. (27,28) be <u>oscillatory</u>. The condition for this, from (28b), is

$$T^2 = [v(d\mu^*/d\dot{\delta})/kd_c + 1)] < 4\mu_{\dot{\delta}} v/kd_c = 4D \tag{31}$$

At neutral stability, $T = 0$, and, from (28), s is given by

$$s = i(v/d_c)((-d\mu^*/d\dot{\delta})/\mu_{\dot{\delta}})^{1/2} \tag{32}$$

and the slip displacement wavelength of the associated stable sinewaves is

$$\text{wavelength} = 2\pi d_c(\mu_{\dot{\delta}}/(-d\mu^*/d\dot{\delta}))^{1/2} \; . \tag{33}$$

The nature of the solutions to (22b,26) is described mostly by the eigenvalues s of (28) and not the "eigenvector" (A_1,A_2) which determines the relative phase and magnitude of $\hat{\theta}$ and $\hat{\dot{\delta}}$ and shall not be discussed further.

Steady state sliding with the spring block is unstable if the steady state velocity dependence is sufficiently negative (eq. 30a). This is different from the strictly velocity dependent law, for which steady sliding was unstable for any amount of velocity weakening. Near the condition of neutral stability oscillatory solutions are expected that may grow or die depending on the sign of T (eqs. 31-33). The wavelength of the oscillations is on the order of $2\pi d_c$, depending on other constitutive parameters.

A few complications may be added to the problem without complicating the solution. If a dashpot is added, parallel to the spring in figure (1), the solution is only modified by the addition of the dashpot constant to both $\mu_{\dot{\delta}}$ and $d\mu^*/d\dot{\delta}$. This further stabilizes the motion. Incidentally one may note that for purely viscous loading (no spring) the solutions can qualitatively be obtained graphically from the likes of fig. 2 since the loading could be represented as a single constraint curve (between $\dot{\delta}$ and μ) on the graph. If a direct displacement dependence is added to the law it can be included so long as it can be modelled as constant slope, $\mu = cx$. The corresponding solution is modified by linearizing about the steady state solution $\dot{\delta}^* = kv/(k+C)$ instead of $\dot{\delta}^* = v$ and replacing k by $k + C$ in results that depend on k. An added positive displacement dependence is stabilizing since it increases the effective stiffness of the spring.

I have not found a simple rationalization of the main result of this section (30a), except in the somewhat singular case where there is no explicit velocity dependence, $\mu_{\dot{\delta}} = 0$. Imagine the block sliding at the steady state solution corresponding to the load point velocity. If the velocity of the block were suddenly to change to a new, say greater speed, the friction law would require that the friction force must begin changing. However if the block speed has changed, the spring begins to relax since the load point speed is constant. Eqn. (30a) is a statement of the fact that the spring force drops less quickly than the friction law force for imagined jumps in slip velocity and that force is thus imbalanced towards the direction of motion. A similar argument applies for imagined sudden drops in velocity.

A surprising feature of the stability criterion (30a) is that it contains no explicit reference to the direct velocity dependence $\mu_{\dot{\delta}}$ neglected in the last paragraph. That is, only the amount of the steady state velocity weakening, and the characteristic distance d_c, determine whether or not stable sliding is possible in a given system, no matter how large the transient velocity strengthening. However, as can be seen from (29-33), the direct velocity dependence $\mu_{\dot{\delta}}$ does determine whether or not oscillations can occur, what their wavelength is, and, if sliding is unstable, at what rate instabilities grow.

The solution of (28b) for $k \to \infty$ implies that for a very stiff system perturbations decay to steady state as is in the constitutive law for constant slip velocity. As the stiffness goes to zero the constant force loading results are approached (compare (30) to (24)). At intermediate stiffnesses the characteristic lengths v/s in the exponential solutions are not d_c. For example for compliances not quite large enough to cause oscillatory approach to steady state (equality in (31)) the characteristic distance of the approach to steady state v/s is, by (29), $2d_c\mu_{\dot{\delta}}/(-d\mu^*(\dot{\delta})/d\dot{\delta})$.

Simultaneous solution of the spring-block constraint (25) with the simplified Dieterich law (not-linearized) (14) has been carried out numerically and is shown in fig. 4a. The steady state solution is perturbed by suddenly changing the load point velocity a small amount. In a very stiff machine the effect is small and gives the result that would be predicted by a sudden change in the rigidly controlled slip velocity. With more compliance decaying oscillations are observed. At neutral stability, equality in (30b), oscillations persist with the wavelength of about $2\pi d_c$ as predicted by (33) with $C = 2B$ in (14). With still a more compliant machine oscillations grow, beyond the applicability of the previous linearizations, to a massive instability. This is indicated by the nearly vertical slope in force vs. load point displacement implying nearly infinite slip velocities. This indicates the failure of the quasistatic calculation and the onset of dynamic instability.

Oscillations like we discuss here were first noticed by Scholz, et al. (1972) and were seen to occur in the transition from stable sliding to dynamic stick slip as the normal stress was increased. A sample of their results is reproduced in figure (4b). Assuming μ is independent of normal stress σ_n, as discussed, increases in normal stress are equivalent to decreases in stiffness $\hat{k}$ since both decrease the normalized stiffness $\hat{k} = k/N$ which governs the stability of motion (see also Dieterich 1978, 79a). We have carried out experiments where the stiffness was electronically controlled (Ruina, 1980). Perturbations were imposed by changing, stepwise, the motion of the 'load point' (an imagined point in the electronic control). As the stiffness was decreased there was a transition from steady state to decaying oscillations to large sustained oscillations bordering on dynamic stick slip. Results are shown in fig. (4c).

Oscillations of this type are seen in some of the results of Summers and Byerlee (1976) as well as Shimomoto (1977). That these works do not show these small quasistatic oscillations more frequently may be due to the fact that in fault gouge layers instabilities of the type discussed thus far are mixed with instabilities associated with localization of deformation in the gouge layer.

Localization of Deformation

The slip displacement δ may represent the deformation of a layer of finite thickness. In rock friction experiments, surfaces of intact rock are often separated by a layer of gouge, either generated by sliding or introduced artificially. If the deformation in this layer is homogeneous, μ, $\dot{\delta}$ relationships like (14) measured in experiments are actually measurements of μ as a function of shear strain history. In the constitutive laws (11) (and following) the slip displacement δ should then be replaced by the average shear strain in the layer $\gamma = \delta/h$, where h is the thickness of the layer. The characteristic distance(s) in the law d_c should be replaced by a characteristic strain γ_c as suggested implicitly by Dieterich (1978). The laws (11) then take the form

$$\mu = \mu(\theta(y), \dot{\gamma}(y)) \tag{34a}$$

$$\dot{\theta}(y) = \dot{\theta}(\theta(y), \dot{\gamma}(y)) \tag{34b}$$

where y is distance in the direction perpendicular to the macroscopic surface. A restriction like (6c) is still assumed to apply. The question arises as to when deformation governed by (34) leads to localized deformation. With large restrictions on possible 2- and 3-dimensional effects, we now consider the conditions for the stability of homogeneous simple shearing deformation.

Assume that the shear strain $\gamma = \partial\delta/\partial y$ depends only on y and that there is no deformation in any plane parallel to the surface. The normal stress is still assumed to be constant or irrelevant, so effects of elongation strain perpendicular to the surface are, if important, incorporated in (34).

The simplest criterion for localization is the growth in time of small perturbations from homogeneous deformation for a given macroscopic loading. We restrict our attention to a homogeneous material and thus only discuss perturbations of state or deformation rate. Although possibly of interest, perturbations of properties are not considered here. The obvious variable to consider, the shear strain γ, is in fact not sensible because it does not appear in the constitutive law (34). That is, the growth or diminution of shear strain perturbations would depend on an arbitrarily associated perturbation in the state θ or strain rate $\dot{\gamma}$.

Consider a homogeneous deformation $\mu_h(t)$, $\theta_h(t)$, $\dot{\gamma}_h(t)$ that satisfies (34) as well as the given remote boundary conditions. The perturbation is the difference between the values of relevant variables and their values in the reference homogeneous deformation:

$$\partial\mu\,(t) = \mu(t) - \mu_h(t) \tag{35a}$$

$$\partial\theta\,(y,t) = \theta(y,t) - \theta_h(t) \tag{35b}$$

$$\partial\dot{\gamma}\,(y,t) = \dot{\gamma}(y,t) - \dot{\gamma}_h(t) \tag{35c}$$

where force balance requires μ and $\partial\mu$ to be independent of y. This set includes both homogeneous and inhomogeneous perturbations. The degree of inhomogeneity in the perturbation can be measured by the quantities $\Delta\dot{\gamma}$, $\Delta\theta$ and $\Delta\mu$ which are the differences between the values $\dot{\gamma}$, θ and μ at two points in the layer, (e.g. $\Delta\theta\,(t) = \theta\,(y_1,t) - \theta\,(y_2,t)$ where we assume y_1 and y_2 are such that $\Delta\theta$ is positive. Force balance (or the first of (35)) requires $\Delta\mu$ to be zero. For small perturbations the quantities $\Delta\theta$, $(\dot{\Delta\theta})$, $\Delta\dot{\gamma}$ and $\Delta\mu$ can be related by linearization of (34):

$$0 = \Delta\mu = \mu_\theta \Delta\theta + \mu_{\dot{\gamma}} \Delta\dot{\gamma} \tag{36a}$$

$$(\dot{\Delta\theta}) = \dot{\theta}_\theta \Delta\theta + \dot{\theta}_{\dot{\gamma}} \Delta\dot{\gamma} \tag{36b}$$

where subscripts denote partial derivatives of the functions on the right sides of (34) evaluated at the values θ_h, $\dot{\gamma}_h$ in the corresponding homogeneous deformation.

The equations (35a,b) can be re-expressed as

$$(\Delta\dot{\theta}) = (\dot{\theta}_\theta - \dot{\theta}_{\dot{\gamma}}(\mu_\theta/\mu_{\dot{\gamma}}))\Delta\theta \tag{37a}$$

$$= (\dot{\theta}_\theta/\mu_{\dot{\gamma}})(d\mu/d\gamma)_{\dot{\theta}\ \text{fixed}}\ \Delta\theta \tag{37b}$$

$$\Delta\dot{\gamma} = -(\mu_\theta/\mu_{\dot{\gamma}})\Delta\theta \tag{37c}$$

where, again, all derivatives are to be evaluated at the corresponding values of $\theta_h(t)$ and $\dot{\gamma}_h(t)$. Equation (37b) follows from (37a) by evaluating $d\mu(\theta,\dot{\gamma})$ subject to the constraint $d\dot{\theta}(\theta,\dot{\gamma}) = 0$. Since we assume throughout that $\mu_{\dot{\gamma}} > 0$ (or else localization follows immediately without any discussion of state variables) and $\dot{\theta}_\theta < 0$ (by a constraint like (6c) that allows the concept of steady state) we have that $(\dot{\Delta\theta}) > 0$ if $(d\mu/d\dot{\gamma})_{\dot{\theta}\text{fixed}} < 0$. Since $\Delta\theta$ is positive by definition $(\dot{\Delta\theta}) > 0$ implies growth of inhomogeneities in state θ and thus the <u>localization condition</u>

$$(d\mu/d\dot{\gamma})_{\dot{\theta}\ \text{fixed}} < 0 \tag{38a}$$

$$\mu > \mu^*(\dot{\gamma}) + (B-C) \tag{38b}$$

$$\mu^*(\dot{\gamma}) = \mu_o + (B-C)\ln(\dot{\gamma}) \tag{38c}$$

where (38b,c) is (38a) evaluated for the strain analog of the simplified Dieterich slip law (14c). Since the inhomogeneity in strain rate $\Delta\dot{\gamma}$ can be found from the inhomogeneity of state $\Delta\theta$ from (35c) we can view (38a) as the condition for localization of strain rate. This is not strictly true, however, since the coefficient of $\Delta\theta$ in (37c) may change in time if the reference homogeneous deformation is not at steady state.

The localization condition (37) can be found graphically on figures (2a,b). Where lines of constant $\dot{\theta}$ (dotted lines) have negative slope localization takes place. In the case where the reference homogeneous deformation is at steady state the localization condition (38) reduces to:

$$d\mu^*/d\dot{\gamma} < 0 \tag{39a}$$

$$B-C < 0 \ . \tag{39b}$$

Thus steady state homogeneous deformation is unstable if the material is strain rate weakening for steady state deformation. This result could be obtained directly from the analysis of constant force loading for the slip law, equations (24), by considering a perturbation that has zero average strain and thus keeps the layer under constant force loading no matter what the boundary conditions.

A few interesting features of the localization results should be mentioned. Localization is possible even when the load μ is below the steady state value for the current state. Thus localization can occur even when $\dot{\theta} > 0$ and the layer is strengthening (i.e. the level of required for macroscopic slip is increasing). If the load μ is below the level required for localization (i.e. is in the region of fig. 2 where lines of constant $\dot{\theta}$ have positive slope) then a homogenization process takes place. Finally, localized deformation is possible even in a material that allows stable steady sliding for force controlled loading. If, as in figure 2b, B is greater than C then a suddenly applied μ much greater than $\mu^*(\theta)$ brings the material into the region of localization in fig. 2b (i.e. negative slope of the constant $\dot{\theta}$ lines).

Relationship of Localization to Instability

With homogeneous deformation of the gouge zone, stability of an elastic system with finite compliance is determined largely by $\dot{\gamma}(d\mu^*/d\dot{\gamma})/d_c$ where the characteristic relaxation distance $d_c = \gamma_c h$. If localization of deformation reduces the thickness of the most actively deforming region the effective relaxation distance will also be diminished and instability is encouraged. The thickness of the localized deformation could mathematically go to zero but must be limited physically by the deformation mechanism. Presumably fully localized deformation of a gouge layer has similar constitutive properties to the friction of relatively intact surfaces. So homogeneous or continuum response is to fully localized response like the shear of a deck of playing cards is to the slip between two cards. Macroscopic response and stability of a system like in fig. 1 will be different for homogeneous deformation than for localized deformation and may depend on the transition from one to the other. A few possibilities will be considered.

The stability of the spring block system depends on the degree of steady state velocity weakening (eqs. 30). Any strain rate weakening whatsoever for all relevant strain rates leads to localization that must, in order to be compatible with constant slip rate on the boundary, lead to fully localized deformation (i.e. most of the deformation is only in one minimally thin layer). So homogeneous deformation does not persist at steady state and stability of steady state sliding depends on the fully localized constitutive behavior. In the case of constant force loading the condition for stability of steady state motion is identical with the condition for localization. The exponential growth of fluctuations in macroscopic slip velocity $\dot{\delta}$ is in fact independent of whether or not there is associated localization. So, near the steady state, fully or partially localized deformation leads to identical response as homogeneous deformation for constant force loading. The thinner the actively deforming layer, however, the sooner the much faster than exponential instabilities like that given by (19) can occur.

If homogeneous deformation leads to stability in the spring block model for a given macroscopic loading and fully localized deformation leads to instability then the localization process and instability might be intertwined (as might also be the case if both are unstable). For example consider an initially homogeneous layer deforming at steady state with the load point velocity u about equal to the slip rate $\dot{\delta}$ in the model of fig. 1. Any through the thickness perturbation with zero average strain must have some accelerating region in order to keep the through the thickness tangential stress independent of position. So long as the material is strain rate weakening for steady state deformation at all deformation rates, the localization process thus initiated must lead to fully localized deformation, and possibly elastic unloding instability. As the localization process takes place, the steady state force required to maintain steady state diminishes since the actively deforming region must have higher strain rates for a given macroscopic displacement rate $\dot{\delta}$. There is then an unloading process due to the localization that is different from the unloading that could have occurred if the deformation was fully localized from the start. If fully localized deformation leads to instability then the localization process itself may lead to instability.

After an instability with large elastic unloading the force drops possibly into the region where $(\Delta\dot{\theta})$ is negative (positive slope of dotted lines in fig. 2). This leads to homogenization that may leave the layer more or less homogenized at the next reloading up to the steady state level (the level required before any macroscopic instabilities are possible). The next instability may or may not be localized in form depending on whether the layer is effectively homogenized by the time at low load level or whether the fully localized layer still has a distinctly different state than its surroundings (or its properties have been altered by the localization).

Consider now the steady state deformation of a layer that has a positive steady state velocity dependence, as in fig. 2b (positive slope of heavy line). If, as some recent experiments indicate, (Dieterich 1980, Ruina 1980a), there is a gradual transformation in properties with deformation from positive to negative steady state rate dependence, then steady sliding becomes gradually unstable. There are two possibilities for instability. The first is similar to that just discussed. At the point where slip first becomes velocity weakening at steady state (e.g. $d\mu^*/d\dot{\gamma} < 0$) localization begins. The localization process, being a concentration of deformation, could enhance the trend towards rate weakening and instability could occur as part of the localization process. Alternatively, the deformation could become fully localized without elastic unloading instability, then when the constitutive law has sufficient negative velocity dependence instabilities could begin, starting with the oscillations described previously.

As was implicit in (29) - (33) and the related discussion, gradual change of friction parameters, normal load or stiffness during steady sliding can lead to massive instabilities only by means of a transient oscillatory instability (so long as $\mu_{\dot\delta}$ and k are both positive). The only exception to this rule, within the context of the laws we describe here, seems to be if the instability occurs simultaneously with localization as just discussed.

A fuller consideration of localization incorporating 2 or 3 dimensional effects could only lead to the possibility of the onset of localization even before it is anticipated in our 1 dimensional (deck of cards) analysis. In general it seems then, that except in cases where an initially homogeneous sample (either due to virginity or substantial rehealing) is deformed or there is a gradual change of parameters during 'steady state' deformation, that instabilities can be assumed to occur only in samples where the deformation is previously fully localized to a very thin layer.

Dieterich's (1980) recent experiments with fault gouge showed no scaling of characteristic distances with layer thickness implying localized deformation as predicted here. However his characteristic distances were an order of magnitude greater than with surfaces polished to the same approximate roughness of his gouge particle size. Thus the mechanism of deformation apparently does not localize as much as might have been expected. The characteristic distances might then be governed by the sizes of particle aggregates that move more or less rigidly rather than the size of individual particles as proposed by Dieterich (1979a).

Application to Fault Mechanics

A popular qualitative model for earthquakes is the occurence on a fault of a frictional instability similar to that observed in laboratory 'stick-slip.' In order to flesh out the model a particular friction law must be applied to a fault of specified geometry in a medium with specified properties and with specified loading.

Efforts in this direction are frequently based on the spring block model of fig. 1. For example if motion modelled as roughly homogeneous on a planar fault acted on by normal stress σ_n with characteristic distance δ in a medium with modulus G the average slip displacement δ is related approximately to the average imposed μ by $\mu = (G/\ell\sigma_n)(\ell\gamma_\infty - \delta)$ where γ_∞ is the far field shear strain (Walsh 1971). This relationship makes obvious the association $u = \ell\gamma_\infty$, $k = G/\ell\sigma_n$ for spring block modelling (remember we use a stiffness based on μ). Or if homogeneous motion is assumed on a very large fault that is constrained at a distance ℓ away from the fault to a displacement u, as in the geometry of fig. 4, then the spring-block model may be used with the stiffness $k = G/\ell\sigma_n$ again.

The two geometries just mentioned lend themselves to analysis with the spring-block model so long as no effects are missed by the 'lumped' displacement. In particular the stability results discussed previously may be applied to these 'zero-dimensional' faults.

Earthquake rupture, however, need not correspond to homogeneous slip on a surface. The application of a Dieterich-like law to an elastic continuum leads to difficult problems that may only be answered numerically.

Such numerical work was initiated by Dieterich (1979b) and is now being pursued by Gary Mavko (U.S.G.S. Menlo Park, CA) and Kosloff and Liu (also USGS). The primary discovery of Dieterich's experiment work with large rock blocks was the spatial propagation of a slipping zone precursory to massive instability. In his numerical model of his experiments the propagation of creep (aseismic slip) was fundamentally connected to spatial gradients in the difference between applied load and surface strength (or state). The rate of propagation of these frictional 'fractures' is then given by the ratio of the far field stress loading rate to the strength difference gradient. This kind of propagating slip could be predicted by a friction law that is only displacement dependent as well. In contrast, it is interesting to consider the possibility of spatial propagation of slip for a homogeneous material with no spatial inhomogeneities in initial conditions. For example, spatial inhomogeneities may be generated by the instability processes (the localization of deformation is a kind of analogy). Some rough evidence that this might be so is given in the next section.

Creep Waves

We now endeavor to investigate the possibility of spatial instabilities with the simplest possible model. The long elastic slab of fig. 5 is modelled as 'a rod on an elastic foundation.' This is the continuum equivalent to a long train of blocks separated by springs with each block connected to the displacement controlled boundary by another spring. The interconnecting springs cause a net force on a block if spacing is not even ($\delta_{xx} \neq 0$) and the boundary springs cause a force if slip displacement is less than boundary displacement. This model should well approximate an elastic layer for deformations with wavelength large compared to ℓ, the slab thickness.

The governing force balance equation is

$$\mu = k(u-\delta) + (k\ell^2/\alpha)\,\delta_{xx} \tag{40}$$

where $k = G/\sigma n\ell$, $\alpha \approx 3G/E = 3/2(1+\nu) \approx 1$. The stiffness k is defined to be consistent with previous discussions. G, E are shear and elongation moduli, ν is Poisson's ratio. The assigned values for k and α in (37) may be determined by assuming that planes normal to the x direction do not deform (Simons, 1979). For homogeneous deformation $\delta_{xx} = 0$ and (40) reduces to (25) and all spring-block results apply. The general single internal variable friction law, eq. (11) is used. If the imposed displacement rate is constant, $u = vt$, then homogeneous slip at speed v, with θ and μ at steady state values, is a solution. We look for solutions near this steady state solution. Linearizing as for the spring block model (26)

$$\mu_\theta \hat{\theta} + \mu_{\dot{\delta}} \hat{\dot{\delta}} = -k\hat{\delta} + (k\ell^2/\alpha)\hat{\delta}_{xx} \tag{41}$$

with the '^' variables defined as before and with (22b) still applying for linearization of $\dot{\theta}$. Assume a solution of the form

$$\hat{\delta} = \exp(\lambda\alpha^{1/2}x/\ell + qvt/d_c) \tag{42a}$$

$$\hat{\theta} = a\hat{\delta} \tag{42b}$$

where λ, q and a are to be determined. Application of the assumed solution (42) to the linearized equations (41) and (22b) yields an equation quadratic in both q and λ. It can be solved for λ in terms of q or q in terms of λ

$$\lambda = \pm(1+Tq+Dq^2)^{1/2}/(1+q)^{1/2} \tag{43a}$$

$$q = ((\lambda^2-T) \pm ((\lambda^2-T)^2 + 4D(\lambda^2-1))^{1/2})/2D \tag{43b}$$

where T and D are defined as in (28c,d).

Small perturbations can be decomposed into a sum (or integral) of terms that are spatially sinewaves and thus correspond to λ as pure imaginary. Stability of a perturbation of given λ is determined by whether or not the corresponding q has a positive real part. Let $\lambda = i\beta$ (β real) then from (43b) motion is <u>unstable</u> if

$$-T > \beta^2 \quad \text{or} \quad -(d\mu^*/d\dot{\delta})/d_c > k(1+\beta^2)/v \tag{44}$$

The most relaxed condition is for $\beta = 0$ which corresponds to waves of infinite wavelength and the solution of the spring block model. For a given $T < 0$ there is a corresponding β for which sliding is neutrally stable ($\mathrm{Re}\{q\} = 0$). Waves of greater wavelength are unstable, those with smaller wavelength are stable. The critical spatial wavelength is, from (44) and (42)

$$\text{crit. wavelength} = 2\pi\ell(-\alpha T)^{-1/2} \quad . \tag{45}$$

The velocity of propagation of such a wave is given from (42) by $(qv/di)/(\beta\alpha^{1/2}/\ell)$ which can be found from (43) to be

$$\text{wave speed} = \pm(v\ell/d_c\alpha^{1/2})[(d\mu^*/d\dot{\delta})/\mu_{\dot{\delta}}T] \tag{46a}$$

or

$$\text{wave speed} = (v/2\pi d_c)[(-d\mu^*/d\dot{\delta})/\mu_{\dot{\delta}}]^{1/2}\ (\text{wavelength}) \tag{46b}$$

The results (44-46) can also be derived directly from the spring-block model by observing that for a propagating sine wave of spatial wavelength $2\pi\ell/\lambda d^{1/2}$ in a slab, as modelled, the slip displacement is related to the friction coefficient by $\mu = -k(1+\lambda^2)\delta$. Each point of the surface may be then treated as with the spring-block model with the stiffness k replaced by $k(1+\lambda^2)$. The velocity of propagation is then determined from the frequency as determined by the ratio of the nominal slip velocity to the slip displacement wavelength from (33).

The propagation of a small disturbance of any shape can, in principle, be determined by decomposition into sinusoidal components with the above results used for superposition to obtain the general solution. Qualitatively one can note that the components with wavelength smaller than the critical will decay, possibly in an oscillatory manner (if close to the critical wavelength). Components with larger wavelength will grow in time. The disturbance will propagate, if at least some wavelengths are greater than critical. The growth of instabilities is, then, insensitive to small wavelength (stiff) perturbations and very sensitive to long wavelength (soft) perturbations.

Further investigation of slip on a continuous boundary is left for future work.

A Rate Scaling Rule

Consider a friction law of the form

$$\mu = \mu_o + \sum C_i \ell n\theta_i + B\ell n\dot{\delta} \tag{47a}$$

$$\dot{\theta}_i = \dot{\theta}_i(\dot{\delta}\theta_i) \quad \text{or} \quad d\theta/d\delta = (1/\dot{\delta})\dot{\theta}_i(\dot{\delta}\theta_i) \tag{47b}$$

where, in this case, each $\dot{\theta}_i$ is taken to depend only on the product $\dot{\delta}\theta_i$. The apparently special friction law (47) includes as special cases the simplified Dieterich law (12, 13 or 14), the simplest truly time dependent law (13,17), the velocity average law (13,16) and the two internal variable law (18). The ℓn function may also be a useful approximation for friction laws described by a power law with a small exponent.

Consider also a displacement field $\underline{u}(\underline{x},t)$ and associated velocity and stress fields $\underline{v}(\underline{x},t)$, $\underline{\underline{\sigma}}(\underline{x},t)$ in a linear elastic solid, where $\underline{x}$ is the position of solid material points in a reference configuration. On a frictional boundary the elastic displacement is equal to the slip $\underline{u} = \underline{\delta}$ and $v = \dot{\underline{\delta}}$ (or $\underline{\delta}$ is the jump in u across an interior slip surface). If changes of direction change the constitutive description (47) then it is assumed that $\underline{u}$ and $\underline{v}$ are restricted to fields that only have slip in one direction on the slip surfaces. Assume that $\underline{u}(\underline{v},t)$, $\underline{v}(\underline{x},t)$ $\underline{\underline{\sigma}}(\underline{x},t)$ satisfy the equations governing the deformation of a linear elastic solid. Assume also that a friction law of the form of (47) as computed with the velocity field $\underline{v}(\underline{x},t)$ is in mechanical equilibrium with the elastic field $\underline{\underline{\sigma}}(x,t)$ (i.e. $\mu = \tau/\sigma_n$ where τ and

σ_n are shear and normal tractions). The slip surface need not be homogeneous, but a law of the form of (47) is assumed to hold at each point.

Now consider the same velocity field viewed in fast or slow motion and a compatible field,

$$\tilde{\underline{v}}(\underline{x},t) = s\ \underline{v}(\underline{x},st) \tag{48a}$$

$$\tilde{\underline{u}}(\underline{x},t) = \underline{u}(\underline{x},st) + \underline{u}_s(x) \tag{48b}$$

where s is the ratio of velocities at corresponding points in the motion and $u_s(\underline{x})$ is a displacement field that is constant in time.

Under a few interesting loading conditions a constant field $\underline{u}_s(\underline{x})$ can be found so that $\tilde{\underline{u}}(\underline{x},t)$, $\tilde{\underline{v}}(\underline{x},t)$ leads to an elastic stress field that is in mechanical equilibrium with the friction force as computed from (47). That is, corresponding to the velocity field $\underline{v}$ that satisfies the elasticity and friction laws, is a family of rate scaled fields $\tilde{\underline{v}}$ which also satisfy the elasticity and friction laws.

Application of the scaled field (48) to a linear elastic body yields (on the slip surface)

$$\frac{\tilde{\tau}(\underline{x},t)}{\tilde{\sigma}_n(\underline{x},t)} = \frac{\tau(\underline{x},st) + \tau_s(\underline{x})}{\sigma_n(\underline{x},st) + \sigma_{n_s}(\underline{x})} \tag{49}$$

where τ_s and σ_{n_s} are the shear and normal traction due to the constant in time field u_s. Application of the rate scaled field to the friction law (47) gives as one solution

$$\tilde{\theta}_i(t) = \theta_i(st)/s \tag{50a}$$

$$\tilde{\mu}(t) = \mu(st) + (B-\Sigma C_i)\ell ns \tag{50b}$$

where it is assumed that sufficient slip has occurred that initial conditions don't matter or that initial conditions are altered according to (50a).

The scaling rule is then the statement that $\tilde{\mu}(\underline{x},t) = \tilde{\tau}(\underline{x},t)/\tilde{\sigma}_n(\underline{x},t)$ for an appropriate $\underline{\underline{\sigma}}_s(\underline{x})$.

If the loading is such that the normal traction on the slip boundary does not depend on time (i.e. $\sigma_n(x,t) = \sigma_n(x)$) then take for u_s, $\underline{\underline{\sigma}}_s$ any static field that satisfies the following traction boundary conditions on the slip boundary:

$$\sigma_{n_s}(\underline{x}) = 0 \ , \ \tau_s = \sigma_n(B-\Sigma C_i)\ell ns \quad . \tag{51}$$

In this case equation (49) with the assumed solution $\underline{u},\underline{v},\underline{\underline{\sigma}}$ implies that the rate scaled field $\tilde{\underline{u}},\tilde{\underline{v}},\tilde{\underline{\underline{\sigma}}}$ also satisfies the elastic and friction laws. Note that the rate scaled field is not strictly rate scaled since the field $\underline{u}_s$, $\underline{\underline{\sigma}}_s$ has been added to the strictly scaled field. However in observing instabilities and transient motions the absolute value of field quantities is of little interest: it is the variations in these quantities during the motion that is interesting. In the scaling rule just derived, the velocity field $\underline{v}$ and stress rate field $\dot{\underline{\underline{\sigma}}}$ do strictly scale. So, in terms of motion the rule says the following: Any solution for motions occurring in an elastic solid with constant normal stress on a frictional surface governed by (47) is also a solution when viewed in fast or slow motion. A curve of stress at any point as a function of strain or displacement at any other point is changed only by a constant due to the rate scaling.

Motions occurring near a steady state motion in an elastic system loaded remotely at constant rate also must scale according to this rule. Therefore, conditions for stability of steady state motion in a constant normal stress system governed by (47) must be independent of remote loading rate. Condition (30b) is an example of this rate independence.

In systems where the normal stress depends on slip displacement and where slip is homogeneous the spring-block model (normalized by normal force) may appear as a linearization if the normal force does not vary by a large fraction of its value. The scaling rule is easily seen to apply to a system described by $\mu = k(u-\delta)$ where k and u represent effective stiffness and displacement as explained in the earlier discussion of the spring block model. Thus, if (47) applies for surfaces with varying normal stress, "bi-axial" experiments like those of Scholz et al. (1972) or sawcut "tri-axial" experiments like those of Johnson (1980) also lend themselves to application of the scaling rule.

The strongest application of the rule is to the results of Johnson (1980). In these "time dependent friction" experiments a load point (analogous to the load point in figure 1) was controlled to move at a speed v_o while slip was taking place, then the load point was stopped for time t^* after which the load point motion was resumed at speed v_o. Both the friction force and a displacement nearly equal to the slip displacement were measured. Johnson found that the force drop during the hold time t^* (due to "creep" of the surface) depended, to a good approximation, on the product $v_o t^*$. He also found that the difference between the friction force after the load point was stationary for time t^* and the peak force occurring after the load point motion was resumed at speed v_o was dependent, roughly, on the product $v_o t^*$.

Two motions of the load point which have the same product $v_o t^*$ are related by the rate scaling relation (48). Thus, if the friction law (47) holds, the scaling rule also applies (We neglect questions of whether or not solutions are unique here). Therefore all variations of stress should vary in rate but not magnitude between experiments with the same value of $v_o t^*$. In particular, the force changes reported by Johnson should be the same for different experiments with the same $v_o t^*$ or, in other words, the force changes should depend on the product $v_o t^*$.

Johnson's experiments highlight the fact that "time-dependence" is only a manifestation of a general load history dependence, since, in these experiments slip velocity has an equally prominent role. It was, incidentally, these experiments that prompted investigation of the scaling rule for the friction laws (47).

In a bi-axial experiment Scholz et al. (1972) found that during the accelerating motion preceeding dynamic slip the slip displacement as a function of time scaled with remote load point velocity. That is, the amount of precursory slip was independent of remote load point speed but occurred at a rate proportional to load point speed (see fig. 9 of Scholz et al. 1972). They inferred that the accelerating motion was basically stable and that, implicitly, the friction force was only displacement dependent. However, the friction law (47) leads to the same result by the scaling rule. That is, corresponding to a slip history due to a particular loading rate is a family of slip histories scaled with a family of loading rates. The slip occurring could be very unstable in that it would, for example, continue even if the load point was stopped but still be consistent with the experimental observations of Scholz et al. The initial conditions required for the scaling rule are not strictly satisfied by the experiments, however, since the slip in the instability preceding the noted observations is apparently dynamic and thus does not satisfy the equations of elastostatics that were assumed here. This apparent shortcoming might be overcome by the fact that, since the dynamic slip is very fast, the memory (θ_i) is dominated by the slip history occurring during the nominally still part of the instability cycle as the force increases.

The scaling rule is also consistent with the results of Dieterich (1979b) that propagation rate of slip events is proportional to the load rate. Here again, however, there is some doubt about the applicability of the appropriate initial conditions as well as the varying normal stress in a sample with inhomogeneous slip.

Concluding Remarks

We have started with the ideas of Rabinowicz (1958) and Dieterich (1978a) about representation of memory dependence in friction laws. The approach of Dieterich is equivalent to an internal variable representation, that is in turn expressible in terms consistent with ideas of Rabinowicz as a dependence on the weighted average of some function of the velocity. Several consequences of friction laws of this type have been shown with some reference to experimental results. The stability results have been obtained by assuming that one internal variable yields a sufficiently accurate constitutive description. All of the stability results highlight the central role of $\dot{\delta} d\mu^*/d\dot{\delta}$, the slope of the steady state dependence of the coefficient of friction on slip velocity. However, as highlighted in Dieterich (1978, 1979a), stability is determined as much by the stiffness of the loading apparatus and by normal force as by the frictional laws.

A one internal variable law suggests that continuous deformation is localized to a narrow band before elastic unloading instabilities are possible and thus that, for stability analysis, friction parameters should not scale with the thickness of the frictional deformation zone.

It should be emphasized that the linearizations used in the analysis here preclude knowledge of the full range of instabilities that are possible. Full non-linear analysis may show that what is observed as "stick-slip" may be possible, for example, even when steady sliding is stable.

A property that seems to be approximated by many experiments is the linear dependence of the friction coefficient on the log of the slip velocity. The resultant scaling law should highlight the fact that the phrase 'time-dependence' does not have an obvious meaning in laws of the type discussed here.

The range of experimental evidence is too slim to know which aspects of the description here can be generalized and employed for useful prediction of frictional phenomena. This paper will have served its purpose, however, if it demonstrates that constitutive relations for friction warrant more attention than has apparently been applied previous to the paper of Dieterich (1978).

APPENDIX

Experiments on Friction

Introduction

Following similar work with metals Dieterich (1972) demonstrated a dependence on the time of nominally stationary contact of the friction force required to initiate macroscopic slip. This time dependence, with later experimental observations of displacement dependence (Dieterich, 1978, 1979b) serves as the basis of the constitutive description that is the starting point of this thesis. These experiments, were performed in a testing machine in which slip velocity was manually controlled at a point somewhat removed from the slip surface. The interaction of the testing machine and the slip surface along with the lack of detailed control inhibited investigation of the details of the constitutive description. Particular details of interest are, 1) the applicability of a description based on a single internal variable, 2) high and low slip speed limiting forms of the constitutive description, and 3) the form of the constitutive description with particular interest in the implications of the form for nominally stationary contact experiments.

The "time dependence of static friction" aspect is of particular interest for two reasons: 1) unless machine-block interactions are included, the constitutive description used by Dieterich does not yield time-dependent static friction as a low slip speed limit. 2) Experiments with metals (Johannes, et.al., 1973, Green and Brockley, 1974) unknown to Dieterich, showed that the apparent time dependence of static friction was really an apparent dependence on the rate of increase of friction force previous to macroscopic slip.

The concept of time dependence in Dieterich's original conceptualization in terms of average contact time is incompatible with the load rate dependence just mentioned. The empirical law used by Dieterich as well as the law used here are, however, in at least qualitative agreement with both "time-dependent" and "load-rate dependent" static friction if the details of the experiments used are taken into account.

This appendix will discuss experimental methods and results for experiments very much like those of Dieterich (1978, 1979a) only with finer control. Following will be a brief speculative discussion about time dependence of static friction.

Experimental Method

Experiments were conducted in the apparatus used by Dieterich (1978, 1979a) as modified for use with servo-control and with slight change of sample geometry and support. A similar arrangement was used by Dieterich (1980) though a few differences are noted here.

The sample geometry is as in Figure 6. A central piece of rock is pushed between two outer pieces. The outer pieces are supported by a thin piece of copper as close as possible to the central block. This reduces the moment arm of the couple due to the friction force and support force on the side blocks so that the normal traction can be uniform and still maintain static equilibrium. The uniformity of the normal traction can be indirectly checked by the uniformity of the wear particles generated by slip. Figure 7 shows two samples, one before slip (left) and one after slip (right) and the wear is seen to be fairly uniform. Nonuniformity in normal stress is, however, only significant to the extent that the measured frictional forces are not proportional to normal stress since force measurements are a total of friction forces over the whole surface.

The arrows in Figure 6 show where force is applied by means of hydraulic pistons in series with load cells (measuring force). The horizontal force (normal force) is kept constant by connection with an accumulator (gas reservoir). The vertical force is controlled by a servo-valve (MOOG) as will be explained. The coefficient of friction is given by one-half the ratio of vertical to horizontal force.

The Servo System

The servo system controls a valve connnecting the vertical force piston to either a drain or a high pressure oil reservoir. The valve (i.e. rate of draining or filling) is roughly proportional to an electric signal, provided by the comparator, which is proportional to the difference between a reference (intended displacement) signal and the measured displacement. Neglecting inertia and time lags the control system can be thought of as causing a correction rate proportional to the deviation from the intended displacement. Depending on the gain in the comparator, the response time of the system is on the order of 30×10^{-3} seconds. This estimate is based on unstable resonances of about 30 c.p.s. that occur if the comparator gain was too high relative to the effective stiffness of the displacement measure (the effective stiffness is the ratio of force change to measured displacment with no slip).

The Reference Signal

The intended displacement history was provided by a reference signal from the 12-bit digital to analogue converter ("D to A") on a PDP-11 computer. The computer was thus used as a versatile signal generator. The minimum displacement step for a 12-bit D to A is 1/4000 of the full-scale slip distance. This causes noticeable steps in force if the displacement measure is stiff. These steps are visible in Figure 3(a) for slip at .01 μ/sec. but not at higher slip rates due to the slow (≈.1 sec.) response of the vertical load cell amplifier; later experiments used a faster load cell. The more recent experiments have been conducted (following a suggestion of Dave Lockner) using the sum of two D to A channels (at a relative gain of 16) for a reference signal. This reduced the step size to 1/64000 of full scale with 16 interspersed steps of about 1/4000 of full scale. Electronic integration of a computer specified rate was found to be a less useful way to avoid steps because of problems with drift (due to capacitor leakage) at low load rates. A single higher resolution (16-bit) D to A was not used because of cost restrictions.

Displacement Measure

The displacement signal used for control was provided either by a Hewlett-Packard Linear Voltage Displacement Transducer (LVDT) or a small contilever beam. In either case the signal could be modified to vary the effective stiffness.

The LVDT was mounted as in Dieterich (1978, 1979a, 1980) and, if the signal was unmodified, led to an effective stiffness of about

42 lb./μm or (over 8 in.^{2} surface) about .37 bars/μm or, in the normal force normalized stiffness k used in the main paper (for 30 bars normal stress), $\kappa \approx .012/_{\mu}$m. However, if the difference between the LVDT measured displacement and the frictional slip is due to linear elastic deformation, the effective stiffness of the machine can be modified. The stiffness is modified by either adding or subtracting a fraction of the load cell output (in a summing amplifier) to the LVDT output and using the summed signal for control. Subtraction of a fraction of the load stiffens the machine, addition softens the machine. Figure 4(c) shows an experiment with the LVDT and a progressively reduced stiffness. A constant must be subtracted from the load cell signal so that changes in gain, as the stiffness is altered, do not cause alterations in the displacement signal.

In the finer experiments (less than .1 mm full scale) displacement was measured using a small cantilever transducer (Fig. 8). The cantilever is mounted on one rock and a pin on the adjacent rock (see Fig.6). Slip causes the pin on one rock to bend the cantilever mounted on the other. Semi-conductor strain gauges on opposite sides of the cantilever make up half of a 4-arm resistance bridge, the output of which is a signal proportional to the cantilever beam tip deflection. This transducer has the features that: 1) It is small and mounted next to the slip surface and, thus, records a displacement very close to the slip displacement. 2) It is sensitive to very small displacements (on the order of 10^{-8} meters). 3) Because it is small, it is fast (slowest vibration frequency $\approx$ 20 Khz.). The compliance in the system under cantilever control is due to shear of the rock between the pin and beam supports as well as elongation of the sample if the pin and cantilever are not mounted directly opposite each other. The effective stiffness of the machine with control on cantilever measure displacement is about 2200 lb./μm. In terms of the normal force normalized stiffness this is about k = .6/μm. That is, the full elastic loading, leading to slip at a friction coefficient equal about .6, involves a measured displacement of about 1μm. Artificial increases in stiffness, by subtraction of a fraction of the load signal from the displacement signal lead eventually to large drifts in the force. That is, small displacement noise (in the transducer or electric noise) leads to large force fluctuations in a stiff machine. Stiffness is inherently limited by displacement measurement (and reference signal) accuracy. Without artificial stiffening by load subtraction 1/60 μm displacement steps caused quite finite jumps in load. For similar reasons the LVDT could not be used for very (artificially) stiff control since there was a hysterisis of about .5 μm in the LVDT readings for reversed loading.

<u>Sample Preparation and Experimental Conditions</u>

All experiments were performed with Eureka Quartzite at a normal stress of about 30 bars. The samples were lapped with #90 abrasive in water before any slip and flatness was only checked by feel during preparation and indirectly by uniformity of wear after the friction experiment. The abrasive was washed off with water and then the rocks were rinsed with acetone and air dried. Electron microscope pictures of the surface after preparation (right) and after about 1 cm. of slip (left) are shown in Figure 9.

Experiments

Following the approach of Dieterich (1978, 1979a) experiments were performed with step changes in "load point" velocity. Because of the high stiffness of the system with cantilever measured displacement servo controlled, one can think of the slip displacement as being controlled, for most experiments. The friction coefficient μ is plotted for several such experiments in Figs. 10(a),(b). The range of slip speed is .01 to 2 μm/sec.. This range was limited on the low side by the minimum step size and patience, and on the high side by not understood problems with stability of the servo-control system.

Several features of these experiments are, perhaps, worth noting. 1) No matter what the previous slip history, the friction coefficient approached a steady state value that depended on slip velocity. We denote this steady state velocity dependence $\mu^*(v)$ and it is plotted in Figure 11. The steady state dependence is approximated by negative linear dependence on the log of the slip velocity. The existence of a steady state dependence of the friction coefficient μ on slip velocity v is strongly suggestive of a state (as described in the body of the thesis) that also has a steady state. This is further supported by the consistency of the results of step change in velocity experiments, independent of previous slip history. 2) Coincident with sudden changes of slip velocity is a jump in the friction coefficient. This jump $\Delta\mu$ depends, approximately, on the ratio of the new slip velocity to the previous steady state velocity V^*. This result is shown in Figure 12. 3) After sudden changes in slip velocity there is a relaxation to steady state. This relaxation seems well approximated by the sum of two exponentials, as illustrated in Figure 10(b). As noted before, the fact that the two curves in Fig. 10(b) do not retrace implies that a single internal variable is not sufficient for accurate description of the friction law. The constitutive law (18) is constructed to match the noted features and the simulation of Figure 3(b) shows that, at least for experiments with step changes slip velocity, a good approximation has been obtained.

A further experiment, though not systematic in nature, indicated other features of possible interest.

Using the LVDT for displacement control slip was continued for several centimeters beginning with a freshly prepared sample. Following Dieterich (1980) the slip velocity was changed about every 100 μm. from fast to slow (or visa-versa). The response to the velocity change, as noted by Dieterich, changes slowly as slip progresses. In one case, the sample was used within 1/2 hour of an acetone rinse. In this case the frictional behavior changed slowly from velocity strengthening to velocity weakening with displacement. In a later, similar test, the sample was left for a day in open room air before testing. Here the sample was velocity weakening from the start. A possible cause of this difference is that the presence of absorbed water is responsible for the weakening. The change from velocity strengthening to weakening may be a process related more to surface contamination then comminution as Dieterich (1980) suggested.

Speculations About Time Dependence

The friction law that approximates these experiments (18), as well as the Dieterich law (4), the simplified Dieterich law (14) and the velocity average law (13,16) have the property that no state changes occur in the limit as slip velocity goes to zero. Consider a friction experiment of this form: A load point in a spring-block model is moved steadily at speed v_0, stopped for time t^* and then moved again at speed v_0. What is the peak force after the load point motion has resumed? A numerical simulation is shown in Fig. 13 using the friction law (18). This plot shows a definite apparant time-dependence in the static friction. In fact, however, the friction law used is not time-dependent. The peak forces observed are due to state changes due to slip when the load point was stationary.

Therefore, it is seen that results similar to those reported in studies of "time-dependent" frictional strengthening in "stationary contact" (Dieterich, 1972) are, qualitatively, consistent with the predictions of a slip-rate and surface-state dependent law. It remains for future work to make quantitative comparisons and also to examine effects of the rate of load application. For example, predictions of the rate and state dependent law are also qualitatively consistent with Johannes, et.al. (1973) observation that friction strength decreases with increasing rate of load application.

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FIGURE CAPTIONS

FIGURE 1: Spring-Block Model. The load point moves a distance u causing a force $\hat{F} = k(u-\delta)$ where δ is the rigid block slip distance. Alternatively, the load point moves u causing an imposed coefficient of friction $\mu = k(u-\delta)$. The slip may represent deformation of a layer with thickness h.

FIGURE 2: Simplified Dieterich Friction Law. The friction law of Eqs. (14) is plotted. The light lines give μ vs. $\dot{\delta}$ for fixed state μ_m. The heavy solid line gives μ vs. $\dot{\delta}$ at steady state (i.e. $\mu^*(\dot{\delta})$). The arrows indicate the rate of change of state $\dot{\mu}_m$, μ_m increases below the solid line and decreases above. Lines of constant $\dot{\mu}_m$ are dashed.

a) $.01 = C > B = .005$, instabilities near steady state are possible depending on loading.

b) $.02 = B > C = .01$, steady state is stable.

FIGURE 3: Friction vs. Displacement for step changes in load point velocity in a very stiff machine. The load point velocity is suddenly changed from either .01μm/sec. or .1μm/sec. to 1μm/sec.. The dotted line shows μ vs. u for no slip (slope = k).

a) Quartzite finished with 60#-90# abrasive at approximately 30 bars normal stress in room environment (from Ruina 1980).

b) Numerical simulation of (3a) using $\mu = \mu_o+\mu_1+\mu_2+B \ln \dot{\delta}$.

$\mu_o = .545$, $B = .011$, $\dot{\mu}_1 = -(\dot{\delta}/.25\mu)(\mu_1 + .011 \ln \dot{\delta})$,

$\dot{\mu}_2 = -(\dot{\delta}/5.2)(\mu_2 + .0092 \ln \dot{\delta})$, $\dot{\delta}$ measured in μ/sec.

FIGURE 4: Frictionally Induced Oscillations

a) Numerical model of stiffness effect on steady sliding of spring-block (Fig. 1) with simplified Dieterich law (Eq. 14) and C = 2B. The load point speed is suddenly reduced a small amount at u = 0. Slip goes from stable to unstable as $(C-B)kd_c$ goes from less to greater than 1.

b) Experimental record of displacement (something between u and δ from Fig. 1) vs. time (from Scholz et.al. 1972). This oscillation observed at the transition from stable sliding to 'stick-slip' with increasing normal stress.

c) Stiffness effect in experiment with quartzite at about 30 bars normal stress. Stiffness is controlled electronically in a servo-controlled testing machine. Transition from stable sliding through oscillations to fast instabilities is observed as stiffness is decreased. At higher stiffnesses (preceding this graph) perturbations decayed to steady state. The load point velocity was alternated between .8 and 1μm/sec..

FIGURE 5: Creep Wave Model. An infinitely long elastic layer of thickness ℓ is deformed in shear by the controlled displacement u . Slip of amount $\delta(x,t)$ occurs in accordance with the simplified Dieterich friction law and the elastic law of the layer. The layer is idealized as satisfying the relation

$$\mu = k(u-\delta) + (\ell^2/\alpha)k\partial^2\delta/\partial x^2 \text{ , where } k = G/\ell\sigma_n \text{ , } \alpha \approx 1 .$$

FIGURE 6: Sample Geometry. Two rocks on outside squeeze inner rock which is pushed down.

FIGURE 7: Sample Wear. Left: before slip; Right: after about 1 cm. slip at 30 bars.

FIGURE 8: Cantilever displacement transducer (with beard hair for scale).

FIGURE 9: Scanning Electron Micrographs of surface before (right) and after (left) about 1 cm. of slip.

FIGURE 10: (a),(b). Friction coefficient vs. cantilever measured displacement with step changes in velocity.

FIGURE 11: Friction as a function of steady state velocity after a variety of slip histories.

FIGURE 12: Jump in friction due to step changes in slip velocity.

FIGURE 13: "Time-dependent friction" from law with no time dependence. Friction law is the same as in Figure 3. Machine stiffness in this simulation is about 1/2 that of the servo-controlled system with cantilever control.

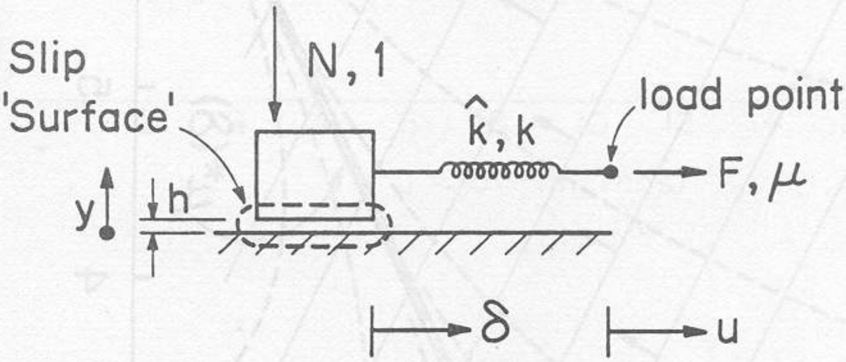
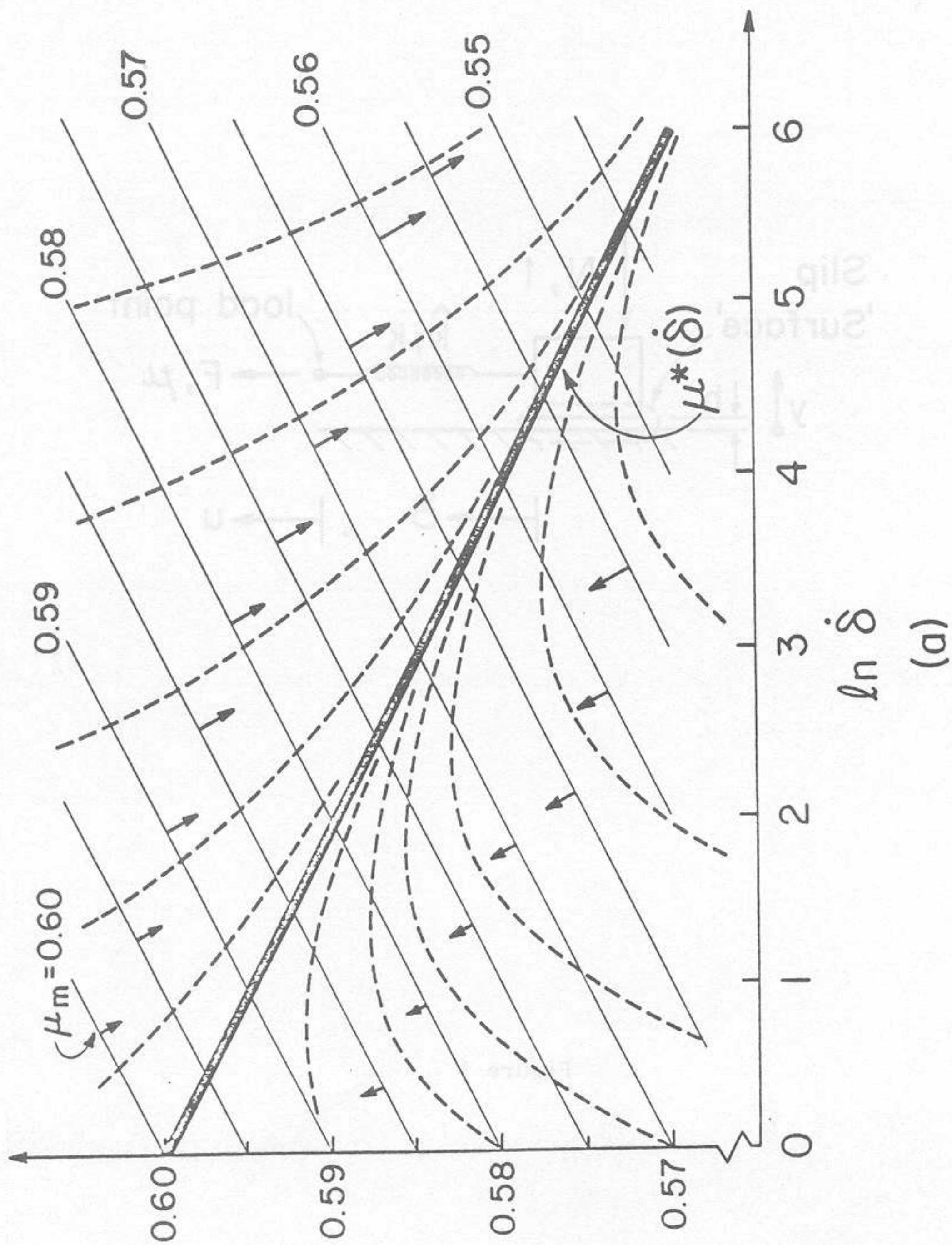
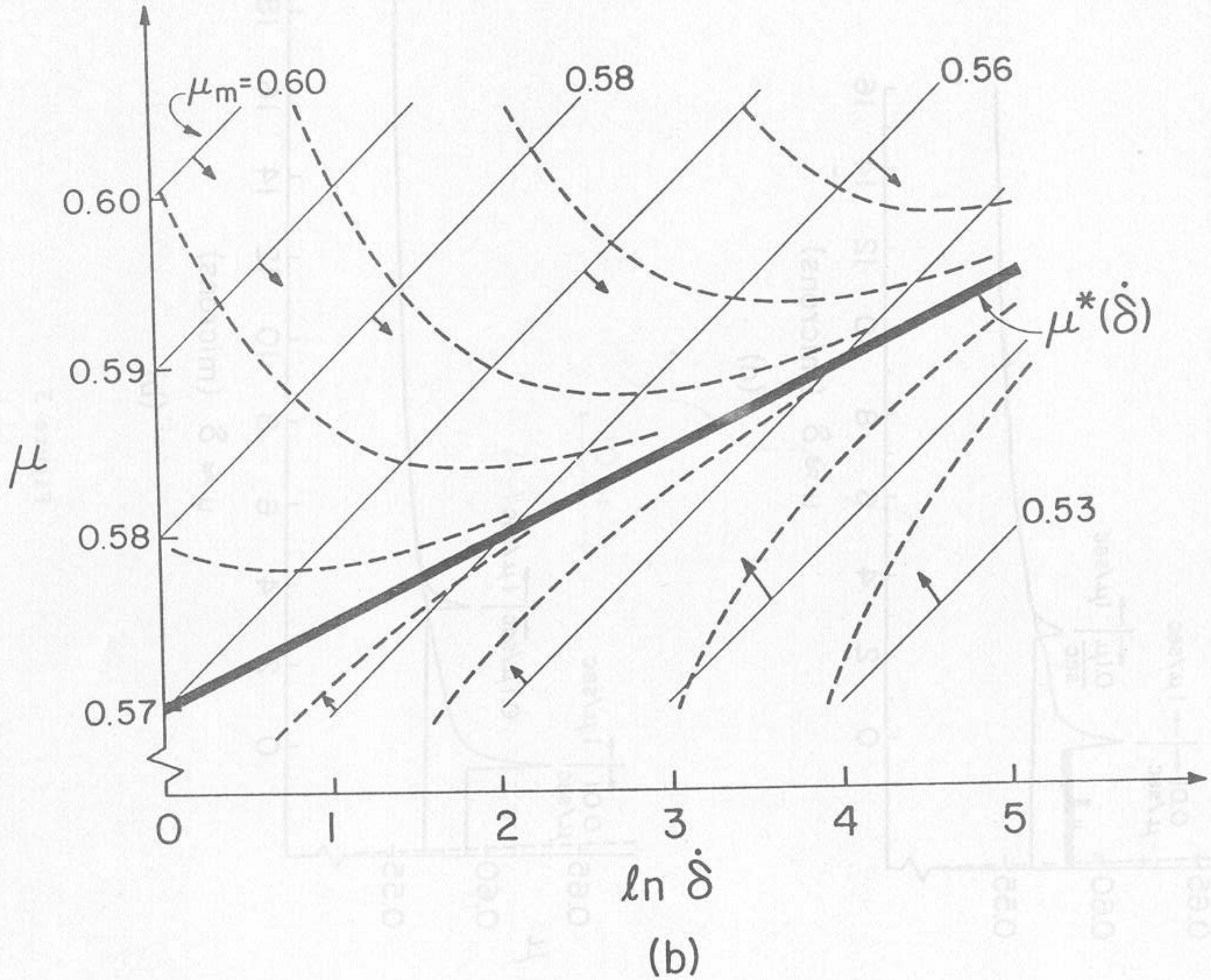


Figure 1



μ
0.60
0.59
0.58
0.57
$\mu_m = 0.60$
0.59
0.58
0.57
0.56
0.55
$\mu^*(\dot{\delta})$
0
1
2
3
4
5
6
$\ell n\ \dot{\delta}$
(a)

Figure 2



μ
0.57
0.58
0.59
0.60
$\mu_m = 0.60$
0.58
0.56
0.53
$\mu^*(\dot{\delta})$
0
1
2
3
4
5
$\ell n\ \dot{\delta}$
(b)

Figure 2

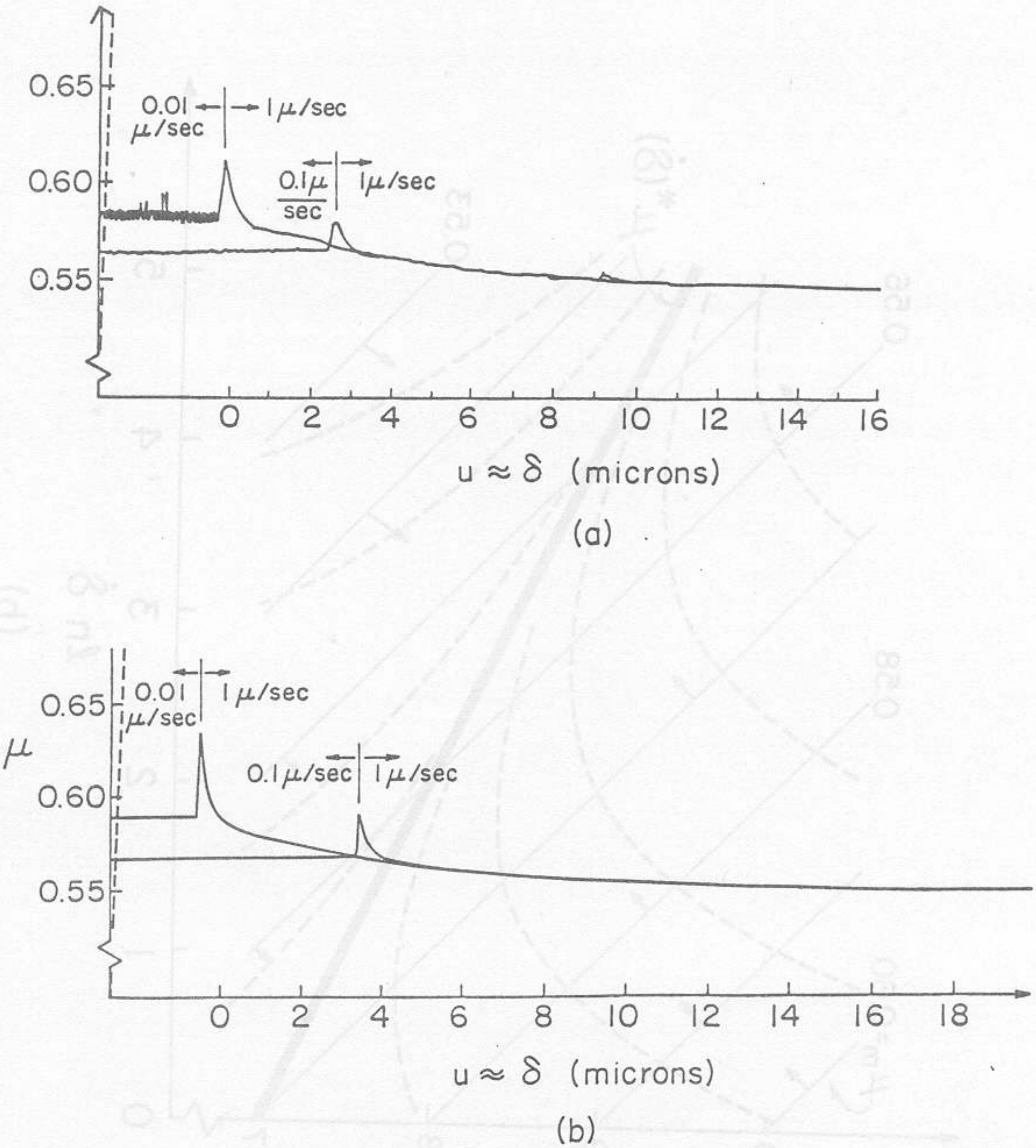
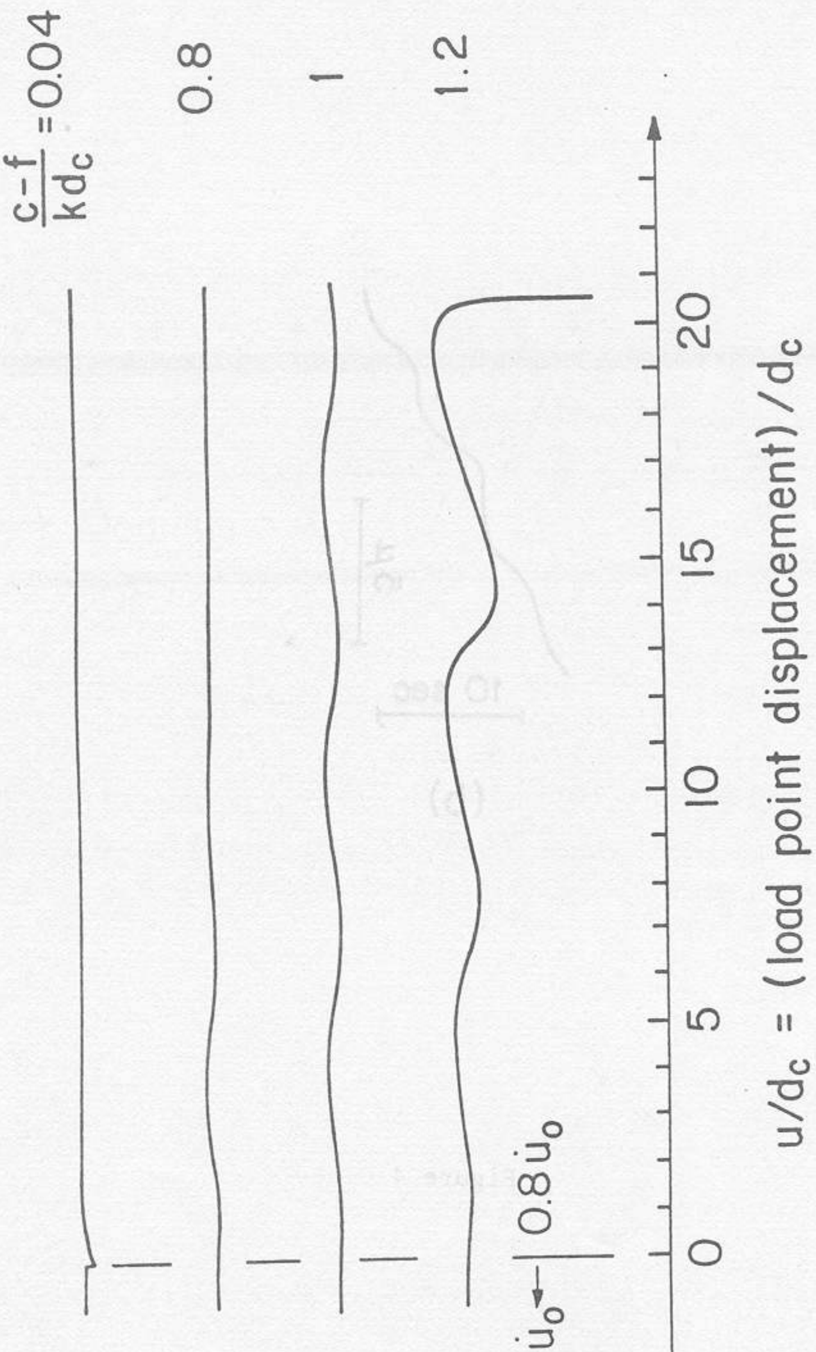
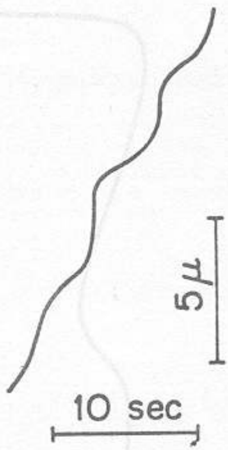

0.65
0.60
0.55
0.01 μ/sec
1 μ/sec
0.1μ sec
1μ/sec
0 2 4 6 8 10 12 14 16
u ≈ δ (microns)
(a)
μ
0.65
0.60
0.55
0.01 μ/sec
1 μ/sec
0.1 μ/sec
1 μ/sec
0 2 4 6 8 10 12 14 16 18
u ≈ δ (microns)
(b)

Figure 3



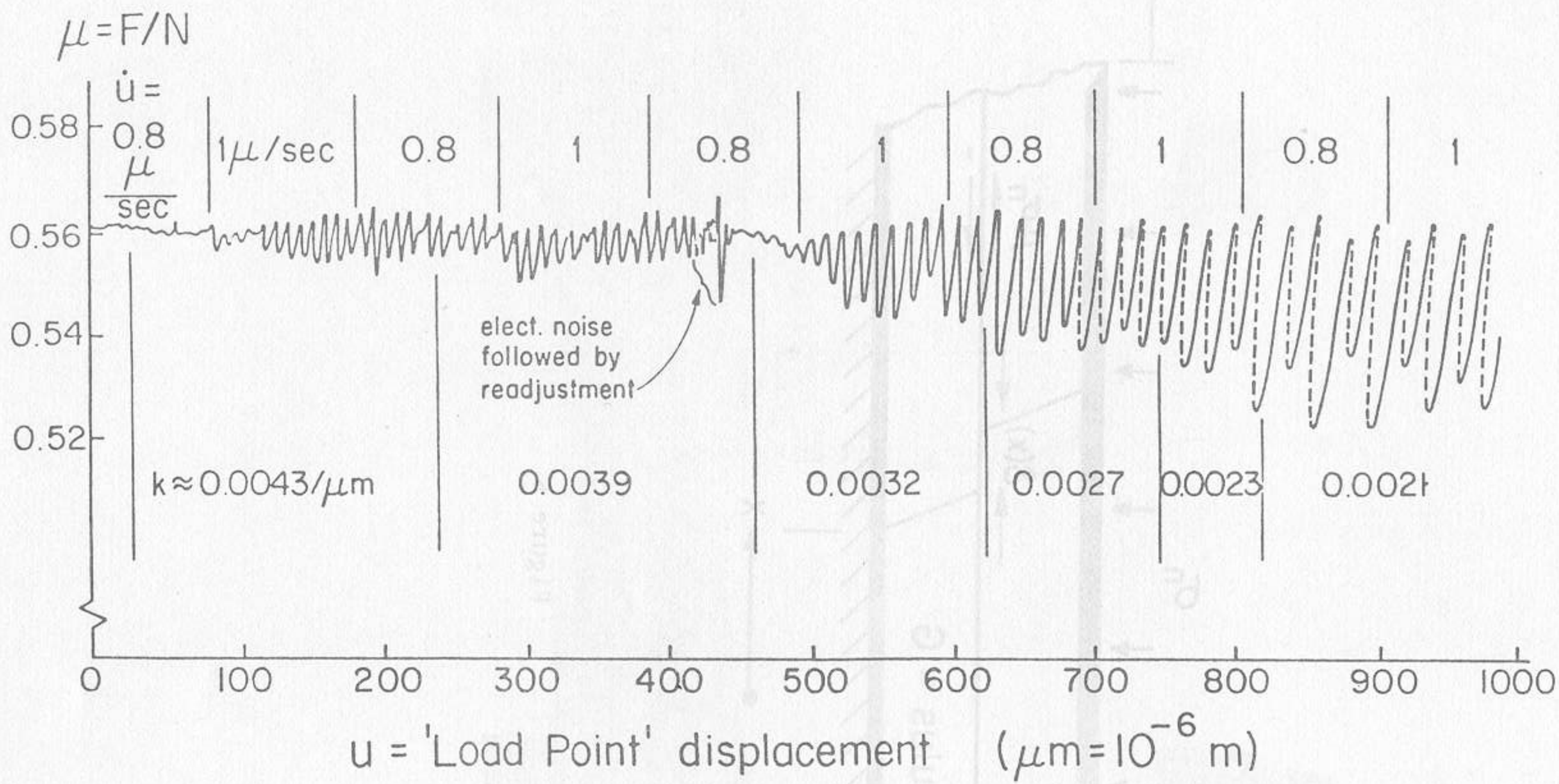
$\frac{c-f}{kd_c} = 0.04$
0.8
1
1.2
μ
$\dot{u}_o$
$0.8\dot{u}_o$
0
5
10
15
20
u/d_c = (load point displacement) / d_c

Figure 4a



(b)

Figure 4


$\mu = F/N$
$\dot{u} =$ 0.8 μ/sec
1μ/sec
0.8
1
0.8
1
0.8
1
0.8
1
0.58
0.56
0.54
0.52
elect. noise followed by readjustment
k ≈ 0.0043/μm
0.0039
0.0032
0.0027
0.0023
0.0021
0
100
200
300
400
500
600
700
800
900
1000
u = 'Load Point' displacement (μm = 10^{-6} m)

(c)

Figure 4

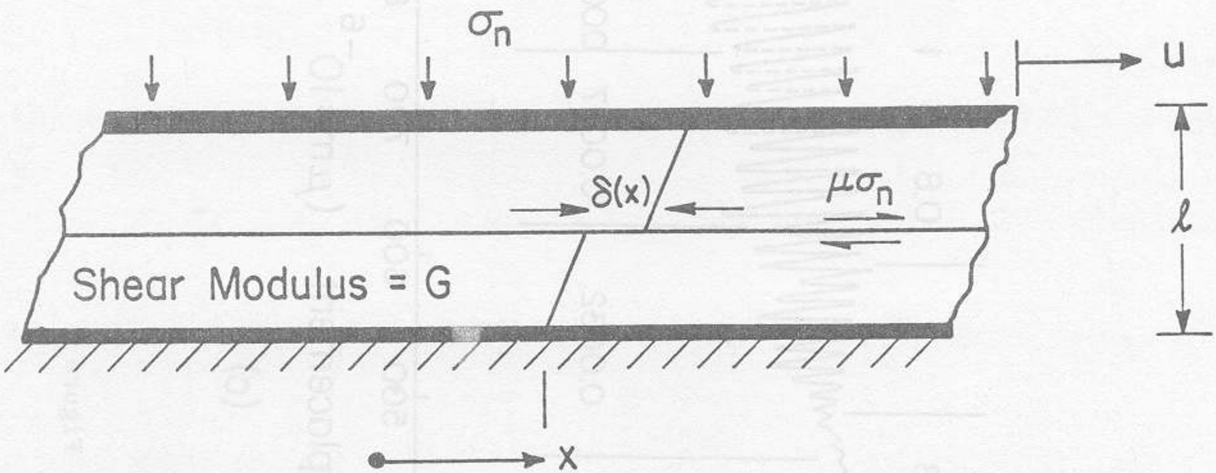
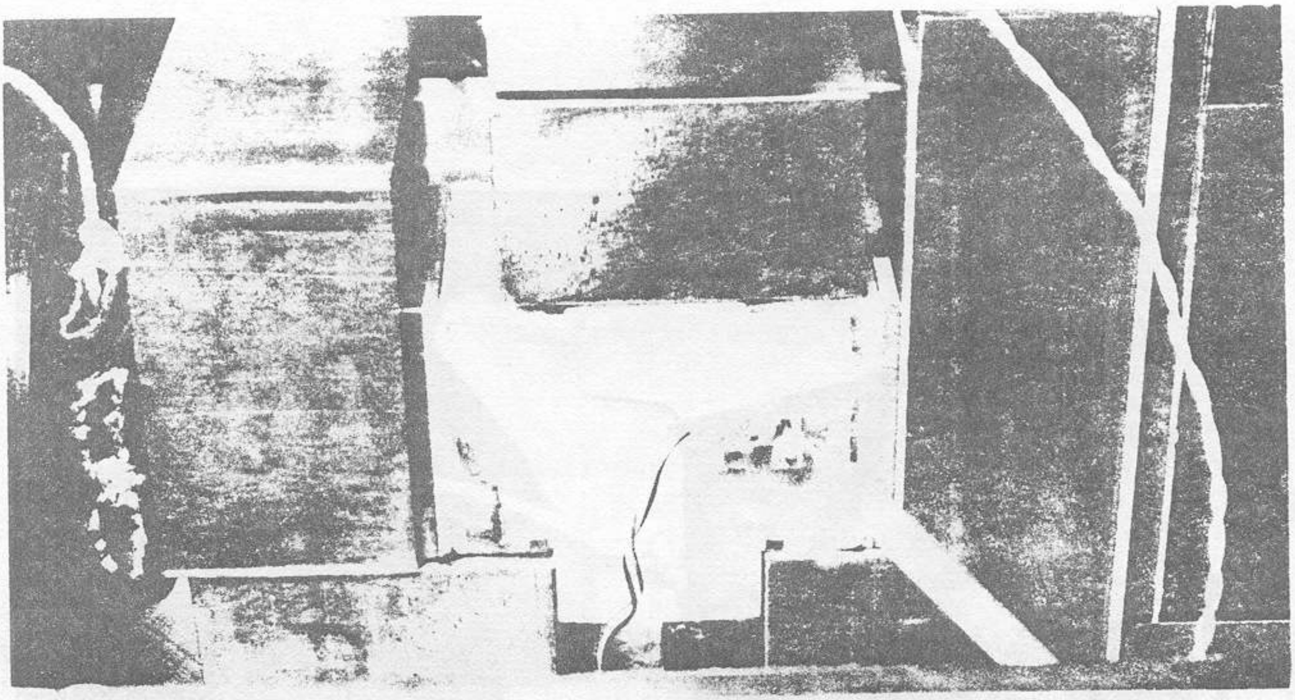


Figure 5



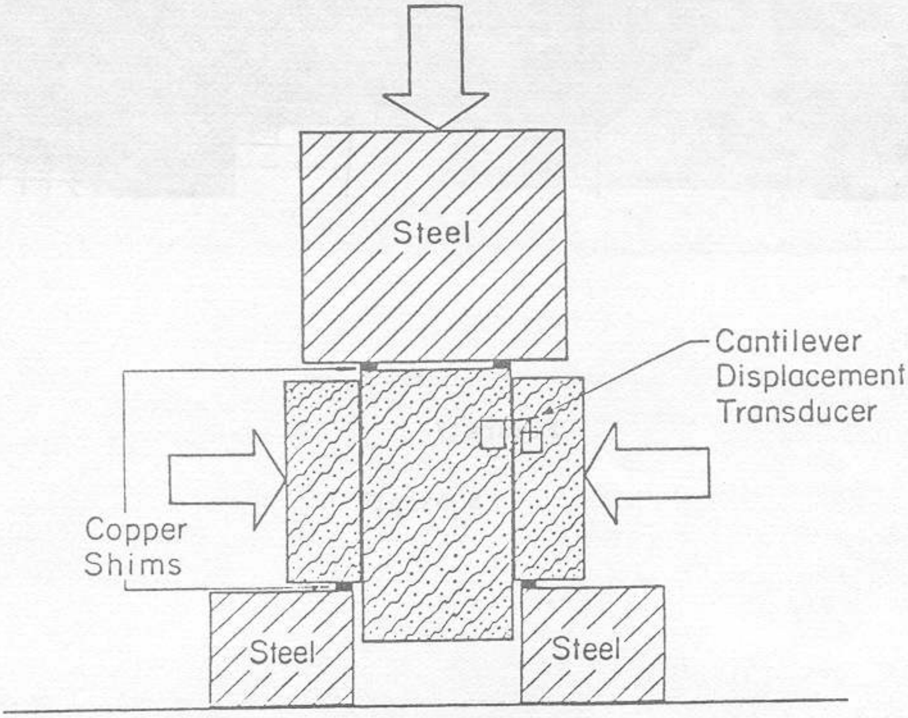

Steel
Cantilever
Displacement
Transducer
Copper
Shims
Steel
Steel

Figure 6

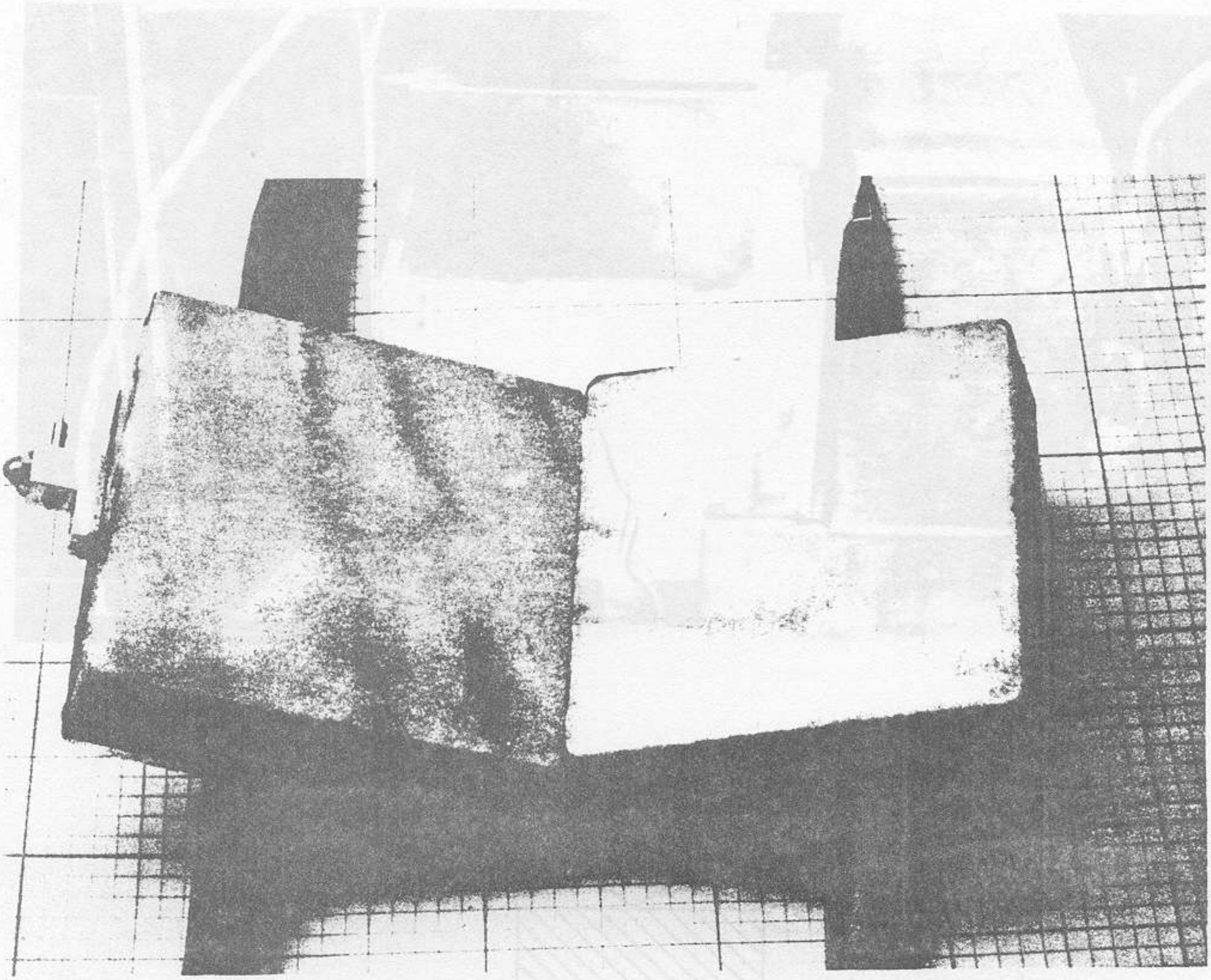


Figure 7

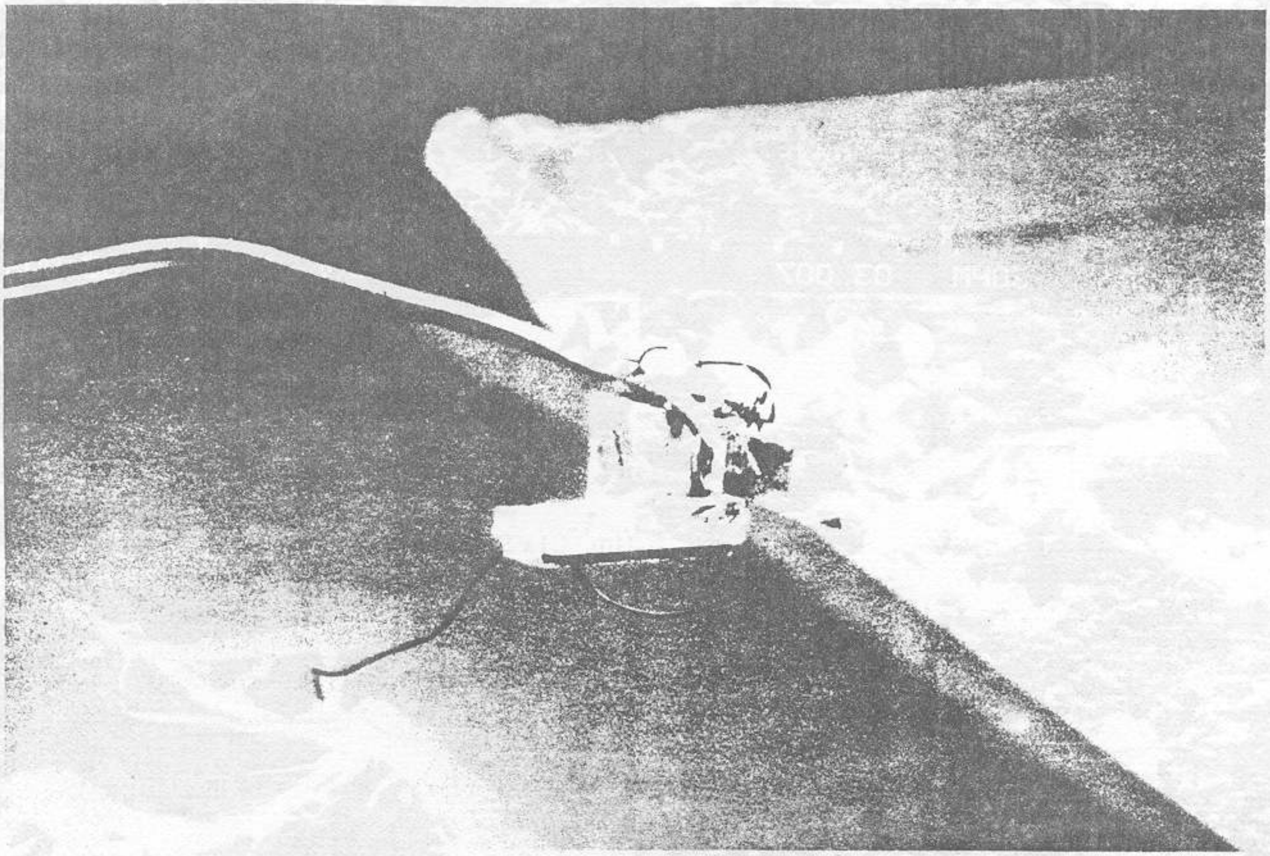


Figure 8

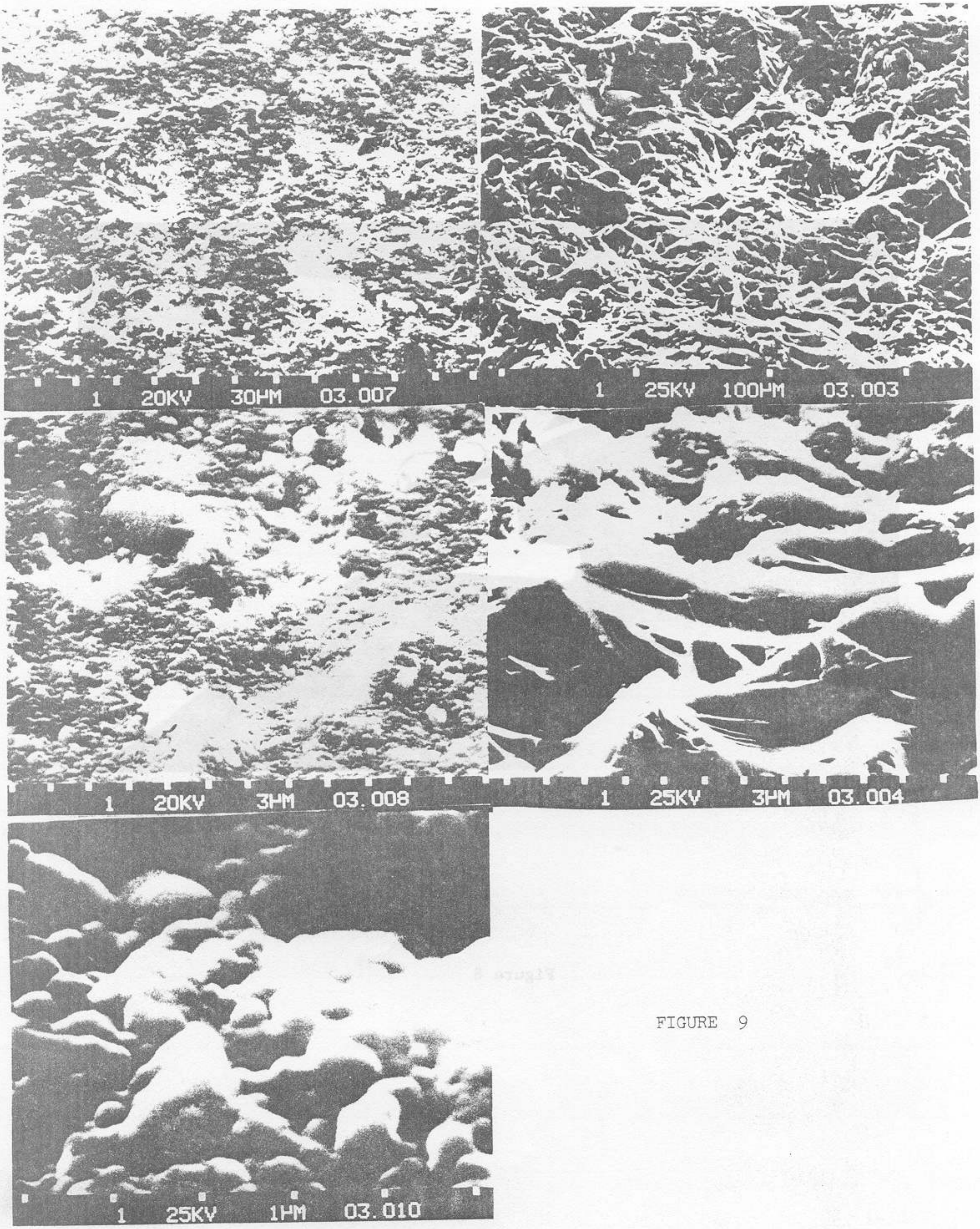
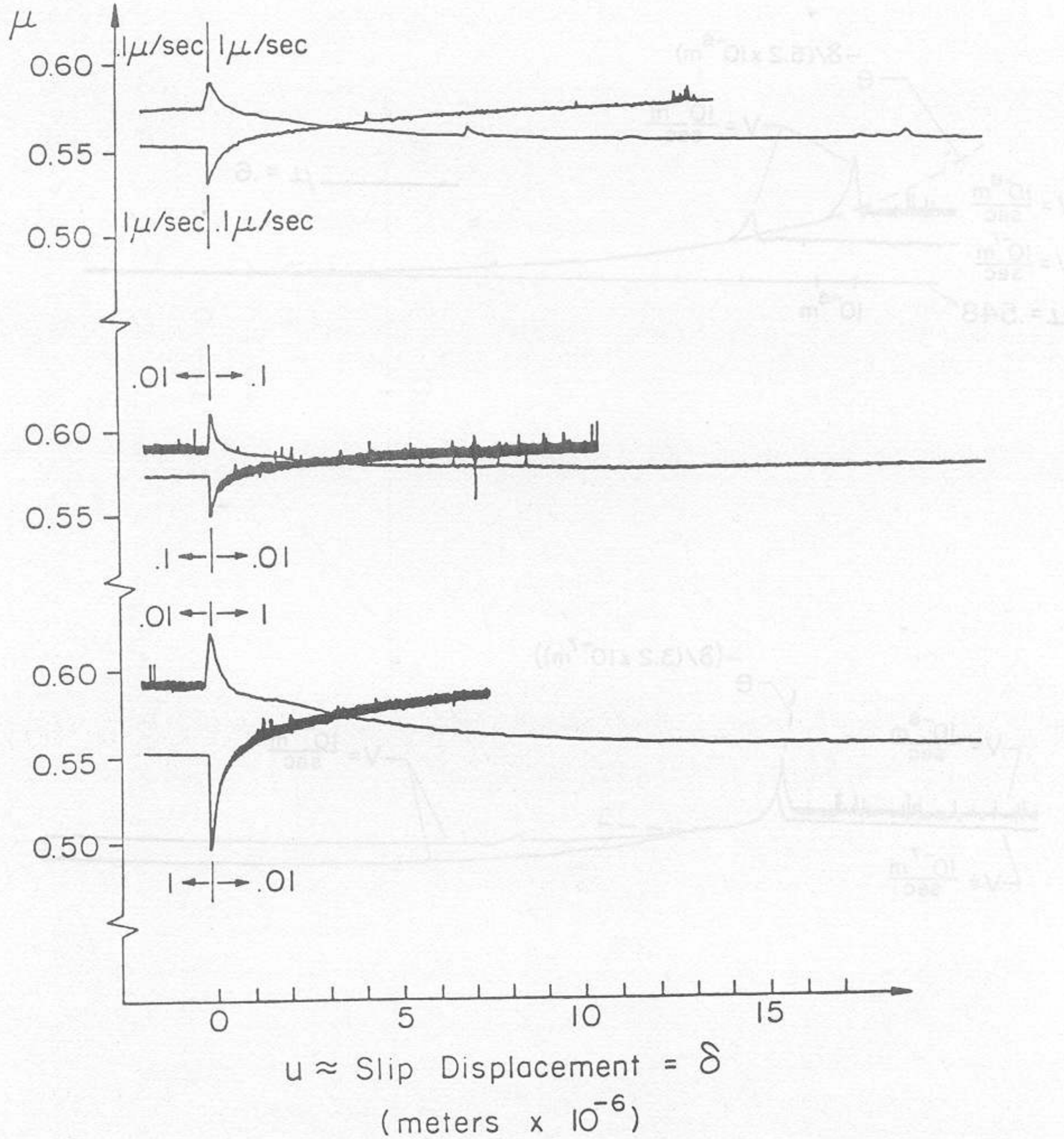
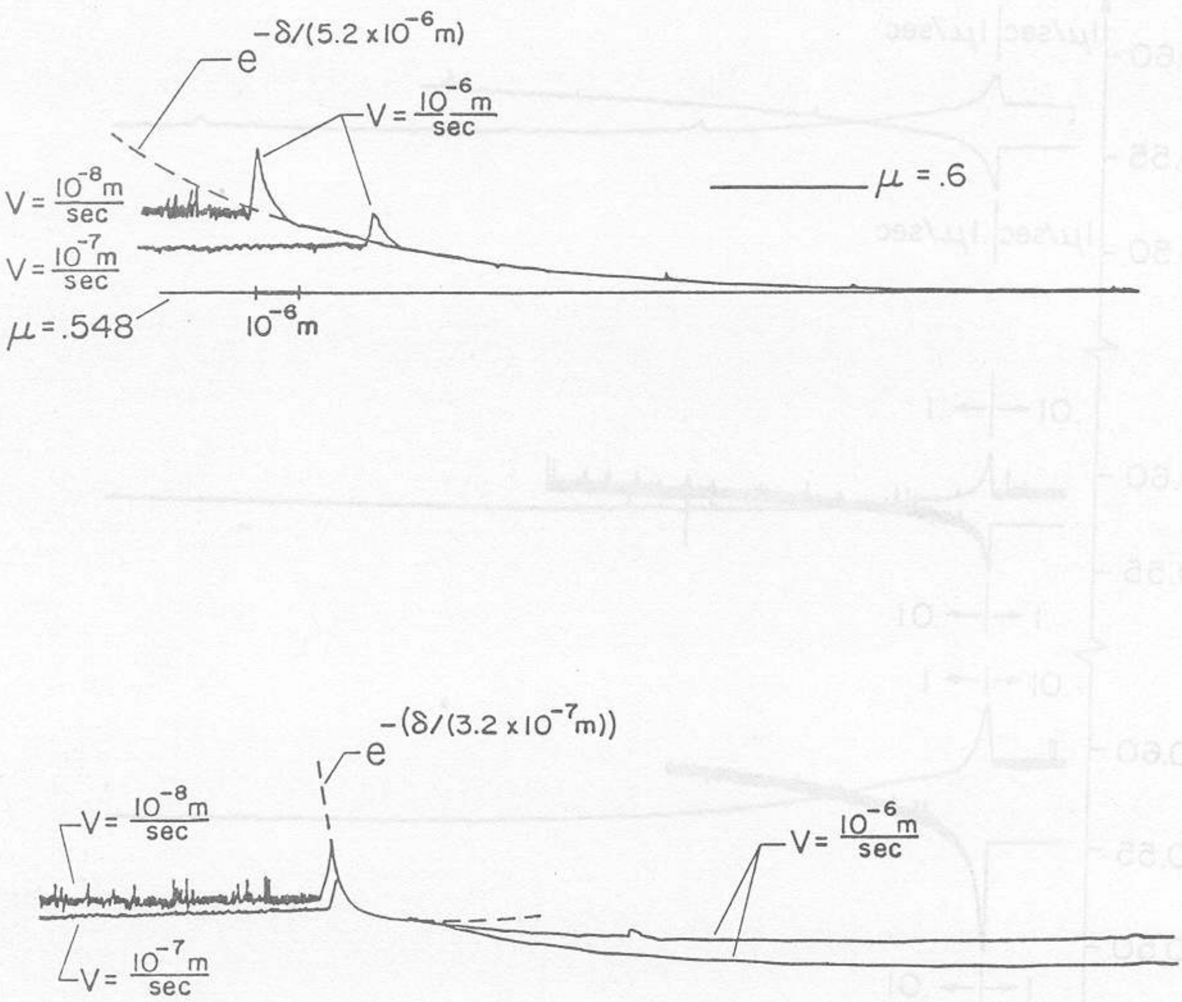

1 20KV 30µM 03.007
1 25KV 100µM 03.003
1 20KV 3µM 03.008
1 25KV 3µM 03.004
1 25KV 1µM 03.010

FIGURE 9



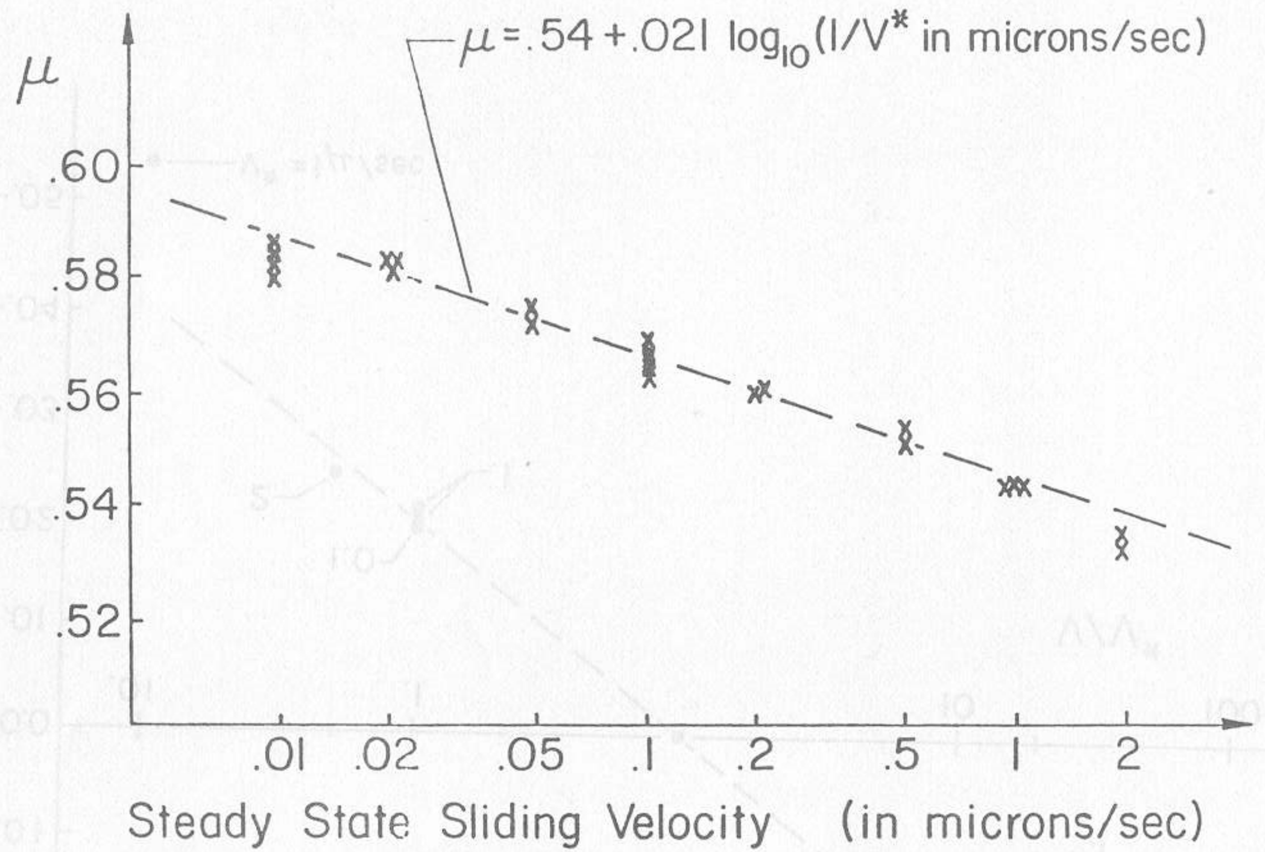
μ
0.60
0.55
0.50
.1μ/sec
1μ/sec
1μ/sec
.1μ/sec
.01 ← → .1
.1 ← → .01
.01 ← → 1
1 ← → .01
0
5
10
15
u ≈ Slip Displacement = δ
(meters x 10^-6)

Figure 10(a)



$e^{-\delta/(5.2 \times 10^{-6} m)}$
$V = \frac{10^{-6} m}{sec}$
$\mu = .6$
$V = \frac{10^{-8} m}{sec}$
$V = \frac{10^{-7} m}{sec}$
$\mu = .548$
$10^{-6} m$
$e^{-(\delta/(3.2 \times 10^{-7} m))}$
$V = \frac{10^{-8} m}{sec}$
$V = \frac{10^{-6} m}{sec}$
$V = \frac{10^{-7} m}{sec}$

Figure 10(b)



$\mu = .54 + .021 \log_{10}(1/V^*$ in microns/sec)
μ
.60
.58
.56
.54
.52
.01
.02
.05
.1
.2
.5
1
2
Steady State Sliding Velocity (in microns/sec)

Figure 11

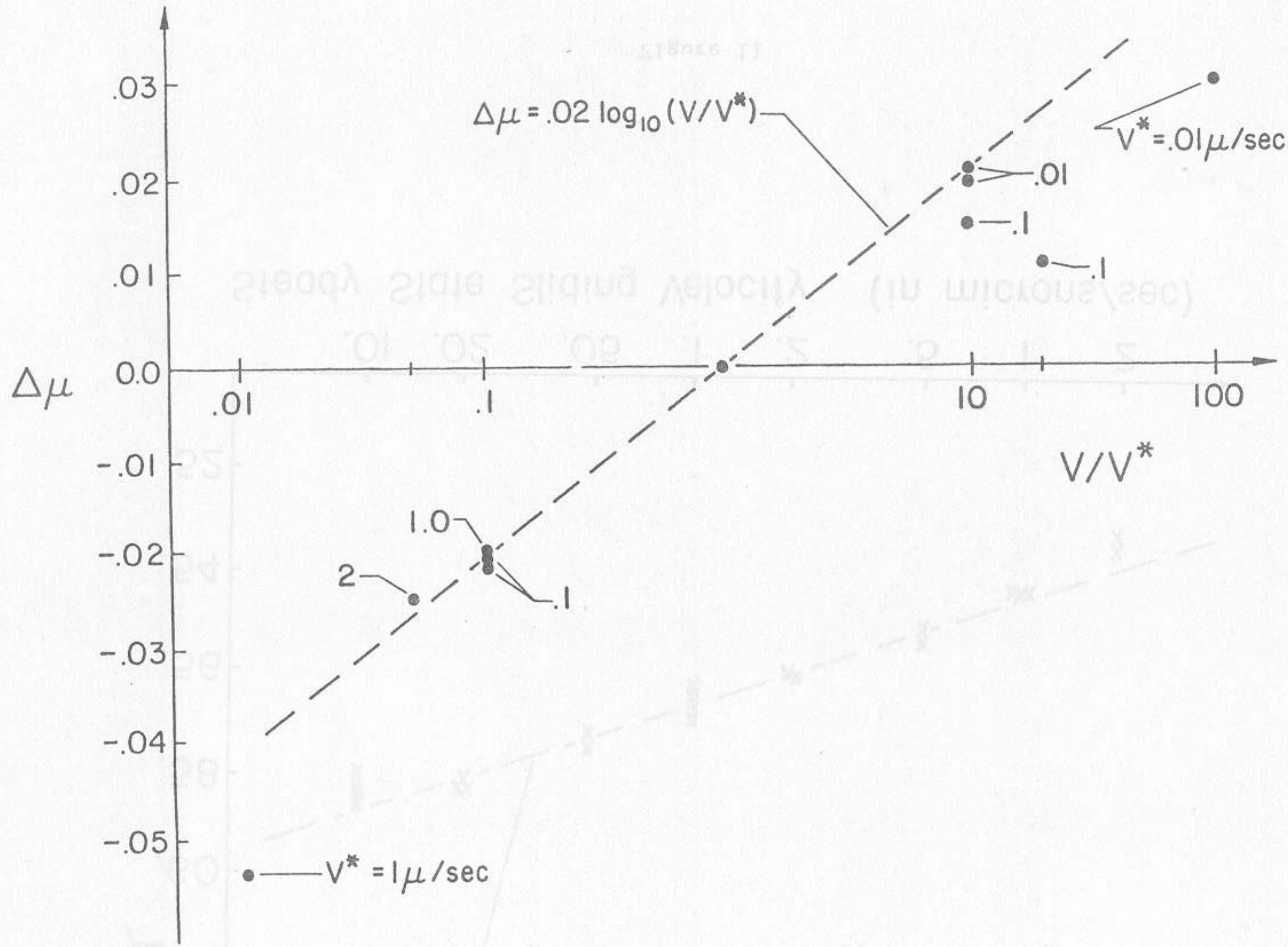

Δμ = .02 log10 (V/V*)
V* = .01μ/sec
.01
.1
.1
Δμ
.03
.02
.01
0.0
-.01
-.02
-.03
-.04
-.05
.01
.1
10
100
V/V*
1.0
2
.1
V* = 1μ/sec

Figure 12

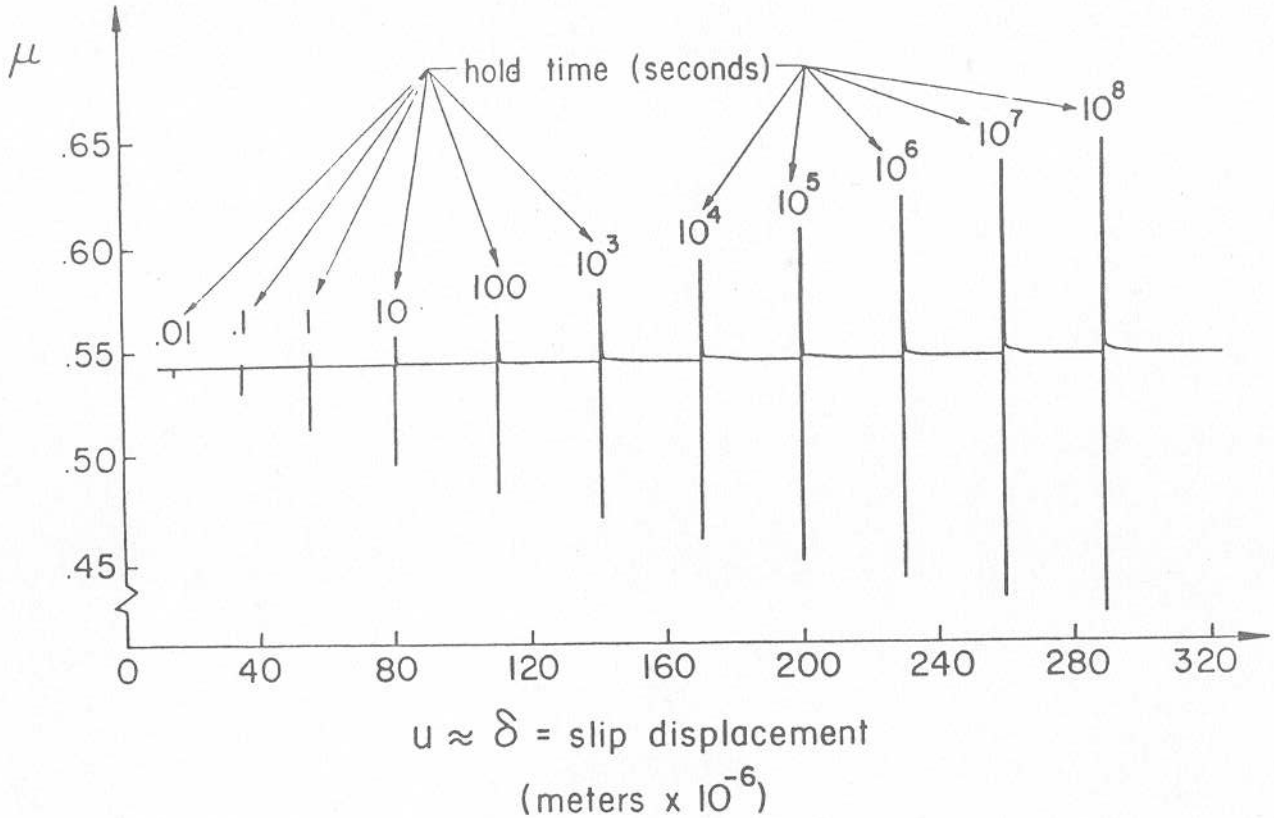

μ
hold time (seconds)
.01
.1
1
10
100
10³
10⁴
10⁵
10⁶
10⁷
10⁸
.65
.60
.55
.50
.45
0
40
80
120
160
200
240
280
320
u ≈ δ = slip displacement
(meters x 10⁻⁶)

Figure 13