3.8.2 If a tennis ball is dropped from a height of 100 in and it rebounds more than 53 in and less than 58 in, what is the range of $e$?

**Solution**

Note: The tennis ball collides onto the ground in $(-\hat{j})$ direction. Before the collision, the velocity of the ball is $V_1(-\hat{j})$, and after the collision, the velocity of the ball is $V_2\hat{j}$. The velocity of the ground is $\hat{0}$ before and after the collision.
Thus, in \((-\hat{y})\) direction,
\[
\mathbf{C} = \frac{\mathbf{O} \cdot (-\hat{y}) - (\mathbf{u}_2 \cdot \hat{y}) \cdot (-\hat{y})}{[\mathbf{u}_1 \cdot (-\hat{y})] \cdot (-\hat{y}) - \mathbf{O} \cdot (-\hat{y})} = \frac{\mathbf{u}_2}{\mathbf{u}_1}.
\]

In addition,
\[
\mathbf{u}_1^2 - \mathbf{O}^2 = 2(-\hat{y}) \cdot (-h_1)
\]
\[
\Rightarrow \quad \mathbf{u}_1 = \sqrt{2g \cdot h_1}
\]
\[
\mathbf{O}^2 - \mathbf{u}_2^2 = 2(-\hat{y}) \cdot h_2
\]
\[
\Rightarrow \quad \mathbf{u}_2 = \sqrt{2g \cdot h_2}
\]

Thus,
\[
\mathbf{C} = \frac{\sqrt{2g \cdot h_2}}{\sqrt{2g \cdot h_1}} = \frac{h_2}{h_1}
\]

\[
\Rightarrow \quad \frac{\sqrt{53 \text{ in.}}}{100 \text{ in.}} \leq \mathbf{C} \leq \frac{\sqrt{58 \text{ in.}}}{100 \text{ in.}}
\]

That is,
\[
0.728 \leq \mathbf{C} \leq 0.762
\]
3.8.3 A 5000 lbm SUV plows into the rear of a 2200 lbm Sports car. \( E = 0 \). Just before the collision the SUV was moving at 20 mph and the sports car was stationary. The entire collision takes 0.3 s.

a) What are the two vehicles' speeds immediately after the collision?

Solution

\[
\begin{align*}
\text{before collision} & \\
M_{\text{suv}} & M_{\text{sc}} \\
0 & 0 \\
V_{\text{suv}} & V_{\text{sc}} = 0
\end{align*}
\]

\[
\begin{align*}
\text{after collision} & \\
M_{\text{suv}} & M_{\text{sc}} \\
0 & 0 \\
V'_{\text{suv}} & V'_{\text{sc}}
\end{align*}
\]

\[
E = \frac{V_{\text{sc}} - V_{\text{suv}}}{V_{\text{suv}} - V_{\text{sc}}} = \frac{V'_{\text{sc}} - V'_{\text{suv}}}{V_{\text{suv}}} = 0
\]

\[\Rightarrow V_{\text{sc}} = V'_{\text{suv}}\]

In addition, the total momentum of the system is conserved, i.e.,

\[
M_{\text{suv}} \vec{V}_{\text{suv}} + M_{\text{sc}} \vec{V}_{\text{sc}} = M_{\text{suv}} \vec{V'}_{\text{suv}} + M_{\text{sc}} \vec{V'}_{\text{sc}}
\]

\[\Rightarrow M_{\text{suv}} \vec{V}_{\text{suv}} = M_{\text{suv}} \vec{V'}_{\text{suv}} + M_{\text{sc}} \vec{V'}_{\text{sc}} \quad (\star)\]
\[ (*) \cdot \hat{c} \Rightarrow M_{suv} \dot{V}_{suv} = M_{suv} \dot{V}_{suv} + M_{c} \dot{V}_{c} \]

\[ \Rightarrow M_{suv} \dot{V}_{suv} = M_{suv} \dot{V}_{suv} + M_{c} \dot{V}_{suv} \]

\[ \dot{V}_{suv} = \dot{V}_{suv} \]

\[ \Rightarrow \dot{V}_{suv} = \frac{M_{suv}}{M_{suv} + M_{c}} \dot{V}_{suv} \]

\[ = \frac{5000 \text{ lbm} / (32.2 \text{ ft/s})}{(5000 \text{ lbm} + 2200 \text{ lbm}) / (32.2 \text{ ft/s})} \cdot 20 \text{ mph} \]

\[ = \frac{5000}{7200} \cdot 20 \text{ mph} \]

\[ \approx 13.89 \text{ mph} = 20.37 \text{ ft/s} \]

Thus, \[ \dot{V}_{c} = \dot{V}_{suv} = 20.37 \text{ ft/s} \]

b) What time-average accelerative loads do the two vehicles experience?

Solution

\[ \bar{A}_{suv} = \frac{V_{suv} - V_{suv}}{\Delta t} = \frac{20.37 \text{ ft/s} - 20 \text{ mph} \times 1.467 \text{ ft/s}}{0.3 \text{ s}} \approx 29.9 \text{ ft/s}^2 \]

\[ \bar{A}_{c} = \frac{V_{c} - V_{c}}{\Delta t} = \frac{20.37 \text{ ft/s} - 0 \text{ ft/s}}{0.3 \text{ s}} \approx 67.9 \text{ ft/s}^2 \]