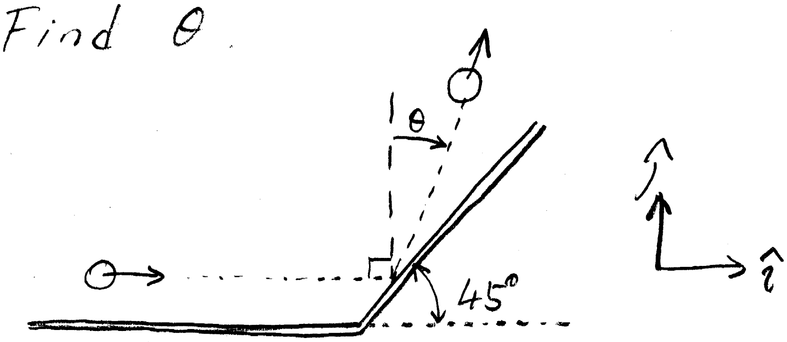
ENGRD/TAM 203 Spring 2006

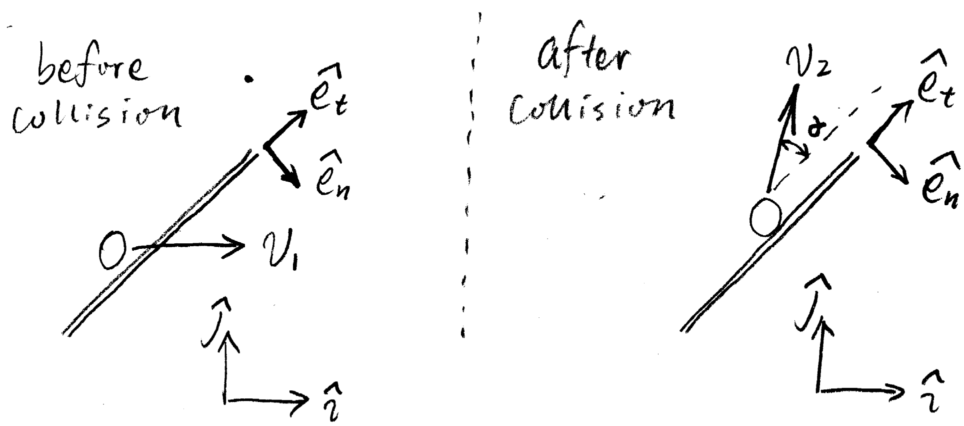
HW12 (Assigned Mar. 2, due on Mar. 9)

Solution by Dennis Yang

3.8.14 A ball is aimed in the $\hat{\jmath}$ direction and bounces off the inclined side wall as shown. $e = 0.86$. Find θ.



Solution



$$e = \frac{0 - \vec{v}_2 \cdot \hat{e}_n}{\vec{v}_1 \cdot \hat{e}_n - 0} = \frac{-\vec{v}_2 \cdot \hat{e}_n}{\vec{v}_1 \cdot \hat{e}_n} = \frac{-\vec{v}_2 \cdot \hat{e}_n}{(v_1 \hat{\imath}) \cdot \hat{e}_n} = \frac{-\vec{v}_2 \cdot \hat{e}_n}{v_1 \cos 45^\circ}$$

$$\Longrightarrow \quad \vec{v}_2 \cdot \hat{e}_n = -e v_1 \cos 45^\circ \quad (1)$$

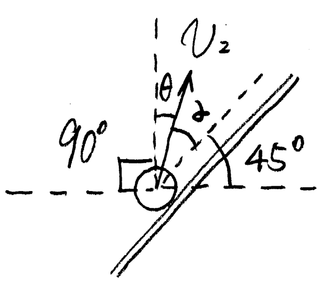
In addition, there is NO change in the velocity component in $\hat{e}_t$ direction, i.e.,

$$\vec{v}_2 \cdot \hat{e}_t = \vec{v}_1 \cdot \hat{e}_t = (v_1 \hat{\imath}) \cdot \hat{e}_t = v_1 \cos 45^\circ \quad (2)$$

Thus, $$\vec{v}_2 = (\vec{v}_2 \cdot \hat{e}_n)\,\hat{e}_n + (\vec{v}_2 \cdot \hat{e}_t)\,\hat{e}_t$$

$$= -e v_1 \cos 45^\circ\, \hat{e}_n + v_1 \cos 45^\circ\, \hat{e}_t \quad (\text{by } (1) \,\&\, (2))$$

$$\Longrightarrow \tan\alpha = \left| \frac{-e v_1 \cos 45^\circ}{v_1 \cos 45^\circ} \right| = e$$



as shown, $90^\circ + \theta + \alpha + 45^\circ = 180^\circ$

$$\Longrightarrow \theta = 45^\circ - \alpha$$

Thus, $$\tan\theta = \tan(45^\circ - \alpha)$$

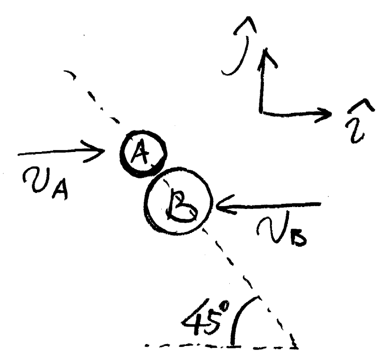
$$= \frac{\tan 45^\circ - \tan\alpha}{1 + \tan 45^\circ \cdot \tan\alpha}$$

$$\boxed{\tan\theta = \frac{1-e}{1+e}}$$

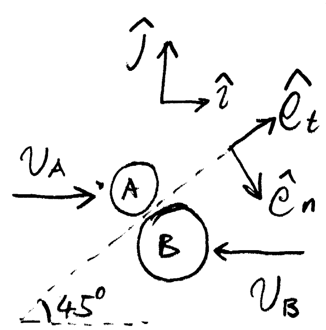
$$\tan\theta = \frac{1-0.86}{1+0.86} \approx 0.075 \Longrightarrow \boxed{\theta \approx 4.3^\circ}$$

$\square$

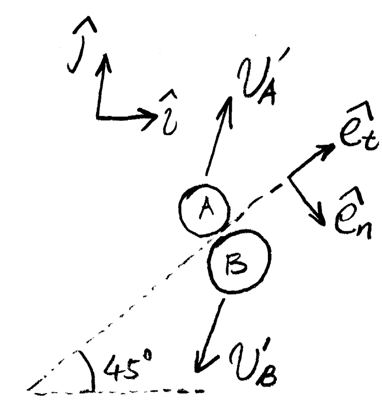
3.8.25 Two particles A & B strike each other as shown. $m_A = 5\ kg$, $m_B = 8\ kg$ just before the collision, $\vec{v}_A = 10\ m/s\ \hat{\imath}$ and $\vec{v}_B = -7\ m/s\ \hat{\imath}$. $e = 0.4$. What are their velocities after the collision?



Solution



before collision



after collision

Conservation of linear momentum :

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}_A' + m_B \vec{v}_B' \qquad (*)$$

$$(*) \cdot \hat{e}_n \Longrightarrow m_A v_A \hat{\imath} \cdot \hat{e}_n + m_B v_B (-\hat{\imath}) \cdot \hat{e}_n = m_A \vec{v}_A' \cdot \hat{e}_n + m_B \vec{v}_B' \cdot \hat{e}_n$$

$$\Longrightarrow \boxed{m_A v_A \cos 45° - m_B v_B \cos 45° = m_A \vec{v}_A' \cdot \hat{e}_n + m_B \vec{v}_B' \cdot \hat{e}_n} \qquad (1)$$

On the other hand,

$$e = \frac{\vec{V}'_B \cdot \hat{e}_n - \vec{V}'_A \cdot \hat{e}_n}{\vec{V}_A \cdot \hat{e}_n - \vec{V}_B \cdot \hat{e}_n} = \frac{\vec{V}'_B \cdot \hat{e}_n - \vec{V}'_A \cdot \hat{e}_n}{V_A \hat{i} \cdot \hat{e}_n - V_B(-\hat{i}) \cdot \hat{e}_n} = \frac{\vec{V}'_B \cdot \hat{e}_n - \vec{V}'_A \cdot \hat{e}_n}{V_A \cos 45° + V_B \cos 45°}$$

$$\Longrightarrow \boxed{e \cos 45° (V_A + V_B) = \vec{V}'_B \cdot \hat{e}_n - \vec{V}'_A \cdot \hat{e}_n} \quad (2)$$

$$(1)/m_A + (2) \Longrightarrow \left(V_A - \frac{m_B}{m_A} V_B\right) \cos 45° + (V_A + V_B)\, e \cos 45°$$

$$= \left(\frac{m_B}{m_A} + 1\right) \vec{V}'_B \cdot \hat{e}_n$$

$$\Longrightarrow \boxed{\vec{V}'_B \cdot \hat{e}_n = \frac{(1+e) \cos 45° V_A + \left(e - \frac{m_B}{m_A}\right) \cos 45° V_B}{\frac{m_B}{m_A} + 1}} \quad (3)$$

$$(1)/m_B - (2) \Longrightarrow \left(\frac{m_A}{m_B} V_A - V_B\right) \cos 45° - (V_A + V_B)\, e \cos 45°$$

$$= \left(\frac{m_A}{m_B} + 1\right) \vec{V}'_A \cdot \hat{e}_n$$

$$\Longrightarrow \boxed{\vec{V}'_A \cdot \hat{e}_n = \frac{\left(\frac{m_A}{m_B} - e\right) V_A \cos 45° - (1+e) V_B \cos 45°}{\frac{m_A}{m_B} + 1}} \quad (4)$$

In addition, there is no change in the velocities' components in $\hat{e}_t$ direction, i.e.,

$$\vec{V}'_A \cdot \hat{e}_t = \vec{V}_A \cdot \hat{e}_t = V_A \hat{i} \cdot \hat{e}_t = V_A \cos 45° \quad (5)$$

and $\vec{v}_B' \cdot \hat{e}_t = \vec{v}_B \cdot \hat{e}_t = v_B(-\hat{\imath})\cdot\hat{e}_t = -v_B\cos 45°$ (6)

Thus, $\vec{v}_A' = (\vec{v}_A' \cdot \hat{e}_n)\,\hat{e}_n + (\vec{v}_A' \cdot \hat{e}_t)\,\hat{e}_t$

$$= \frac{\left(\frac{m_A}{m_B} - e\right) v_A \cos 45° - (1+e)\, v_B \cos 45°}{\frac{m_A}{m_B} + 1}\,\hat{e}_n + v_A \cos 45°\,\hat{e}_t$$

(by (4) & (5))

$$= \frac{\left(\frac{5\text{ kg}}{8\text{ kg}} - 0.4\right)\cdot 10\text{ m/s}\cdot\frac{\sqrt{2}}{2} - (1+0.4)\cdot 7\text{ m/s}\,\frac{\sqrt{2}}{2}}{\frac{5\text{ kg}}{8\text{ kg}} + 1}\,\hat{e}_n + 10\text{ m/s}\,\frac{\sqrt{2}}{2}\,\hat{e}_t$$

$$\boxed{\vec{v}_A' \approx -3.285\text{ m/s}\,\hat{e}_n + 7.071\text{ m/s}\,\hat{e}_t}$$

or

$$\vec{v}_A' = -3.285\text{ m/s}\,\overbrace{\left((\hat{e}_n\cdot\hat{\imath})\hat{\imath} + (\hat{e}_n\cdot\hat{\jmath})\hat{\jmath}\right)}^{\hat{e}_n \text{ in terms of } \hat{\imath},\hat{\jmath}}$$

$$+ 7.071\text{ m/s}\,\underbrace{\left((\hat{e}_t\cdot\hat{\imath})\hat{\imath} + (\hat{e}_t\cdot\hat{\jmath})\hat{\jmath}\right)}_{\hat{e}_t \text{ in terms of } \hat{\imath},\hat{\jmath}}$$

$$= -3.285\text{ m/s}\left(\cos 45°\,\hat{\imath} + \cos 135°\,\hat{\jmath}\right)$$

$$+ 7.071\text{ m/s}\left(\cos 45°\,\hat{\imath} + \cos 45°\,\hat{\jmath}\right)$$

$$\boxed{\vec{v}_A' \approx 2.677\text{ m/s}\,\hat{\imath} + 7.323\text{ m/s}\,\hat{\jmath}}$$

$$\vec{V}_B' = (\vec{V}_B' \cdot \hat{e}_n)\hat{e}_n + (\vec{V}_B' \cdot \hat{e}_t)\hat{e}_t$$

$$= \frac{(1+e)\cos 45^\circ V_A + (e - \frac{m_B}{m_A})\cos 45^\circ V_B}{\frac{m_B}{m_A}+1}\hat{e}_n + (-V_B \cos 45^\circ)\hat{e}_t$$

(by (3) & (6))

$$= \frac{(1+0.4)\frac{\sqrt{2}}{2}10\,m/s + (0.4 - \frac{8kg}{5kg})\frac{\sqrt{2}}{2}\cdot 7\,m/s}{\frac{8kg}{5kg}+1}\hat{e}_n - 7\,m/s \cdot \frac{\sqrt{2}}{2}\hat{e}_t$$

$$\boxed{\vec{V}_B' \approx 1.523\,m/s\,\hat{e}_n - 4.950\,m/s\,\hat{e}_t}$$

or $\vec{V}_B' = 1.523\,m/s\,((\hat{e}_n \cdot \hat{i})\hat{i} + (\hat{e}_n \cdot \hat{j})\hat{j})$

$$- 4.950\,m/s\,((\hat{e}_t \cdot \hat{i})\hat{i} + (\hat{e}_t \cdot \hat{j})\hat{j})$$

$$= 1.523\,m/s\,(\cos 45^\circ\,\hat{i} + \cos 135^\circ\,\hat{j})$$

$$- 0.495\,m/s\,(\cos 45^\circ\,\hat{i} + \cos 45^\circ\,\hat{j})$$

$$\boxed{\vec{V}_B' \approx -2.423\,m/s\,\hat{i} - 4.577\,m/s\,\hat{j}}$$

$\square$

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% This is an M-file for Problem 3.8.25 in HW12
% It is adopted from Prof. Ruina's M-file "ParticleCollision.m"
% available from the course web:
% http://ruina.tam.cornell.edu/Courses/tam203_spring06/Matlab.html

% Note: "1" refers to "particle A" of the original problem,
%   and "2" refers to "particle B" of the original problem.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 2-Particle collisions
% Andy Ruina, March 2, 2006
% See lecture notes from March 2, 2006 for
% basic problem setup.

theta = -pi/4; %-45 degrees = -pi/4 rad, the angle between n and plus x axis
nx = cos(theta); ny = sin(theta);
n = [nx ny]';    %Impulse direction
v1bef = [ 10 0]'; %vel of m1 before collision, in the unit of "m/s"
v2bef = [ -7 0]'; %vel of m2 before collision, in the unit of "m/s"
m1 = 5;  m2 = 8; %values of two masses, in the unit of "kg"
e = 0.4;           % coefficient of restitution

%Write governing equations in form of Az=b
%where z is a list of unknowns representing
%the particle velocities after the collision
%and the magnitude of the impulse.

A = [ m1   0   m2  0    0    %x comp of lin mom bal
      0    m1  0   m2   0    %y comp of lin mom bal
     -nx  -ny  nx  ny   0    %restitution equation
      0    0   m2  0   -nx   %impulse-momentum for m2, x comp
      0    0   0   m2  -ny]  %impulse-momentum for m2, y comp

b = [m1*v1bef(1) + m2*v2bef(1)  %x comp of lin mom bal
     m1*v1bef(2) + m2*v2bef(2)%y comp of lin mom bal
     -e*dot((v2bef-v1bef),n)%restitution equation, note dot product
     m2*v2bef(1)  %impulse-momentum for m2, x comp
     m2*v2bef(2)] %impulse-momentum for m2, y comp


%Matlab command for solving simultaneous equations
%of form Az=b for z, where A and b are known.
z= A\b; % The greatest command in all of Matlab.

%Type out the solution (crudely).
' v1xaft v1yaft v2xaft v2yaft P'
z'
```

```
% This is the output

>> ParticleCollision

A =

    5.0000         0    8.0000         0         0
         0    5.0000         0    8.0000         0
   -0.7071    0.7071    0.7071   -0.7071         0
         0         0    8.0000         0   -0.7071
         0         0         0    8.0000    0.7071


b =

   -6.0000
         0
    4.8083
  -56.0000
         0


ans =

 v1xaft v1yaft v2xaft v2yaft P


ans =

    2.6769    7.3231   -2.4231   -4.5769   51.7820

>>
```

NOTE

From the output, we have

[v1xaft v1yaft]=[2.6769 7.3231]
[v2xaft v2yaft]=[-2.4231 -4.5769]

These agree with our previous results:

$$\vec{v}_A' \approx 2.677\ m/s\ \hat{i} + 7.323\ m/s\ \hat{j}$$

$$\vec{v}_B' \approx -2.423\ m/s\ \hat{i} - 4.577\ m/s\ \hat{j}$$