"Solutions"

Your Name: ANDY RUINA

TA name and section time: —

T&AM 203 Prelim 1

Tuesday February 28, 2006

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3 problems, 25 points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- reasonable justification, enough to distinguish an informed answer from a guess, is given; you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem 1:	/25
Problem 2:	/25
Problem 3:	/25

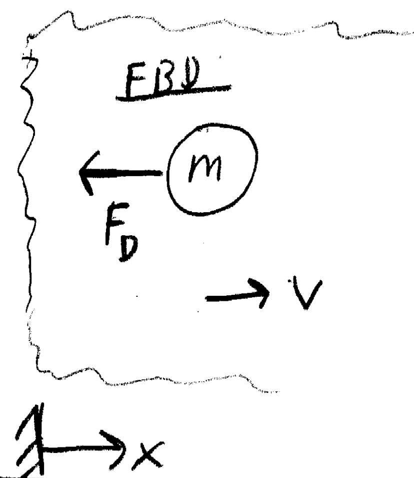
1) **(25 pt)** A shot pellet mass m hits a bird or person's skin at speed v_0. Neglect gravity. Assume m, v_0 and c (below) are given. Assume one dimensional motion in, say, the x direction.

a) (15 points) Assume that the force of the flesh on the pellet is $-cv$, that is the drag force resists motion and is proportional to the speed. How far does the pellet go before it comes to rest? (Please re-read the rules at the front of the exam.) $\Rightarrow F_D = cV$

b) (7 points) Assume that the force of the flesh on the pellet is $-c|\mathbf{v}|\mathbf{v}$, that is the drag force resists motion and is proportional to the speed squared. How far does the pellet go before it comes to rest (the answer is perhaps surprising). $\Rightarrow F_D = cV^2$

(assume $V \geq 0$)

c) (3 points) Given that quadratic drag (b above) is a much more accurate model than linear drag (a above) for fast moving things in air, water and flesh, why does the calculation in b give a patently ridiculous answer? How could you change the calculation to make it more accurate? (It might be possible to get this problem right without getting b right.)



$$V = \dot{X}$$

$$\sum F = ma \Rightarrow \boxed{m\ddot{X} = -F_D} \quad ①$$

a) $m\ddot{x} = -cV$

$$\Rightarrow \ddot{x} + \tfrac{c}{m}\dot{x} = 0$$

$$\Rightarrow \dot{V} + \tfrac{c}{m}V = 0$$

$$\Rightarrow V = V_0 e^{-(c/m)t}$$

$$\Rightarrow x = -V_0 \tfrac{m}{c} e^{-(c/m)t} + C_1$$

$$x(0) = 0 \Rightarrow \quad C_1 = \frac{mV_0}{c}$$

$$\Rightarrow x = \frac{mV_0}{c}\left(1 - e^{-(c/m)t}\right)$$

$$\boxed{\underset{t\to\infty}{X =} \quad mV_0/c} \quad (a)$$

b) ① $\Rightarrow m\dot{V} = -cV^2$

$$\Rightarrow \frac{dV}{V^2} = \frac{-c}{m}dt$$

$$-1/V = -\tfrac{c}{m}t + C_1$$

$$V(0) = V_0 \Rightarrow \quad C_1 = -1/V_0$$

$$\Rightarrow V = \frac{1}{\frac{1}{V_0} + \frac{c}{m}t}$$

$$\Rightarrow dX = \frac{V_0\,dt}{1 + cV_0t/m}$$

$$X = \tfrac{m}{c}\ln(1 + cV_0t/m) + C_1$$

$$X(0) = 0 \Rightarrow \quad C_1 = 0$$

$$\Rightarrow x = m/c \ln(1 + cV_0t/m)$$

$$\boxed{\underset{t\to\infty}{X =} \quad \infty} \quad (b)!$$

c) because eventually the pellet goes slowly. other neglected forces then dominate the quadratic drag.

Some fixes

i) $F_D = C_1V^2 + C_2V$

or ii) $F_D = C_1V^2 + C_2$

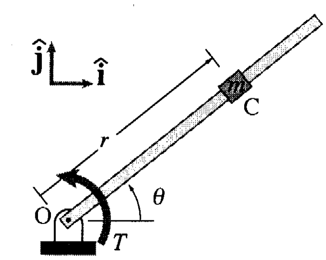
Both of these slow the pellet down enough at low speeds to predict finite (and more accurate) penetration

2) (25 pt) A torque T varies in time as it must in order to rotate a rigid rod at constant rate of $\dot{\theta} = 2$ rad/s. A bead slides on the rod. At the start $t = 0$, $\theta = 0$, $r = 1$m and $\dot{r} = 0$. Neglect gravity and friction.

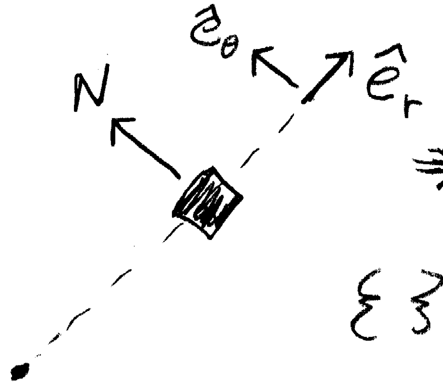
a) (15 points) What is the radius when $\theta = 2\pi$?

b) (5 points) What is the speed $|\underline{\mathbf{v}}|$ when $\theta = 2\pi$?

c) (5 points) When $\theta = 9\pi/4$ what is the direction of $\underline{\mathbf{v}}$. A *very* simple answer is desired which is not exact, but is accurate to within a degree or less.



FBD



$$\underline{F} = m\underline{a}$$

$$\Rightarrow \left\{ N\hat{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\cancel{\ddot{\theta}}^{\,0} + 2\dot{r}\dot{\theta})\hat{e}_\theta \right\}$$

$$\{\ \}\cdot\hat{e}_r \Rightarrow \ddot{r} - r\dot{\theta}^2 = 0$$

a constant,

$$\Rightarrow r = c_1 e^{\dot{\theta}t} + c_2 e^{-\dot{\theta}t}$$

$$\dot{r} = c_1\dot{\theta}e^{\dot{\theta}t} - c_2\dot{\theta}e^{-\dot{\theta}t}$$

note: $\dot{\theta}t = \theta$
const.

$\dot{r}(0) = 0 \Rightarrow c_1 = c_2 \Rightarrow$
$r(0) = r_0 \Rightarrow c_1 + c_2 = r_0 \Rightarrow$

$$r = r_0(e^{\theta} + e^{-\theta})/2$$

$$\dot{r} = r_0\dot{\theta}(e^{\theta} - e^{-\theta})/2$$

a) $r(2\pi) = \underbrace{r_0}_{1m}(e^{2\pi} + e^{-2\pi})/2 = \boxed{[(e^{2\pi} + e^{-2\pi})/2\,]\,m \approx \frac{e^{2\pi}}{2}\ m}$ (a)

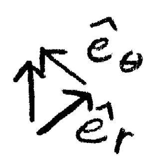
b) $\underline{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = \overbrace{r_0\dot{\theta}\frac{(e^{\theta} - e^{-\theta})}{2}}^{V_r}\hat{e}_r + \overbrace{r_0\dot{\theta}\frac{(e^{\theta} + e^{-\theta})}{2}}^{V_\theta}\hat{e}_\theta$

$$|\underline{V}| = \sqrt{V_r^2 + V_\theta^2} = \frac{r_0\dot{\theta}}{2}\sqrt{(e^{2\theta} - 2 + e^{-2\theta}) + (e^{2\theta} + 2 + e^{-2\theta})}$$

$$= \frac{r_0\dot{\theta}}{2}\sqrt{2(e^{2\theta} + e^{-2\theta})}$$

$$= \frac{2}{2}\sqrt{2e^{4\pi} + 2e^{-4\pi}}\ m/s = \boxed{2\sqrt{\cosh(4\pi)}\ m/s} \quad (b)$$

c) for large θ, $e^{-\theta}/e^{\theta} \Rightarrow \underline{V} \approx \frac{r_0\dot{\theta}}{2}e^{\theta}(\hat{e}_r + \hat{e}_\theta)$ (c)

at $\theta = 9\pi/4$ "$=$" $\pi/4$ $\quad \hat{e}_r + \hat{e}_\theta =$ 

$= \boxed{\underline{V} \text{ in } \hat{\jmath} \text{ direction}}$

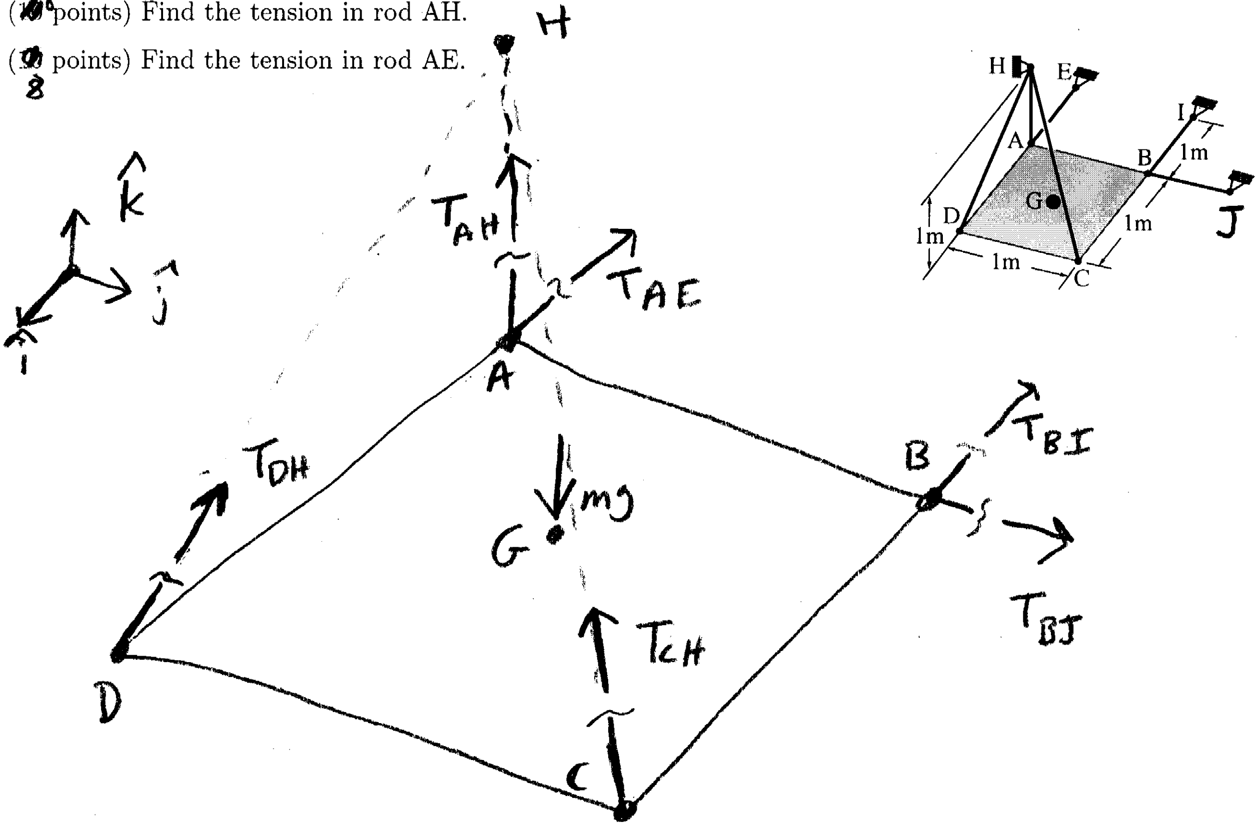
(approx)

3) (25 pt) A uniform square horizontal rigid plate ABCD has weight mg and is held in place by 6 negligible-mass rods. You need not write long vector formulas if you can confidently justify your answers without them. 9

a) (~~10~~ points) Use moment balance about axis AH to find the tension in rod BI.

b) (~~10~~ 8 points) Find the tension in rod AH.

c) (~~5~~ points) Find the tension in rod AE. 8



a) All forces have lines of action $\parallel$ to or intersecting AH except T_{BI} $\Rightarrow$ $T_{BI}\cdot(1m)=0 \Rightarrow \boxed{T_{BI}=0}$ (a)

b) $\sum M_{CD}=0$: only T_{AH} and mg contribute. T_{AH} has twice the lever arm $\Rightarrow$ $\boxed{T_{AH}=mg/2}$ (b)

c) $\sum M_{BH}=0$: only mg & T_{AE} contribute

$$\left\{\sum \underline{M}_{/B}\right\}\cdot \underline{r}_{BH}=0 \Rightarrow 0=\left\{\underbrace{\underline{r}_{BG}}_{\frac{\hat{i}-\hat{j}}{2}m}\times -mg\hat{k} + \underbrace{\underline{r}_{BA}}_{-\hat{j}m}\times T_{AE}(-\hat{i})\right\}\cdot(-\hat{j}+\hat{k})$$

$$\Rightarrow 0=\left[mg\left(\frac{\hat{j}-\hat{i}}{2}\right)+T_{AE}\hat{k}\right]\cdot(-\hat{j}+\hat{k})=\frac{mg}{2}+T_{AE} \Rightarrow \boxed{T_{AE}=-\frac{mg}{2}}$$

(c)