RP 4.87

A uniform 5 kg shelf is supported at one corner with a ball and socket joint and the other three corners with strings. At the moment of interest the shelf is at rest. Gravity acts in the k direction. The shelf is in the x-y plane.

Find the tension in cable AB only.
Solution

Our goal is to find the tension in cable AB only. Thus, we should observe that the reaction force at C (i.e., \( R_{cx}\hat{i} + R_{cy}\hat{j} + R_{cz}\hat{k} \)), \( \hat{R} \) (the force on the shelf from cable HE), and \( \hat{T}_{GD} \) (the force on the shelf from cable GD) all of them pass through line GC.

Thus, the moments produced by them about axis \( \frac{\hat{R}_{GC}}{||\hat{R}_{GC}||} \) is ZERO!

Since the shelf is at rest, we have

\[
\sum_{i} \vec{M}_{i/c} = \vec{0} \Rightarrow \left\{ \sum_{i} \vec{M}_{i/c} \cdot \frac{\vec{R}_{GC}}{||\vec{R}_{GC}||} \right\} = 0
\]

\[
\text{total moments about axis } \frac{\vec{R}_{GC}}{||\vec{R}_{GC}||}
\]

\[
\Rightarrow \left\{ \hat{R}_{BC}/\vec{XAB}(-\vec{j}) + \vec{R}_{MC}/\vec{Xmg}(-\vec{k}) \right\} \cdot \frac{\vec{R}_{GC}}{||\vec{R}_{GC}||} = 0
\]

\[
\Rightarrow \left\{ m\hat{j}/\vec{XAB}(-\vec{j}) + \left( \frac{1}{2}m\hat{i} + \frac{1}{2}m\hat{j} \right)/\vec{Xmg}(-\vec{k}) \right\} \cdot \frac{1m\hat{z} + 1m\hat{k}}{\sqrt{(1m)^2 + (1m)^2}} = 0
\]

\[
\Rightarrow \left\{ m\cdot\vec{XAB}\hat{k} + \frac{1}{2}m\cdot\vec{Xmg}\hat{j} + \frac{1}{2}m\cdot\vec{Xmg}(-\vec{k}) \right\} \cdot (1m\hat{i} + 1m\hat{k}) = 0
\]

\[
\Rightarrow 1m^3\cdot\vec{XAB} - \frac{1}{2}m^3\cdot\vec{Xmg} = 0 \Rightarrow \vec{XAB} = \frac{1}{2}m\vec{Xmg} = \frac{1}{2}\cdot5kg\cdot9.81m/s^2
\]

\[
\Rightarrow \vec{XAB} = 24.525 N
\]
3.1.3 A car is breaking at constant acceleration, causing fuzzy dice hanging from the mirror to take the position shown. Approximate the fuzzy dice as a lumped mass \( m \) and solve for the steady-state value of \( \theta \) if the car is decelerating at \( 0.2g \).

**Solution**

\[ \vec{F}_z = m \vec{a} \implies mg(-\hat{j}) + T(-\hat{e}_r) = m(0.2g) \hat{i} \]

Since we don't care what \( T \) is, we can dot (*) with \( \hat{e}_o \) to get rid of \( T(-\hat{e}_r) \), that is

\[ (*) \cdot \hat{e}_o \implies mg(-\hat{j}) \cdot \hat{e}_o + T(-\hat{e}_r) \cdot \hat{e}_o = m(0.2g) \hat{i} \cdot \hat{e}_o \]

\[ \implies -mg \sin \theta = 0.2mg \cos \theta \]

\[ \implies \tan \theta = 0.2 \implies \theta \approx 11.3^\circ \]
Assume no friction, massless pulley P, and inextensible rope. The spring is initially unstretched. For scenarios (a) & (b), determine what extension of the spring is necessary to support a static equilibrium of the system. Then assume that $m_2$ is pulled downward 0.01 m and released. What is the acceleration of $m_2$ at the time of release?

Solution (a)

FBD

Note: $\delta$ is the extension of the spring. Since the pulley is round, frictionless, and massless, the tensions on both sides of the pulley are equal.
When the system is at static equilibrium

\[ \sum_{i} F_i = 0 \implies T \hat{i} + k \delta (-\hat{i}) + N \hat{j} + m g (-\hat{j}) = 0 \]  
\[ (1) \]

\[ T \hat{i} + k \delta (-\hat{i}) + N \hat{j} + m g (-\hat{j}) = \delta \hat{i} \]
\[ \implies T - k \delta = 0 \implies T = k \delta \]  
\[ (3) \]

for \( m_2 \), \[ \sum_{i} F_i = 0 \implies T \hat{j} + m_2 g (-\hat{j}) = \delta \hat{j} \]  
\[ (2) \]

\[ T \hat{j} + m_2 g (-\hat{j}) = \delta \hat{j} \]
\[ \implies T - m_2 g = 0 \implies T = m_2 g \]  
\[ (4) \]

\[ (3), (4) \implies k \delta = m_2 g \implies \delta = \frac{m_2 g}{k} \]

Thus, \[ \delta = \frac{20 \text{ kg} \cdot 9.81 \text{ N/kg}}{1000 \text{ N/m}} = 0.1962 \text{ m} \]

Next, when \( m_2 \) is pulled downward 0.01 m further

\[ \text{let } L \text{ be the total length of the rope. Then we have} \]

\[ L = (d + x_2) - x_1 \]

\[ L, d \text{ are constant} \]

\[ L = \dot{x}^0 = \dot{d} + \dot{x}_2 - \dot{x}_1 \]

\[ 0 = \dot{x}_2 - \dot{x}_1 \implies \dot{x}_1 = \dot{x}_2 \]
Now for \( m_1 \),
\[
\sum_i F_i = m_1 \ddot{a}_1
\]
\[
\Rightarrow T \ddot{\hat{i}} + K \delta (-\hat{i}) + N_1 \ddot{\hat{j}} + m g (-\hat{j}) = m_1 \ddot{x}_1 (-\hat{j}) \quad (5)
\]
(5) \( \Rightarrow \)
\[
T \dot{i} + K \delta (-\hat{i}) \dot{i} + N_1 \dot{j} \dot{i} + m g (-\hat{j}) \dot{i} = m_1 \ddot{x}_1 \dot{i} \dot{j}
\]
\[
\Rightarrow T - K \delta = m_1 \ddot{x}_1 \quad (6) \quad (\text{Note: now } \delta = 0.1962m + 0.01m)
\]

for \( m_2 \),
\[
\sum_i F_i = m_2 \ddot{a}_2
\]
\[
\Rightarrow T \ddot{\hat{j}} + m g (-\hat{j}) = m_2 \ddot{x}_2 (-\hat{j}) \quad (7)
\]
(7) \( \Rightarrow \)
\[
T \dot{j} + m g (-\hat{j}) \dot{j} = m_2 \ddot{x}_2 (-\hat{j}) \dot{j}
\]
\[
\Rightarrow T - m_2 g = -m_2 \ddot{x}_2
\]
\[
\Rightarrow T = m_2 g - m_2 \ddot{x}_2
\]

Substitute this to (6):
\[
(m_2 g - m_2 \ddot{x}_2) - K \delta = m_1 \ddot{x}_1
\]
\[
\Rightarrow m_2 g - m_2 \ddot{x}_1 - K \delta = m_1 \ddot{x}_1
\]
\[
\ddot{x}_1 = \ddot{x}_2
\]
\[
\ddot{\ddot{x}}_1 = \ddot{\ddot{x}}_2 = \frac{m_2 g - K \delta}{m_1 + m_2}
\]

Thus,
\[
\ddot{x}_1 = \ddot{x}_2 = \frac{20 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 1000 \text{ Nm} \cdot (0.1962m + 0.01m)}{10 \text{ kg} + 20 \text{ kg}}
\]
\[
= -0.3333 \text{ m/s}^2
\]
\[
\ddot{a}_2 = -0.3333 \text{ m/s}^2 (-\hat{j}) = 0.3333 \text{ m/s}^2 \hat{j}
\]
Solution (b)

Note: $\delta$ is the extension of the spring.

Since the pulley is massless, the tension on both sides of the pulley are equal, i.e., $T_1 = K\delta$.

When the system is at rest

for $m_1$, $\Xi_{i} \vec{F}_i = \vec{0} \Rightarrow T_1 \hat{j} + m_1g(-\hat{j}) + T_2(-\hat{j}) = \vec{0}$

(1) \cdot \hat{j} \Rightarrow T_1 \hat{j} \cdot \hat{j} + m_1g (-\hat{j}) \cdot \hat{j} + T_2 (-\hat{j}) \cdot \hat{j} = \vec{0} \cdot \hat{j}

\Rightarrow T_1 - m_1g - T_2 = 0

\Rightarrow K\delta = T_1 = m_1g + T_2 \quad (2)

for $m_2$, $\Xi_{i} \vec{F}_i = \vec{0} \Rightarrow T_2 \hat{j} + m_2g(-\hat{j}) = \vec{0}$

(3) \cdot \hat{j} \Rightarrow T_2 \hat{j} \cdot \hat{j} + m_2g (-\hat{j}) \cdot \hat{j} = \vec{0} \cdot \hat{j}

\Rightarrow T_2 - m_2g = 0 \Rightarrow T_2 = m_2g \quad (4)

Substitute (4) into (2) \Rightarrow K\delta = m_1g + m_2g

\Rightarrow \delta = \frac{(m_1 + m_2)g}{K} = \frac{(10 \text{ kg} + 20 \text{ kg}) \cdot 9.81 \text{ N/kg}}{1000 \text{ N/m}} = 0.2943 \text{ m}
When $m_2$ is pulled downward 0.01 m further

Let $L$ be the length of the rope btw $m_1$ and $m_2$. Then

$$L = x_2 - x_1$$

$$\Rightarrow \ddot{L}^0 = \ddot{x}_2 - \ddot{x}_1$$

$L$ is constant

$$\Rightarrow \ddot{x}_1 = \ddot{x}_2$$

For $m_1$, $\sum_i F_i = m_1 \ddot{a}_1$,

$$\Rightarrow T_1 \ddot{j} + m_1 g (-\ddot{j}) + T_2 (-\ddot{j}) = m_1 \ddot{x}_1 (-\ddot{j})$$  \hspace{1cm} (5)

$$\Rightarrow T_1 \ddot{j} + m_1 g (-\ddot{j}) \dot{\ddot{j}} + T_2 (-\ddot{j}) \dot{\ddot{j}} = m_1 \ddot{x}_1 (-\ddot{j}) \dot{\ddot{j}}$$

$$\Rightarrow T_1 - m_1 g = -m_1 \ddot{x}_1$$  \hspace{1cm} (7)

For $m_2$, $\sum_i F_i = m_2 \ddot{a}_2$,

$$\Rightarrow T_2 \ddot{j} + m_2 g (-\ddot{j}) = m_2 \ddot{x}_2 (-\ddot{j})$$  \hspace{1cm} (6)

$$\Rightarrow T_2 \ddot{j} + m_2 g (-\ddot{j}) \dot{\ddot{j}} = m_2 \ddot{x}_2 (-\ddot{j}) \dot{\ddot{j}}$$

$$\Rightarrow T_2 - m_2 g = -m_2 \ddot{x}_2$$

$$\Rightarrow T_2 = m_2 g - m_2 \ddot{x}_2 \Rightarrow T_2 = m_2 g - m_2 \ddot{x}_1$$  \hspace{1cm} (8)

Substitute (8) to (7) $\Rightarrow T_1 - m_1 g - (m_2 g - m_2 \ddot{x}_1) = -m_1 \ddot{x}_1$

$$T_1 = k\ddot{x}_1$$

$$\ddot{x}_1 = \frac{-k\ddot{x}_1 - (m_1 + m_2) g}{m_1 + m_2}$$

Thus, $\ddot{x}_1 = \ddot{x}_2 = \frac{-1000 \times 10 \times 0.2943 m + 0.01 m - (10 kg + 20 kg) \times 9.81 N/kg}{10 kg + 20 kg} = -0.3333 \text{ m/s}^2$

$\ddot{a}_2 = -0.3333 \text{ m/s}^2 (-\ddot{j}) = 0.3333 \text{ m/s}^2 \hat{j}$
3.1.34 If a car decelerates at 1.1g, how many car lengths will it travel if it decelerates from 60 mph to zero? (the car length is 15 ft)

Solution This is a 1-D ("straight line") motion problem. Since the acceleration is constant (1.1g) we have

$$V(t_2)^2 - V(t_1)^2 = 2a(X(t_2) - X(t_1))$$

$$V(t_1) = 60 \text{ mph} \approx 88 \text{ ft/s}$$

$$V(t_2) = 0$$

$$\implies X(t_2) - X(t_1) = \frac{V(t_2)^2 - V(t_1)^2}{2a}$$

$$= \frac{(88 \text{ ft/s})^2 - 0^2}{2 \cdot 1.1 \cdot 32.2 \text{ ft/s}^2}$$

$$\approx 109 \text{ ft}$$

# of car length = \frac{109 \text{ ft}}{15 \text{ ft}} \approx 7.3