

ENGRD/TAM 203: Dynamics (Spring 2006)

Solution of Homework 8 (assigned on Feb. 16, due on Feb. 23)

by Dennis Yang

1. RP 4.65a

**Statement.** A reel of mass  $M$  and outer radius  $R$  is connected by an inextensible string from point  $P$  across a pulley to a hanging object of mass  $m$ . The inner cylinder of the reel has radius  $r = \frac{1}{2}R$ . The slope has angle  $\theta$ . There is no slip between the reel and the slope. There is gravity. Find the ratio of the masses in terms of  $M$ ,  $m$ ,  $R$ , and  $\theta$ , so that the system is at rest.

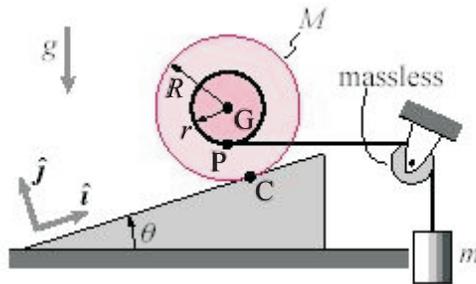


Figure 1

Free Body Diagrams.

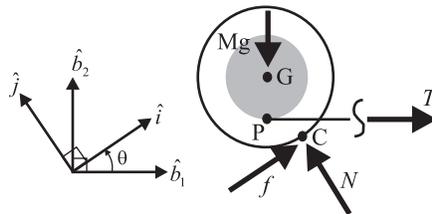


Figure 2

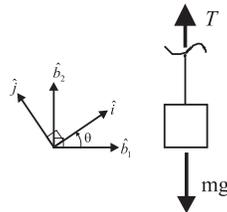


Figure 3

**Solution.** We assume that the pulley is massless, round, and frictionless. Thus on both sides of the pulley, the tensions in the string are identical. Since the system is at rest, the total moments on the reel about point  $C$  is zero. Then we have

$$\begin{aligned}
\sum_i \vec{M}_{i/C} = \vec{0} &\implies \vec{r}_{P/C} \times T \hat{b}_1 + \vec{r}_{G/C} \times Mg(-\hat{b}_2) = \vec{0} \\
&\implies (\vec{r}_{P/G} + \vec{r}_{G/C}) \times T \hat{b}_1 + \vec{r}_{G/C} \times Mg(-\hat{b}_2) = \vec{0} \\
&\implies (r(-\hat{b}_2) + R\hat{j}) \times T \hat{b}_1 + R\hat{j} \times Mg(-\hat{b}_2) = \vec{0} \\
&\implies rT(-\hat{b}_2 \times \hat{b}_1) + RT\hat{j} \times \hat{b}_1 + RMg\hat{j} \times (-\hat{b}_2) = \vec{0} \\
&\implies rT\hat{k} + RT(-\sin(\theta + 90^\circ)\hat{k}) + RMg(\sin\theta\hat{k}) = \vec{0} \\
&\implies (rT - RT\cos\theta + RMg\sin\theta)\hat{k} = \vec{0}. \tag{1.1}
\end{aligned}$$

$$\begin{aligned}
(1.1) \cdot \hat{k} &\implies (rT - RT\cos\theta + RMg\sin\theta)\hat{k} \cdot \hat{k} = \vec{0} \cdot \hat{k} \\
&\implies rT - RT\cos\theta + RMg\sin\theta = 0. \tag{1.2}
\end{aligned}$$

The balance of the forces on the hanging mass  $m$  yields that  $T = mg$ . Substituting this to (1.2), we obtain

$$\begin{aligned}
rmg - Rmg\cos\theta + RMg\sin\theta = 0 &\implies \frac{m}{M} = \frac{R\sin\theta}{R\cos\theta - r} \\
&\implies \frac{m}{M} = \frac{R\sin\theta}{R\cos\theta - \frac{1}{2}R}.
\end{aligned}$$

□

## 2. Problem 3.1.37. (Numerical Solution)

**Statement.** A test sled has a mass of 400 kg. A constant force of 200,000 N acts on it, causing it to accelerate. The sled also experiences a viscous drag with magnitude  $1600 \text{ N} \cdot \text{s/m} |\dot{x}|$ .

- If the sled starts from rest, how long must the force act before it travels 2000 m?
- How fast is it when  $x = 2000 \text{ m}$ ?
- What is the terminal velocity?
- How long does it take to travel to  $x = 2000 \text{ m}$  if we assume it is massless?
- Does the time computed under the massless assumption come close to the answer of (a)?
- Is there a distance over which neglecting the sled's mass leads to a poor approximation of the elapsed time?

**Note:** the above is the original problem statement. In this homework assignment, just redo this problem using numerical integration. A well labeled plot of  $v$  vs.  $x$  is sufficient. You need not answer the various specific questions. Of course hand in your well documented code and explanation of your work with pictures and so on. Your code should include, in the text comments at top, your name (highlight or circle with colored pen).

### Free Body Diagrams.

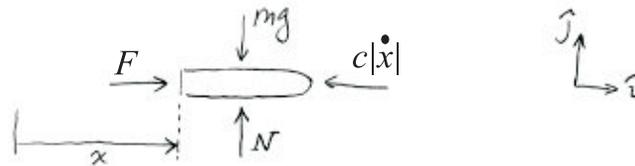


Figure 2

### Solution.

$$\sum_i \vec{F}_i = m\vec{a} \implies F\hat{i} + c|\dot{x}|(-\hat{i}) + mg(-\hat{j}) + N\hat{j} = m\ddot{x}\hat{i} \quad (2.1)$$

$$(2.1) \cdot \hat{i} \implies F - c|\dot{x}| = m\ddot{x}. \quad (2.2)$$

Since the sled travels to the right,  $\dot{x} = |\dot{x}| > 0$ . Thus, (2.2) becomes

$$F - c\dot{x} = m\ddot{x}. \quad (2.3)$$

Let  $v = \dot{x}$ . Then (2.3) becomes

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= \frac{1}{m}(F - cv) \end{aligned}$$

SEE THE NEXT PAGE FOR THE MATLAB CODES!

```

function sled
% Andy Ruina's solution to Tongue and Shepard 3.1.37
% Wednesday Feb 22, 2006

% Initial conditions and time span
tspan= linspace(0,20,100); %Integrate for 20 seconds
z0 = [0 0]'; % initial position and speed both zero

% This one command solves the ODEs
[t z] = ode45(@rhs,tspan,z0);

% Unpack the variables
x = z(:,1); % x is first column of z
v = z(:,2); % v is second column of z

% Plot the results
plot(x,v);
title('Andy Ruina's plot of speed vs position')
xlabel('Position from start in meters')
ylabel('Velocity in meters/s')

end

```

```

%-----
function zdot = rhs(t,z)
% all variables in meters, kg, seconds
x = z(1); v = z(2); % unpack z into readable variables

m = 400; F = 2e5; c = 1600; % given constants

%The next two lines are the heart of this function
%They describe the right hand sides (rhs) of the ODEs.
xdot = v;
vdot = (F - c*v)/m;

z1dot = xdot; z2dot = vdot; % pack up the derivatives

zdot = [z1dot z2dot]'; %This is what the function returns
end
%-----

```

Note to students:

Please type your name as part of your computer commands. Please have part of your computer output be your name. Then circle your name with a colored pen, or highlight it, in both places.

Thanks.

