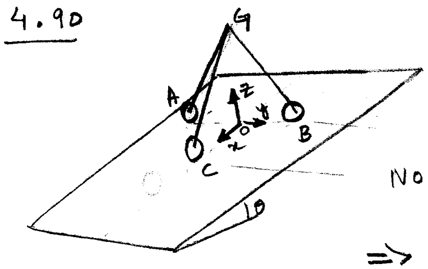
4.90

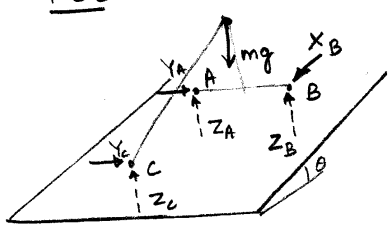


See book for better fig.

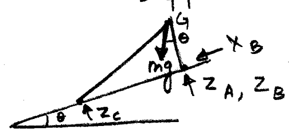
NOTE : Wheels are massless, ideal

$\Rightarrow$ No friction in direction of rolling (eg. C has no friction in x-dirn).

FBD



side view with some forces. (Y_A, Y_c are suppressed)



note mg is not parallel to our z-axis

Q : To find Y_A, Z_A

1. Sum forces in x-dirn.

$\Rightarrow \quad X_B + mg\sin\theta = 0 \Rightarrow \boxed{X_B = -mg\sin\theta}$

2. Sum forces in y-dirn.

$\Rightarrow \quad Y_A + Y_c = 0 \Rightarrow \boxed{Y_A = -Y_c.}$

3. Take moments about axis OG to get rid of all effects of forces except X_B and Y_c (Thus find Y_c since X_B is known).

so first pick a point on OG say O

calculate moments of X_B and Y_c about O

i.e $r_{OB} \times X_B \hat{i} + r_{OC} \times Y_c \hat{j}$

and dot this with unit vector along OG

i.e $\lambda_{OG} = \hat{k}$

$\therefore \quad (r_{OB} \times X_B \hat{i} + r_{OC} \times Y_c \hat{j}) \cdot \hat{k} = 0$

$\Rightarrow \left(\frac{b}{2} \hat{j} \times X_B \hat{i} + L \hat{i} \times Y_c \hat{j} \right) \cdot \hat{k} = 0$

$\Rightarrow \quad Y_c L - \frac{b X_B}{2} = 0 \Rightarrow \boxed{Y_c = \frac{b}{2L} X_B}$

(1.) (2.), (3.) give Y_A

4. Finally to find Z_A we must choose an axis such that most of the forces dont have a moment about it (except ofcourse Z_A)

BC is such an axis

Only Z_A and $mg(-\hat{k})$ have moments about BC.

Again we pick a point on BC say B
calculate moment about B of $\underline{Z}_A$ and $-mg\hat{k}$

i.e $\underline{r}_{BA} \times \underline{Z}_A + \underline{r}_{BG} \times (-mg\hat{k})$

with $\underline{r}_{BA} = -b\hat{j}$ $\quad \underline{r}_{BG} = -\frac{b}{2}\hat{j} + h\hat{k}$

above comes out to be :

$$-bZ_A\hat{i} + mg\left(\frac{b\cos\theta}{2}\hat{i} + h\sin\theta\hat{j} + \frac{b\sin\theta}{2}\hat{k}\right)$$

dot this with unit vector along BC to get moment about BC. (which should equal 0)

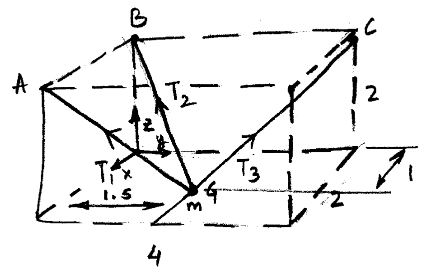
$$\hat{\lambda}_{BC} = \left(-\frac{b}{2}\hat{j} + L\hat{i}\right)\Big/\sqrt{L^2 + b^2/4}$$

this gives us

$$-bL\,Z_A + mg\left[\frac{bL}{2}\cos\theta - \frac{hb}{2}\sin\theta\right] = 0$$

$$\Rightarrow \boxed{Z_A = \frac{mg}{2L}\left[L\cos\theta - h\sin\theta\right]}$$

5.124



See book for better fig.

$m = 2\,kg$

no gravity

acceleration of mass is given

$$\underset{\sim}{a} = (-0.6\hat{i} - 0.2\hat{j} + 2.0\hat{k})\,m/s^2.$$

To find T_1, T_2, T_3 using cross products.

$$\Sigma \underset{\sim}{F} = m\underline{a}$$

gives

$$\underline{T}_1 + \underline{T}_2 + \underline{T}_3 = m\underline{a} \qquad (1)$$

We are supposed to solve this equation using cross products.

Loosely this means performing an operation on (1) which gets rid of all but one variable

So suppose I want to get rid of $\underline{T_2}, \underline{T_3}$ in ONE step. Then this is what I do

$$\left[\underline{r}_{BG} \times \left\{ \underline{T_1} + \underline{T_2} + \underline{T_3} = m\underline{a} \right\}\right] \cdot \hat{\lambda}_{BC} \qquad (2)$$

NO I have not pulled a rabbit out of a hat.

Look at each term in (2) seperately

$\underline{r}_{BG} \times \underline{T_1} \cdot \hat{\lambda}_{BC}$ is simply the moment of T_1 about the axis BC.

Similarly $\underline{r}_{BG} \times \underline{T_2} \cdot \hat{\lambda}_{BC}$ and $\underline{r}_{BG} \times \underline{T_3} \cdot \hat{\lambda}_{BC}$ are the moments of T_2 and T_3 about the axis BC respectively.

But hey – $\underline{T_2}$ and $\underline{T_3}$ intersect BC so they dont have a moment about BC.

Thus these 2 terms drop out in (2).

and we are left with

$$\underline{r}_{BG} \times \underline{T_1} \cdot \hat{\lambda}_{BC} = \underline{r}_{BG} \times (m\underline{a}) \cdot \hat{\lambda}_{BC} \qquad (3)$$

now $\underline{r}_{BG} = \hat{i} + 1.5\hat{j} - 2\hat{k}$

$\hat{\lambda}_{BC} = \hat{j}$ $\quad \underline{a}$ is known

$$\underline{T_1} = T_1 \hat{\lambda}_{GA} = T_1 \frac{(\hat{i} - 1.5\hat{j} + 2k)}{\sqrt{1 + 1.5^2 + 2^2}}$$

Simplifying (3)

$$-\frac{4T_1}{\sqrt{7.25}} = m(-0.8) = -1.6 \qquad (m = 2\text{kg})$$

$$\Rightarrow \boxed{T_1 = 0.4\sqrt{7.25}\ \text{N.}}$$

Next we can play the same game with $\underline{T_2}$ and try and eliminate $\underline{T_1}, \underline{T_3}$ in one shot. . for this we note that T_1 and $\underline{T_3}$ dont exert any moments about the axis AC. So,

$$\underline{r}_{AG} \times \left\{ \underline{T_1} + \underline{T_2} + \underline{T_3} = m\underline{a} \right\} \cdot \hat{\lambda}_{AC}$$

would result in

$$\underline{r}_{AG} \times \underline{T_2} \cdot \hat{\lambda}_{AC} = \underline{r}_{AG} \times (m\underline{a}) \cdot \hat{\lambda}_{AC}$$

since $\underline{r}_{AG} \times \underline{T_1} \cdot \hat{\lambda}_{AC}$ and $\underline{r}_{AG} \times \underline{T_3} \cdot \hat{\lambda}_{AC}$ are both ZERO being moments of $\underline{T_1}, \underline{T_3}$ about AC.

<u>Question</u> What would you do to eliminate $\underline{T_1}, \underline{T_2}$ to get an eq^n only in $\underline{T_3}$?

<u>Note</u> : You can think of $\underline{r}_{BG} \times m\underline{a} \cdot \hat{\lambda}_{BC}$ in (3) as the rate of change of angular momentum about the axis BC. (But this will come later).