Approximate explicit model predictive control for push recovery using mixed-integer convex optimization

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Summary

For legged robots to robustly recover from external disturbances and handle unmodeled environments, they must have control policies which can reason about making or breaking contact with the world at run-time. Such multi-contact control problems can be naturally cast in the hybrid model predictive control (MPC) framework, but at a computational cost which is too high for online use. To overcome this issue, we are exploring results from the explicit MPC literature in order to exploit the known structure of the optimal control policy and derive a concise approximation, which can be queried efficiently online.

Introduction

Recent works in the context of humanoid locomotion [1, 2] have shown that, with plausible assumptions and limited approximations, a legged robot can be modeled as a piecewise affine dynamical system. Such a model, if combined with a convex control objective function, allows searching for a globally optimal set of contacts and inputs in an MPC fashion, using mixed-integer convex optimization. Such optimizations, however, typically take hundreds of milliseconds to tens of seconds to run [1, 2]. Explicit MPC attempts to solve this problem by computing the entire optimal policy as an explicit function of the initial state [3], but at a cost which grows rapidly with the size of the state. Instead, if we treat the optimal policy as something which is too large to model completely, but from which we can sample by running mixed-integer convex programs, then we hope to be able to approximate that optimal policy well.

Explicit Hybrid MPC

MPC for hybrid systems has been a very active research area over the last 20 years: various general modeling frameworks have been proposed [4], stability properties have been analyzed in depth [5], and a multitude of successful implementations have been presented (see, *e.g.*, [6]). However, the intrinsic combinatorial nature of these problems makes the underlying mixed-integer quadratic program (MIQP) prohibitively expensive for high-rate control, even for for low-dimensional systems.

Explicit MPC takes advantage of multiparametric optimization techniques to derive the solution of the optimal control problem as an explicit function of the initial state [3], so that the online control computation is reduced to a quick function evaluation.

While this approach has been very successful for linear systems, its extension to hybrid systems is extremely burdensome. This is mainly due to the fact that, to this day, the detection of boundaries in the parameter space between regions associated with different switching sequences is performed by a direct cost comparison approach [7] that, in practice, requires the solution of a multiparametric program for each feasible combination of binary variables. Nonetheless, in recent years, the MPC community has made great efforts to overcome this hurdle and some promising approaches have been proposed. In [8], for example, an algorithm which solves the problem with a user-defined level of suboptimality has been presented, reducing both storage requirements and online computational effort, while ensuring stability of the controlled system.

Model

We have chosen to simplify our model system as much as possible while still capturing the most basic elements of locomotion. We represent the robot in the sagittal plane by a single rigid body plus four limbs (two hands and two feet), modeled as massless point contacts. Approximate kinematic constraints are enforced by linear inequality constraints on the relative positions of the body and limbs. We enforce that all contact undergoes sticking friction and that all collisions are perfectly inelastic.

The robot's environment consists of a fixed floor and one or more fixed planar walls. Our objective is to stabilize the position and velocity of the robot's center of mass from a wide variety of initial states, some of which will require stepping or reaching out to lean on the walls.

Body pose and centroidal angular momentum must be kept within bounds. Unfortunately, the dynamics of angular momentum are bilinear in the decision variables representing contact locations and forces. We handle the bilinearity using piecewise McCormick envelopes [1].

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Mixed-Integer Quadratic Program

At each time step $i \in \{1, \ldots, N\}$, we have decision variables corresponding to the position and orientation q_i^0, and velocity v_i^0 of the robot's center of mass, the position q_i^j of each limb j, the contact force f_i^j exerted by each limb j. We also introduce binary decision variables z_i^j which indicate whether each limb j is in contact with the world, and binary variables m_i^k which indicate whether each piece k of the McCormick envelope approximation is active. To ensure that the solution is physically realistic, we must ensure that when $z_i^j = 0$ (no contact), we have $f_i^j = 0$, and when $z_i^j = 1$ (contact), we have f_i^j inside the friction cone and limb j actually in contact with the appropriate surface and stationary. This can be imposed with a set of linear inequality constraints using a big-M or convex hull formulation [1]. Additionally, we add linear constraints to impose time-stepping dynamics of q_i^0, and v_i^0, kinematic reachability constraints on $q_i^j - q_i^0$ for each limb j, and any further linear state constraints we may need to impose on the robot. We employ a quadratic objective function to penalize deviation from a desired final position, velocity, and posture.

Exploiting Problem Structure

The optimization problem presented above is very general, however there are specifics about the problem structure that can be exploited to make the problem much more tractable. Consider the possible assignments of a the integer variable $\{z_i^j\}_{i=1}^N$ corresponding to a the j^{th} contact point. Although there are 2^N possible integer assignments, only a small number of these are likely relevant for multi-contact control. Specifically, the optimization allows $\{z_i^j\}_{i=1}^N$ to continually alternate between 0 and 1, continually making and breaking contact with the j^{th} contact point. We conjecture that for time horizons of a few seconds, one or two switching events per limb will be sufficient to stabilize the robot. Allowing just one switching event per limb drastically simplifies the combinatorial optimization, reducing the number of possible assignments of the z_i^j from 2^{LN} to just N^L, where L is the number of limbs. Allowing two contact switches per limb increases the number of assignments to $\mathcal{O}(N^{2L})$. The restriction that the j^{th} limb makes at most S contact switches can be enforced with the constraint $\sum_{i=1}^{N-1} |z_i^j - z_{i+1}^j| \leq S$, which can be represented with a set of linear inequalities and slack variables.

An additional structure which we will seek out and attempt to exploit is *submodularity*. Submodularity encodes a particular form of diminishing returns: informally, a function $f : S \subseteq M \mapsto \mathbb{R}$ is *submodular* if adding an element k to S will increase the value of $f(S)$ by an amount that diminishes as S grows [9]. We hypothesize that the set of contact modes switches in this simple model may exhibit submodularity, i.e. that the stability benefit of a new contact mode switch diminishes with each additional switch. The problem of maximizing a submodular function can be solved by a greedy approach with only bounded suboptimality [9]. By analogy, we will investigate whether additional contact mode switches can also be treated greedily.

Conclusion

We believe that mature control techniques such as explicit MPC, combined with approximation techniques from machine learning, can have great impact on the success of walking robots. Our experiments are ongoing, but we look forward to reporting on both our successes and failures at the meeting.

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