Contact-dependent balance stability of biped systems

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Introduction

Instinctively, for tasks involving multiple time-varying contacts interactions (e.g., walking, climbing stairs, etc.) humans plan their predictable contact configurations such that balance stability during the given task is not compromised (Padois et al., 2016). On the other hand, designing balance controllers that allow biped robots to exploit the available contact interactions, while predicting their effects on the system's balance stability, is still a challenge.

In this work, a novel computational approach (Mummolo et al., 2017) is extended to investigate the effects of various contact configurations on the balancing capabilities of constrained biped systems. A balance stability region is identified in the center of mass (COM) state space that includes all states (position and velocity) from which the biped system can be balanced while satisfying physical constraints and maintaining the specified contact configuration. The stability region boundary is constructed via an optimization-based algorithm and provides a contact-dependent threshold between balanced and falling states for the given system. A COM state outside of the stability region boundary represents the sufficient condition for a falling state, from which a change in the specified contact configuration is inevitable.

Experimental trajectories of robotic and human walking are analyzed relative to their corresponding contact-dependent stability regions, to gain insights on the different balance stability strategies between robot and human in various phases of the walking motion.

Methods

The relative notions of balanced and falling states for a biped system are herein interpreted with respect to a specified contact configuration (Fig. 1). For instance, let's consider a biped system balancing on a single foot (Fig. 1; top-left). If, at a given COM position $\bar{\mathbf{r}} = (\bar{x}, \bar{y}, \bar{z})$, the COM velocity is within a certain threshold $\dot{\bar{\mathbf{r}}}_{SS}^{\lim} = (\dot{\bar{x}}_{SS}^{\lim}, \dot{\bar{y}}_{SS}^{\lim}, \dot{\bar{z}}_{SS}^{\lim})$, then balance can be maintained while satisfying necessary physical constraints and preserving the single support (SS) contact configuration. In this case, the system is said to be balanced with respect to SS, e.g., the system can reach a final static equilibrium (Koolen at al., 2012) without altering its contacts, but simply enabled by its initial conditions and available actuation. Conversely, if the velocity perturbations at the COM surpass the threshold $\dot{\bar{\mathbf{r}}}_{SS}^{\lim}$, then the biped won't be able to stop unless SS contact is altered. In this case, the initial COM state is said to be falling with respect to the intended SS contact (Fig. 1), and will end up in a fallen state relative to SS, e.g., double support (DS). The threshold $(\bar{\mathbf{r}}, \dot{\bar{\mathbf{r}}}_{SS}^{\lim})$ represents the system's balance stability boundary, which is dependent on the specified contact configuration.

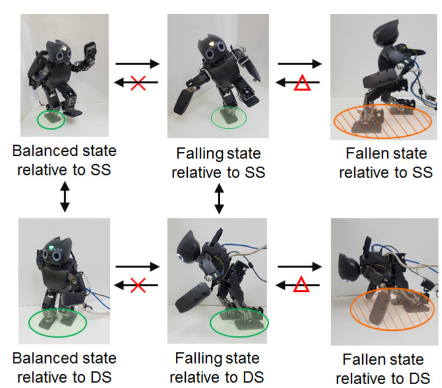

Figure 1. The notion of balance stability states for a biped system are dependent on the specified contact configuration.

For a planar biped model in the (X, Y) sagittal plane, when the COM velocity threshold is searched along the x-direction, the balance stability boundary for a specified contact configuration $(\cdot)$ is the set of points:

$$S_{(\cdot)} = \{(\bar{x}, \bar{y}, \dot{\bar{x}}_{(\cdot)}^{\lim}, \dot{\bar{y}}) \mid \dot{\bar{x}}_{(\cdot)}^{\lim} = \dot{\bar{x}}_{(\cdot)}^{\mathrm{Max}}, \dot{\bar{x}}_{(\cdot)}^{\mathrm{Min}}\} \tag{1}$$

where $\dot{\bar{x}}_{(\cdot)}^{\mathrm{Max}}$ and $\dot{\bar{x}}_{(\cdot)}^{\mathrm{Min}}$ are the COM velocity extrema along x, evaluated at a the COM position through constrained optimization. Using an iterative algorithm, the maximum allowable COM velocity perturbations are evaluated at discretized points of the

COM workspace, by imposing $(\bar{x},\bar{y}) = (x_k, y_j)$ (Fig. 2). Additional constraints include center of pressure bounds for each contact area, positive normal contact forces, friction constraints, joint angle, velocity, and torque limits, final equilibrium at an arbitrary home configuration, and constraints to ensure that the specified contact configuration remains unaltered.

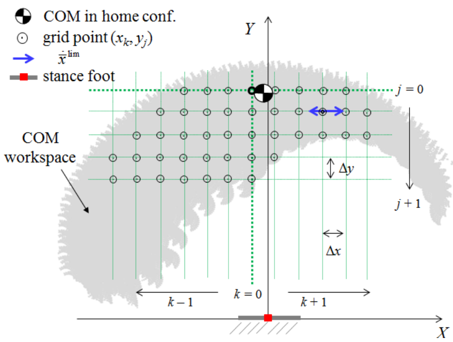

Figure 2. Biped system COM workspace (e.g., in SS), discretized via grid points at which the COM velocity extrema are evaluated.

Since this work is extended to the case of multi-contact conditions, the following challenges are addressed: (1) to formulate a constrained multi-body dynamic model for biped systems in which the indeterminacy between the motion, control, and ground reactions is resolved in a physically consistent manner; (2) to design a numerically efficient and kinematics-consistent method for evaluating and discretizing the complete COM workspace in SS and DS contact configurations; (3) to establish a systematic optimization-based algorithm for the construction of balance stability boundaries that are dependent on the specified contact.

Results and Discussion

The contact-dependent balance stability boundary are evaluated for the biped robot DarwIn-OP and analyzed with respect to its experimental walking trajectory. The stability region in DS contact configuration is smaller than that of SS (Fig. 3). Hence, during SS the robot can recover from larger COM velocity perturbations in the x direction, as opposed to DS, while simultaneously ensuring that the contact status between robot parts and the environment is unaltered.

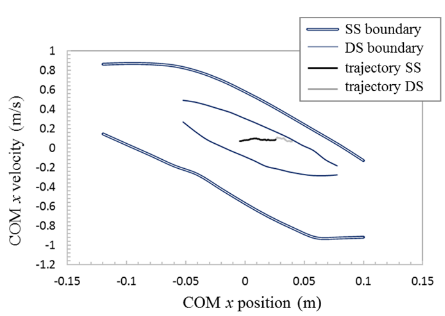

Figure 3. Results of the biped robot stability boundaries and one-step trajectory in the state space of COM x position and velocities.

Every state in the one-step walking trajectories results balanced with respect to SS and DS. This implies that the balance controller available for the robot platform (DarwIn-OP, ROBOTIS) is designed to generate walking motions that are very "conservative" (i.e., far from stability boundaries, except for late DS phase).

The stability boundary results and the experimental one-step trajectory are also shown for a biped model based on a real human subject (Fig. 4), for which a more refined gait segmentation is chosen (SS1, flat foot; SS2, toe contact; DS). The human balancing strategy during SS1 is in contrast to what observed above for the robot SS, since great part of the COM states in SS1 phase lies outside of the corresponding stability boundary. These results reveal the well-known basic principle that natural human walking is characterized by series of "controlled forward falls", from one foot stance to the next foot contact.

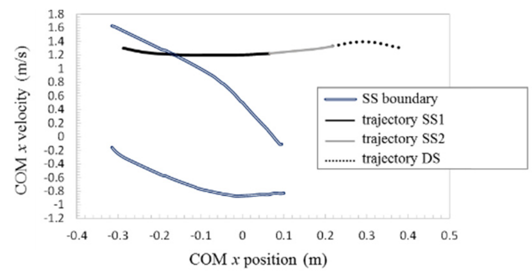

Figure 4. Results of the human subject stability boundaries in the state space of COM x position and velocities, during SS1. Boundary results for SS2 and DS are part of on-going work.

For general robotic gait control applications, the balance stability region provides a "map" of balanced states relative to a given contact configuration. This map could be a useful reference for the (re-)design of system- and contact-specific balance controllers, whose performance region can be analyzed in the state space, relative to the contact-dependent stability regions evaluated beforehand. This could provide, for instance, guidelines for more advanced balance controller for robots that are able to perform a less conservative and more efficient passive dynamic walking and achieve more human-like efficient locomotion.

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Sensitivity analysis of the balance stability region in legged mechanisms

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Summary

In order to determine how close a legged system is to actual falling, the simultaneous influence of multiple factors must be taken into account, such as system parameters (e.g., actuation and angle limits, foot size, etc.), physics initial and final conditions , and center of pressure limits. These factors are responsible for setting a boundary to all the possible balanced state trajectories that can be generated for a given legged system. In this work, we address the question of how sensitive this balance-stability region is to variations in its different system parameters, e.g., to perturbed torque limits and to perturbed foot size limits. We use dynamics to obtain numerical solutions, and optimization tools (sensitivity analysis of the optimal solution, which are in progress) to obtain analytical solutions to this question.

Introduction

The mechanical design of legged systems is not a trivial task due to the high complexity and interplay of the parameters to be considered, especially when considering a design purpose, e.g., energy-efficient vs. highly stable (Collins & Ruina 2005). The question of stability of the system often comes at hand in the design process, especially when making changes in the mechanical design, which is often dependent not only on simulations but also on heuristics. Common parameters disputed in the design process are the selection of the motors, often based on their weight and torque and joint speed limitations and the dimensions and the weight of the foot. The work presented here is focused on the development of theoretical constructs that make use of tools from dynamics and optimization theory to provide incentive of the different impact and interplay of some of these design parameters.

Based on a novel computational framework for the balance stability regions of legged system in the center of mass (COM) state space (Mummolo et al. 2017), this work proposes a sensitivity-based method to predict the effects that variations on system parameters have on the dimensions of the balance stability boundary. In particular the biped parameters considered are foot size, also called foot support region (FSR) and torque limits, which have direct impact on the limits of center of pressure displacement. Numerical results shows that the capability to attain a balanced condition (e.g., avoiding to fall) is enhanced by increasing either the foot support base or the torque capacity at the joint (i.e., by having a stronger joint).

Methods

A simple model used to characterize a bipedal system is the inverted pendulum model, described by:

$$\ddot{\theta} = f(\theta, \tau) \tag{1}$$

with joint angle acceleration $\ddot{\theta}$ being a nonlinear function of joint angle θ and joint torque τ. This model is considered jointly with the biped constraints:

$$F_y \geq 0 \tag{2}$$

$$|F_x| \leq \mu F_y \tag{3}$$

$$\mathrm{FSR}_l \leq x_{\mathrm{COP}} \leq \mathrm{FSR}_r \tag{4}$$

where F_x and F_y are the horizontal and vertical ground reaction forces, respectively, μ is the coefficient of static friction, x_{COP} is the location of the Center of Pressure (COP), and FSR_l and FSR_r are the left and right ends of the FSR, respectively.

The above constrained system, with its specified parameters, is implemented in an optimization formulation to find the minimum and maximum value of $\dot{\theta}$ at a certain time instant t_0, such that the following constraints are satisfied: constraints (1) – (4) for $[t_0, T]$, specified position θ at t_0, final condition of static balance at $t_0 + T$, and the system's joint angle, speed, and torque limitation for $[t_0, T]$. Computing these velocity extrema for each discretized value of θ, the points in the balance stability region can be evaluated (Mummolo et al. 2017). The resulting region for a specified legged system identifies the set of states from which a final balanced state can be attained given proper, unspecified controller.

In the work presented here, these results are extended to study the effects of: (*i*) joint torque limits variation on the stability region, and (*ii*) of FSR variation on the stability region. Through a parametrical study, these effects are studied numerically first, by iteratively computing the boundaries of a system of interest with nominal system parameters m, L, τ^{LB}, τ^{UB}, FSR_l, and FSR_r, and then computing the boundary for a system with one modified parameters at a time (e.g., m, L, $\tilde{\tau}^{\mathrm{LB}}$, $\tilde{\tau}^{\mathrm{UB}}$, FSR_l, and FSR_r). In addition, a sensitivity-based method to predict the results of the parametrical study is proposed. Since the points of the balance stability boundary is a direct result of a constrained optimization problem, the effects that variations on system parameters have on the dimensions of the balance stability boundary could be predicted using nonlinear constrained optimization theory. A sensitivity analysis can be performed on the optimization problem solved at each point of the balance stability boundary, where the cost function being minimized is:

$$f_0(x) = \dot{x}(t_0) \tag{5}$$

where $\dot{x}(t_0)$ denotes the initial velocity of the COM in the x-direction. For instance, in the case of the sensitivity of the boundary with respect to the FSR variation, constraint (4) is written in standard form with:

$$g_1 = x_{\mathrm{COP}} - \mathrm{FSR}_r \leq 0 \tag{6}$$

$$g_2 = \mathrm{FSR}_l - x_{\mathrm{COP}} \leq 0 \tag{7}$$

where the sensitivity equation is given by:

$$\delta f_0^* = -u_1^* e_1 - u_2^* e_2 \tag{8}$$

with u_1^* and u_2^* being the Lagrange multipliers (of the optimal solution) associated to g_1 and g_2, respectively, while e_1 and e_2 being the variation of the RHS of g_1 and g_2, respectively. Hence, the perturbed balance stability regions for perturbed FSR's can be directly computed through (8).

Results

For a nominal system, modeled as a nonlinear inverted pendulum subject to the biped constraints (2) - (4), the parametrical study on the stability boundary region is performed first by varying torque limits at the ankle joint (Figure 1) and then by varying the FSR dimensions (Figure 2), while the rest of the parameters are held constant. It can be seen that as the torque limit decreases, the stability boundary of the system shrinks. Similarly, as the torque limits increase, the stability boundary expands.

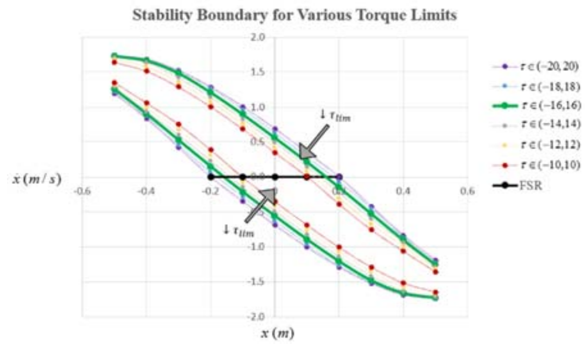

Figure 1. Balance stability boundary with perturbed torque limits.

Slight changes in the FSR have a greater impact on the balance stability region, as compared to the changes seen in Figure 1.

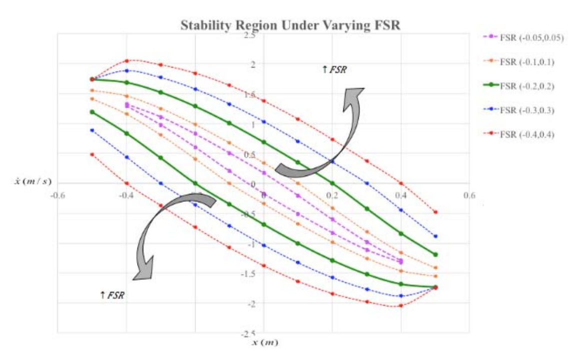

Figure 2. Balance stability boundary with variations on FSR's.

Discussion

This work makes use of a recent boundary stability framework for legged systems to investigate the sensitivity of stability boundary regions in a perturbed system. The results, either from parametrical study of predicted from sensitivity analysis, could provide insights in the model-based design of stable legged mechanisms.

References

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