Using Equilibria and Virtual Holonomic Constraints to Generate Families of Walking Gaits

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I. Summary

We present a principled approach to generating walking gaits for point- and curved-feet, underactuated, multi-degree-of-freedom, 2D and 3D biped walkers subject to a biped's hybrid dynamics and user-defined virtual holonomic constraints (constraints that are enforced through feedback control). Specifically, we combine numerical continuation methods (NCM) and the hybrid zero dynamics (HZD) framework to generate families of gaits starting from equilibria of the biped model that satisfy the hybrid dynamics and virtual holonomic constraints (VHC) [1], [2].

The main contribution of the NCM-HZD framework is a topological approach to gait generation. Under this formulation, gaits form connected components in a finite parameter space. As we are interested in generating gaits from equilibria, we give a sufficient condition for when the connected component of an equilibrium point contains a branch of walking gaits. We cannot prove that these conditions hold for all bipeds, but we give a fast and simple computational test that reports whether the condition holds for a given robot. We have had success generating gaits for a representative set of bipeds, including a point-foot model of Boston Dynamics' Atlas robot.

II. Introduction

A challenging and fundamental problem in bipedal locomotion is the gait generation problem: given a model of a biped robot generate periodic motions subject to its hybrid dynamics and other constraints (e.g., VHCs). This critical step has been identified as a difficult and open problem for underactuated bipeds as there exists states throughout a walking trajectory that evolve open-loop without any feedback control during a step [3].

The standard approach to generating gaits is to solve a nonconvex constrained nonlinear optimization problem subject to periodicity, hybrid dynamical, and other constraints on the trajectory of the biped. While optimization-based methods (OBM) have successfully found feasible walking gaits, gait generation optimization problems can be challenging to solve because of convergence issues [3], [4]. In order for OBMs to generate a walking gait, many approaches rely on state-of-the-art solvers, like SNOPT, IPOPT, or fmincon, to converge to a gait from a randomly generated trajectory [4]–[6]. To our knowledge, there has not been any work published on the efficacy of random guessing as a good strategy for generating multiple walking gaits. **We claim that gait generation routines do not have to resort to random guesses or specialized knowledge to generate a family of walking gaits**. For many biped models, starting from equilibria is enough.

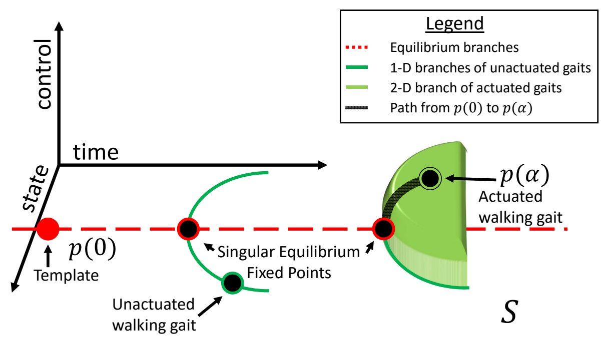

Fig. 1. The connected component of an equilibrium point in a state-time-control space $\mathcal{S}$ of a biped walker. The space $\mathcal{S}$ is a parameterized space of gaits. For example, a point $c = (x^+, \mu, T) \in \mathcal{S}$ is not a trajectory, but a specification of the post-impact state of the biped x^+, the period of the gait T, and control parameters μ. A specific combination of (x^+, μ, T) lead to periodic motion (red and green curves and surfaces).

The NCM-HZD framework is a conceptually different approach to the gait generation problem. We view gaits as forming disjoint connected components in an appropriately defined ambient space $\mathcal{S}$. In this space, equilibria are gaits with zero motion that can collide with the ground at any time. The connected components of equilibria often contain branches of walking gaits. We use numerical continuation methods (NCM) to trace these connected components.

Figure 1 is a conceptual depiction of the connected component of an equilibrium point. The branches of the connected component are the green and red-dashed curves and the light green surface. The red branches consist of stationary gaits of equilibrium points, the green curves are branches of passive dynamic walking gaits —unactuated gaits that walk downhill under the influence of gravity, and the light green surface is a branch of actuated gaits. We can switch from a red branch onto a green branch of passive gaits at special points on the connected component, known as singular points [7] (depicted as black dots with thick red borders). Once we are on a branch of passive gaits, we use the biped's actuators to extend the 1D branch into a higher-dimensional surface of actuated gaits.

III. Methods

We directly address the problem of generating underactuated periodic motions of a biped robot from a *single* seed

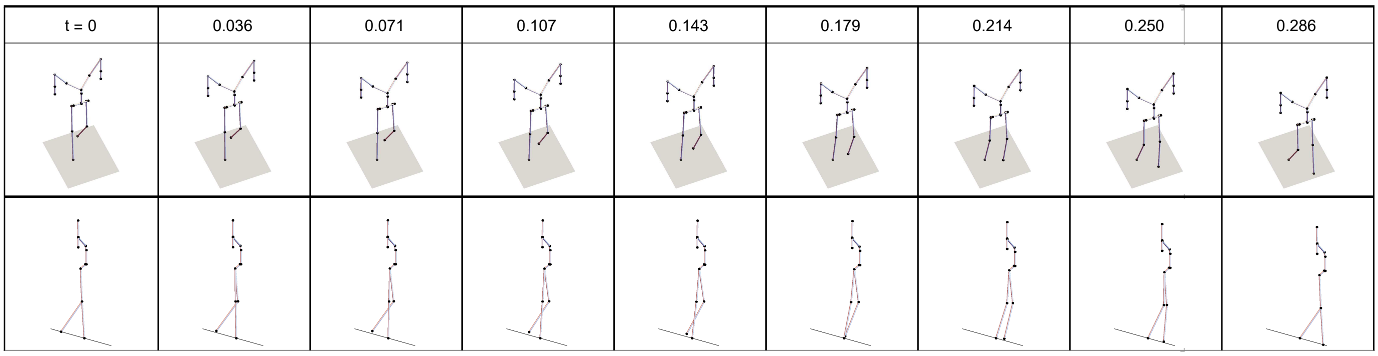

Fig. 2. Boston Dynamics' Atlas Robot walking passively in 3D (top). The same gait projected onto the sagittal plane (bottom).

value, including the case where all joints are unactuated. Specifically, we focus on generating period-one gaits of n-degree-of-freedom biped walkers with point- or curved-feet subject to physical and virtual constraints. The bipeds are physically symmetric about the sagittal plane and have k actuators ($k \geq 0$) powering the internal joints of the biped. For this class of bipeds, it is possible to achieve periodic walking after one step because of the physical symmetry of the mechanism. More formally, given

- a biped model with hybrid dynamics Σ subject to virtual holonomic and physical constraints,
- a finite-dimensional parameter space $\mathcal{S}$, where a point in $\mathcal{S}$ uniquely defines a step of the biped (i.e., $c \in \mathcal{S}$ defines a valid hybrid trajectory satisfying Σ),
- a continuously-differentiable periodicity map $P : \mathcal{S} \to \mathbb{R}^{2n}$, where a point $c^* \in \mathcal{S}$ defines a period-one gait if $P(c^*) = 0$, and
- a set of connected components comprised of implicitly defined curves $c : \mathbb{R} \to \mathcal{S}$, where points on a curve $c(s) \in \mathcal{S}$ satisfy $P(c(s)) = 0$,

trace an implicitly defined curve $c : \mathbb{R} \to \mathcal{S}$ of period-one walking gaits starting from equilibria $x_{\text{eq}} \in \mathbb{R}^{2n}$ of the hybrid dynamics Σ using numerical continuation methods (NCM) to trace the curve c and a hybrid zero dynamics (HZD) controller to enforce the VHCs.

The existence of curves c forming connected components is guaranteed as long as $\frac{\partial P}{\partial c}$ has maximal rank [7]. This is a result of the implicit function theorem. However, we have yet to prove that equilibria of bipeds always contain branches of walking gaits. We can prove that a branch of passive gaits exist if a point on a stationary branch (the red curves in Figure 1) satisfies

$$I(T) = \det\left(\frac{\partial P}{\partial x} \circ (x_{\text{eq}}, T)\right) = 0,$$

assuming a map P that takes a state $x \in \mathbb{R}^{2n}$ and the gait's period $T \in \mathbb{R}$ as part of its input. There exist fast and robust methods for computing roots of the univariate function I [8].

IV. Results and Discussion

This work is a continuation of [1], where the equilibria of representative point-feet planar bipeds are continuously deformed into families of passive dynamic walking gaits. We extend [1] in three important ways:

1) We specify feedback control laws using the HZD framework. In [1], gaits were previously unactuated and the control law was left to the user.
2) We can start from equilibria and generate the passive dynamic walking gaits of a virtually constrained biped. For example, we can make a five-link biped mimic the unactuated gaits of a two-link biped.
3) We can search for gaits on a connected component with specific properties, such as a desired walking speed, step length, or slope.

These results form the core steps of our framework, where we are able to generate passive dynamic walking gaits from equilibria and then extend the passive gaits to gaits with desired properties (e.g., gaits that walk on level ground or walk at a desired average velocity). Figure 2 illustrates a passive gait for a point-foot, 3D version of Boston Dynamic's Atlas robot using the NCM-HZD framework. The biped is walking completely unacuated on an incline of $15.6°$. In other words, we are able to generate an open-loop motion without any feedback control for a 20-degree-of-freedom biped with arms, torso, and legs.

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