An 80 kg skier travels down a ski jump having a profile given by

\[ y = y_0 - ax + bx^3; \]
\[ y_0 = 50 \text{ m}, \quad a = 1, \quad b = \frac{1}{12500} \text{ m}^{-2} \]

**Goal:** Plot the normal force exerted by the slope on the skier as a function of time from \( x = 0 \) to \( x = 50 \text{ m} \).

**Given:**
1. \( y = 50 - x + \frac{x^2}{12500} \)
2. Gravity, \( g \)
3. All units measured in SI units: \( m, \text{ kg}, \text{ s} \)

**Solution:**

1. FBD of skier idealized as a particle

\[ \vec{N}(\hat{e}_n) \quad \downarrow \quad \vec{g} \]

- Profile of slope, \( y \)

\[ \vec{m} \quad \vec{g} \quad \hat{e}_n \]

\[ \hat{e}_t \]

\[ \hat{e}_s \]
The question asks for N to be found across OSX50.

For purposes of solution, obey the law of conservation of energy; the solution should show that the maximum height the skier can achieve is 50 m and not more. Hence, I have extended the range of X.
2. Apply Newton's Second law

\[ m \ddot{a}_p = \sum F_i \]

\[ m \ddot{a}_p = N (\hat{e}_n) + mg (-\hat{j}) \]

\[ m (\ddot{V}e_{_t} + \frac{v^2}{R_c} \hat{e}_n) = N (\hat{e}_n) + mg (-\hat{j}) \]

\[ \ddot{V} \hat{e}_t \quad (\text{aim: solve for } \ddot{X}(t)) \]

\[ m \ddot{X}_i = N (\hat{e}_n, \hat{i}) \]

\[ = N \sin \theta \]

\[ m \ddot{X}_i = N \sin \theta \quad (1) \]

\[ \hat{e}_n \quad (\text{aim: find an expression of } N) \]

\[ m (\frac{v^2}{R_c}) = N + mg (-\hat{j} \cdot \hat{e}_n) \]

\[ = N - mg \cos \theta \]

\[ N = m (\frac{v^2}{R_c} + g \cos \theta) \quad (2) \]

3. Using Eq. 2 for the slope

\[ y = 50 - x + \frac{1}{12500} x^3 \]

\[ \frac{dy}{dx} = \frac{3x^2}{12500} - 1 \quad (3) \]

\[ \tan \theta = \frac{dy}{dx} \quad (4) \]

\[ \frac{d^2 y}{dx^2} = \frac{6x}{12500} \quad (5) \]
\[ R_C = \sqrt{\frac{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2}}{\left\vert \frac{d^2y}{dx^2} \right\vert}} \]

\[ = \sqrt{\left[ 1 + \tan^2 \theta \right]^{3/2}} \frac{1}{\left\vert \frac{d^2y}{dx^2} \right\vert} \]

\[ R_C = \sqrt{\left( \sec^2 \theta \right)} \frac{1}{\left\vert \frac{d^2y}{dx^2} \right\vert} \]

\[ V^2 = \left( \frac{dy}{dt} \right)^2 + \left( \frac{dx}{dt} \right)^2 \]

\[ = \left( \frac{dy}{dx} \cdot \frac{dx}{dt} \right)^2 + \left( \frac{dx}{dt} \right)^2 \]

\[ = \left( \frac{dx}{dt} \right)^2 \left( 1 + \left( \frac{dy}{dx} \right)^2 \right) \]

\[ V^2 = \dot{x}^2 (1 + \tan^2 \theta) \]

\[ V^2 = \dot{x}^2 \sec^2 \theta \]

4. Making sense of eqs. 1-7

From (1)

\[ M \ddot{x}_i = N \sin \theta \]

Substitute (2), (3), (4), (5), (6), (7) → (1)

\[ M \ddot{x}_i = M \sin \theta \left[ x^2 \sec^2 \theta \left( \frac{d^2y}{dx^2} \right) + g \cos \theta \right] \]

\[ \dot{x}_i = \sin \theta \left[ x^2 \left( \frac{d^2y}{dx^2} \right) \cos \theta + g \cos \theta \right] \]

\[ \ddot{x}_i = \frac{1}{2} \sin(2 \tan^{-1}(\frac{3}{12500} x^2 + 1)) \left[ x^2 \left( \frac{6x}{12500} \right) + g \right] \]
function skier

% t = time taken
% let time frame be from 0 to 5s
% let 'z' be a 2x1 matrix, where z(1) = x, 'x' is position of the skier in
% the 'i' direction
% w.r.t an arbitrary position. z(2) = xdot = v, 'v' is the velocity of the
% skier in the 'i' direction

% initial conditions
x = 0; xdot = 0;

% column vector to hold initial conditions
zNot = [0,0];

% timespan integrate over 9 seconds
tSpan = [0,9];

% ODE
[t,z] = ode45(@f,tSpan,zNot);

figure(1);

% X vs Y plots the profile of the ski slope
X=(0:0.130);
Y = 50 - X + (X.^3)/12500;

% plot the results
plot(X,Y,'b-');
xlabel('X (m)');
ylabel('Y (m)');
title('Profile of the ski slope. Y vs X curve');

figure(2);

% unpack variables from z
velocityX = z(:,2);
positionX = z(:,1);

% units of Kg and m/s^2
m = 80;
g = 9.81;

% N was broken up into nl and n2 to facilitate multiplication of row vectors
nl = m*(positionX.*velocityX.^2*6/12500+g);
n2 = (cos(atan(3/12500*positionX.^2-1)));
N = nl.*n2;

% plot Normal force vs position
plot(t,N,'color','b');
xlabel('Time (S)');
ylabel('Normal Force (N)');
title('Normal Force of Slope on Skier (N) vs Time (s)');

figure(3);

% S vs yP plots the position of the skier on the 'ski slope'
S = z(:,1);
yP = 50 - S + (S.^3)/12500;

% X vs Y plots the profile of the ski slope
X = (0:130);
Y = 50 - X + (X.^3)/12500;

% plot the results
plot(X,Y,'b-',S,yP,'ro');
xlabel('X (m)');
ylabel('Y(m) curve(1)');

text annotations to differentiate curve 1 and curve 2

% superimposing Normal Force vs Position curve on Y vs X curve
h1 = gca;
h2 = axes('Position',get(h1,'Position'));

% unpack variables from z
velocityX = z(:,2);
positionX = z(:,1);

% units of Kg and m/s^2
m = 80;
g = 9.81;

% N was broken up into n1 and n2 to facilitate multiplication of row vectors
n1 = m*(positionX.*velocityX.^2*6/12500+g);
n2 = (cos(atan(3/12500*positionX.^2-1)));
N = n1.*n2;

% plot Normal force vs position
plot(z(:,1),N,'color','k');
\textbf{A\textbackslash xis ('Square')} \\
ylabel('Normal Force (N). curve(2)'); \\
title('Y (m) vs X (m) graph superimposed with Normal Force (N) vs X(m) graph') \\
set(h2,'YAxisLocation','right', 'Color', 'none', 'XTickLabel',[]); \\
set(h2,'XLim',get(h1,'XLim'), 'Layer', 'top'); \\
set(gcf,'PaperPositionMode', 'auto'); \\
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\texttt{function zdot = f(t,z)}

\texttt{\%desired output}
\texttt{x = z(1); v = z(2);}

\texttt{\%input for ODE}
\texttt{xdot = v;}
\texttt{vxDot = 0.5*(sin(2*(atan(-3/12500*x^2 + 1))))*(xdot^2*(6/12500*x)+9.81);} \\

\texttt{\%pack up input}
\texttt{zdot = xdot;}
\texttt{z2dot = vxDot;}

\texttt{\%this is what the function returns}
\texttt{zdot = [z1dot; z2dot];}
6. Express Normal force, \( N \) in terms of \( x \) & \( \dot{x} \)

From (2)

\[
N = m \left( \frac{v^2}{Rc} + g \cos \theta \right)
\]

Substitute (6) & (7)

\[
N = m \cos(\tan^{-1}(\frac{3}{12500}x^2 - 1)) \left[ x^2 \left( \frac{6x}{12500} \right) + \frac{\dot{x}^2}{12500} \right]
\]

7. Since Matlab has solved for \( x \) & \( \dot{x} \)

I am able to plot \( N \) vs \( t \)

- Look at Figure (2) of Matlab code.
8. Superimposing the Normal Force as a function of \( X \) on the curve of \( Y - X \), it is observed that, \( N_{\text{max}} \) occurs at the base of the slope.

At valley:
- Velocity is greatest and \( Rc \) the least
- \( \frac{v^2}{Rc} \) is maximum
- Curve (2) matches up with reality.

9. If Matlab code is integrated for a longer time period, by increasing \( t \) span, it is observed that the skier oscillates in the manner shown in the following graph. (assuming no energy loss - closed system)

10. Indeed, the skier never goes past \( y = 50 \, \text{m} \).
    This verifies the validity of the solution.
Red circles represents the Skier. This graph shows the skier oscillating between y=50, x=0 and...