"Solutions"

Your Name: ANDY RUINA

T&AM 203 Prelim 2

Tuesday Oct 24, 2000 7:30 — 9:00$^+$ PM

Draft October 16, 2000

3 problems, 100 points, and 90$^+$ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it.

b) Full credit if

- →free body diagrams← are drawn whenever linear or angular momentum balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- * you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot\theta_7$ = 18" instead of, say, "`theta7dot = 18`".

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/40
Problem	2:	/30
Problem	3:	/30
TOTAL:		/100

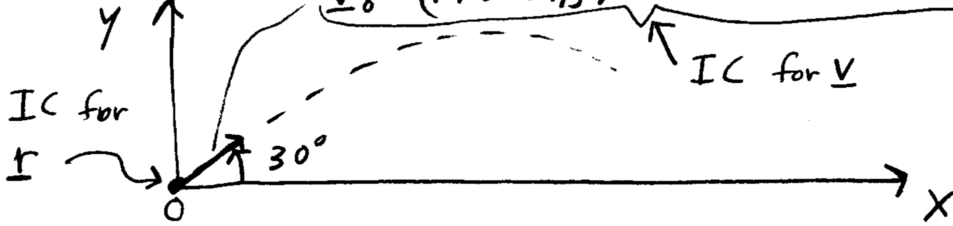
1)(40 pts) **Projectile motion.** Someone in the mideast shot a projectile at someone else. The basic facts:

- Launched from the origin.
- Projectile mass = 1 kg.
- Launch angle 30° above horizontal.
- Launch speed 172 m/s.
- Drag proportional to cv^2 with $c = .01$ kg / m.
- Gravity $g = 10$ m/s.

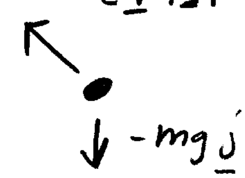
a) (25 pts) Write MATLAB code to find the height at $t = 1$ s. [Hints: sketch of problem, FBD, write drag force in vector form, LMB, 1st order equations, num setup, find height at 1 s].

b) (15 pts) Estimate the height at $t = 1$ s using pencil and paper. An answer in meters is desired. [Hints: Assume g is negligible. Good calculus skills are needed but no involved arithmetic is needed. $1 + 1.72 = 2.72 \approx e$. After you have found a solution check that the force of gravity is a small fraction of the drag force throughout the first second of your solution.]

$$\underbrace{\underline{V}_0 = (172\ m/s)\left[\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}\right]}_{\text{IC for } \underline{V}}$$



FBD:



$$V = |\underline{V}| = (\dot{x}^2 + \dot{y}^2)^{1/2}$$

LMB: $\Sigma \underline{F} = m\underline{a}$

$$\left\{ -c\underline{V}\,V - mg\underline{j} = m(\ddot{x}\underline{i} + \ddot{y}\underline{j}) \right\}$$

$$\{\ \}\cdot\underline{i} \Rightarrow \dot{V}_x = -cV_xV/m, \quad \dot{x} = V_x$$

$$\{\ \}\cdot\underline{j} \Rightarrow \dot{V}_y = -cV_yV/m - g, \quad \dot{y} = V_y$$

ODEs

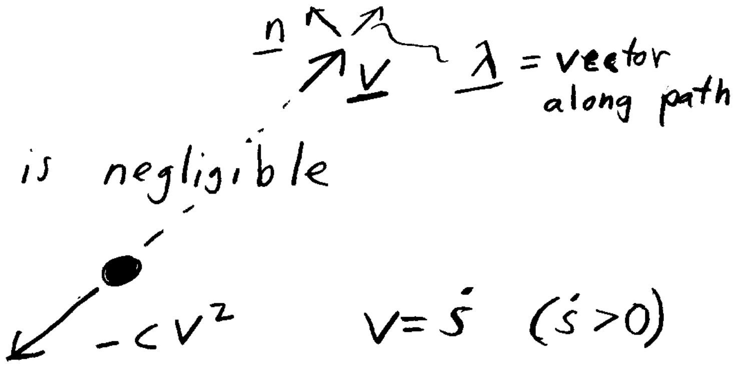
driver

```
x0=0; y0=0;
vx0 = 172*cos(pi*30/180);
vy0 = 172*sin(pi*30/180);
z0=[x0 y0 vx0 vy0]';
tspan=[0 1];
[t z]=ode45('sadam',tspan,z0);
height= z(end,2)
```

(gives ≈ 46.5 m)

in file sadam.m

```
function zdot= sadam(t,z)
c=.01; m=1; g=10;
x=z(1); y=z(2);
vx=z(3); vy=z(4);
xdot=vx; ydot=vy;
v=(vx^2+vy^2)^.5;
vxdot=-c*vx*v/m;
vydot=-c*vx*v/m -g;
zdot=[xdot ydot...
      vxdot vydot]';
```

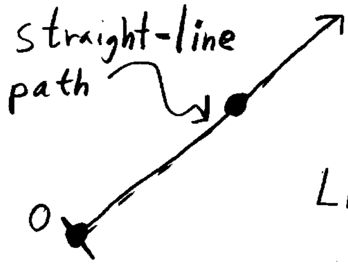


b) Assume gravity is negligible

$\Rightarrow$ FBD:

$V = \dot{s} \quad (\dot{s} > 0)$

No force in $\underline{n}$ dir. ($\perp$ to path) $\Rightarrow$ straight line motion



s = arclength in dir. of motion

LMB: $m\dot{V} = -cV^2$ ①

First solve ①: $\frac{dV}{dt} = \frac{-c}{m}V^2 \Rightarrow \frac{dV}{V^2} = \frac{-c}{m}dt$

$\Rightarrow \quad +V^{-1} = \frac{+c}{m}t + C$

IC: $V(t=0) = V_0 \Rightarrow \quad C = \frac{1}{V_0} \Rightarrow V = \frac{1}{\frac{1}{V_0} + \frac{c}{m}t} = V_0 \frac{1}{1 + \frac{cV_0}{m}t}$

$\Rightarrow \quad \dot{s} = V_0 \frac{1}{1 + \frac{cV_0}{m}t} \Rightarrow ds = V_0 \frac{dt}{1 + \frac{cV_0}{m}t}$

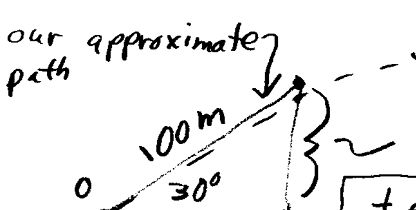
$\Rightarrow \quad s = \frac{V_0 m}{cV_0} \ln\left(1 + \frac{cV_0}{m}t\right) + C$

IC: $s(t=0) = 0 \Rightarrow C = 0 \Rightarrow \boxed{s = \frac{m}{c}\ln\left(1 + \frac{cV_0}{m}t\right)}$

Plug in #s: $s = \frac{1\,\text{kg}}{.01\,\text{kg/m}} \ln\left(1 + \frac{(.01\,\text{kg/m})(172\,\text{m/s})}{1\,\text{kg}}(1\,\text{s})\right)$

$= 100\,\text{m} \cdot \ln(1 + 1.72) = 100\,\text{m} \ln(2.72)$

$\approx 100\,\text{m} \cdot \ln(e) = 100\,\text{m}$



"exact" Matlab ans. is 46.5m

$h \approx 100\,\text{m} \sin(30^\circ) = \boxed{50\,\text{m}}$

at end ($t = 1\,\text{s}$) $|\dot{V}| = cV^2 = .01\left(\frac{172}{2.72}\right)^2 = \left(\frac{1.72}{2.72}\right)^2 172$ (in m/s²)
$|\dot{V}| \gg 10 = g$ (no big error to neglect g)

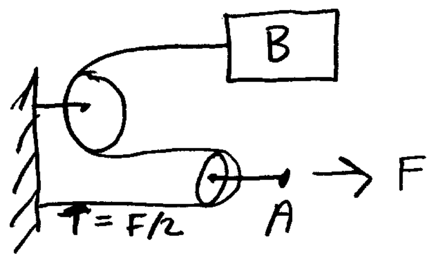
2)(30 pts) **Design a pulley system.** You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\underline{\mathbf{F}} = F\hat{\mathbf{i}}$ pulling it to the right. Mass B has mass m_B. You can connect the point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). You must design the system so that the mass B accelerates to the left with $\frac{F}{2m_B}$ (i.e., $\underline{\mathbf{a}}_B = -\frac{F}{2m_B}\hat{\mathbf{i}}$).

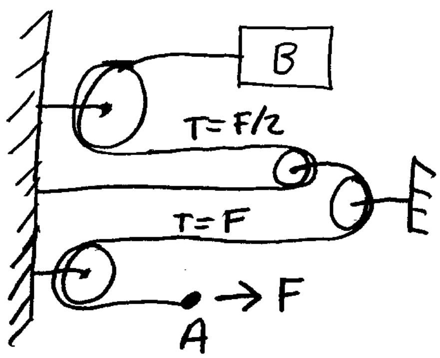
a) (25 pts) Draw the system clearly. Justify your answer with enough words or equations so a reasonable person, say a grader, can tell that you understand your solution.

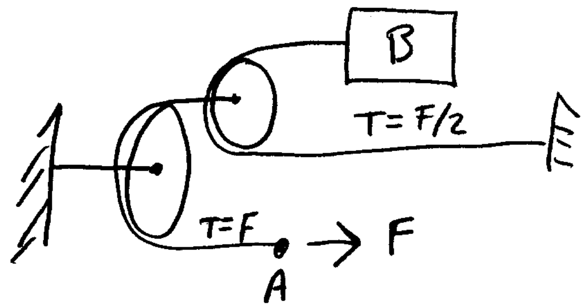
b) (5 pts) Find the acceleration of point A.

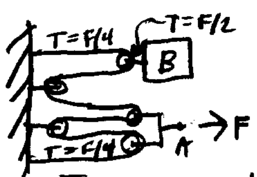
$\rightarrow \underline{\hat{\imath}}$

a) Some solns:









etc...

In all cases m_B pulled to left with tension $= F/2 \Rightarrow a_B = -\frac{F}{2m_B}\underline{\hat{\imath}}$

b) Power balance $\Rightarrow$ $T_B(-V_B) = T_A V_A$ $\quad$ $\boxed{V_B = \text{vel. of B to the right}}$

$$\left\{\frac{F}{2}(-V_B) = F\,V_A\right\}$$

$\frac{d}{dt}(*) \Rightarrow$

$$\frac{F}{2}(-a_B) = F\,a_A$$

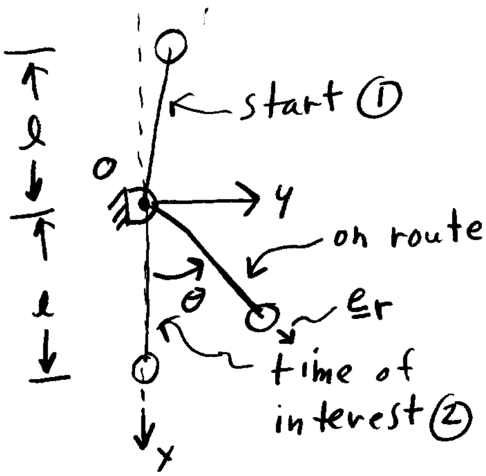
$$a_A = \frac{-a_B}{2} = \frac{-(-F/2m_B)}{2}$$

$$\boxed{a_A = \frac{F}{4m_B}}$$

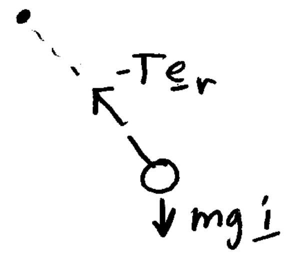
3)(30 pts) **Tension in pendulum.** 2D. A simple pendulum consists of a point mass m connected by a rigid massless rod with length ℓ to a frictionless hinge at O. The only applied force is from gravity. It is released from a vertical orientation with the mass directly *above* the hinge. It is pushed very slightly to the right (with a velocity that you can assume is arbitrarily small) and thus slowly at first falls, then quickly swings through the vertically down orientation and then back up on the left side.

At the instant when the mass passes through the vertically down position (mass directly below hinge) what is the tension in the rod? (i.e., find T in terms of m, ℓ and g).

If you choose a MATLAB solution instead of pencil and paper (not required, just an option) use $m = 3\,\mathrm{kg}$, $\ell = 2\,\mathrm{m}$ and $g = 10\mathrm{m/s}^2$



FBD:



[just like example in lecture]

$$\underbrace{\Delta E_K}_{\text{increase in kin. energy from state ① to state ②}} = \overset{\text{work of gravity force}}{W}$$

(tension does not work because $T\underline{e}_r \perp \underline{v} = \dot{\theta} r \underline{e}_\theta$)

$$\Rightarrow \underbrace{\tfrac{1}{2} m V_2^2}_{\text{K.E. in state 2}} = 2mg\ell$$

$$\boxed{V_2^2/\ell = 4g} \; *$$

Instead of using energy one could do this:

$$\ddot{\theta} + (g/\ell)\sin\theta = 0$$
$$\ddot{\theta}\dot{\theta} + (g/\ell)\dot{\theta}\sin\theta = 0$$
$$\frac{d}{dt}(\dot{\theta}^2/2) + \frac{g}{\ell}\frac{d}{dt}(-\cos\theta) = 0$$
$$\dot{\theta}^2/2 - \frac{g}{\ell}\cos\theta = \text{Const}$$
$$\left.\begin{matrix}\text{at } \theta = \pi \\ \dot{\theta} = 0\end{matrix}\right] \Rightarrow \dot{\theta}^2/2 - (g/\ell)\cos\theta = g/\ell$$

LMB in state ②:

$$\sum \underline{F} = m\underline{a}$$

$$\left\{ -T\underline{i} + mg\underline{i} = m\left[\frac{V_2^2}{\ell}(-\underline{i}) + \ddot{\theta}\ell\underline{j}\right] \right\}$$

$$\{\}\cdot\underline{i} \Rightarrow T = mg + \frac{mV_2^2}{\ell} = mg + 4mg = \boxed{T = 5mg}$$

Tension is 5 times the weight

Alternative Matlab soln. to 3

(Use FBD from before)

AMB/0: $\sum \underline{M}_{/0} = \dot{\underline{H}}_{/0} \Rightarrow \ell \underline{e}_r \times (mg\underline{i}) = \ell \underline{e}_r \times m[\ell\ddot{\theta}\underline{e}_\theta - \dot{\theta}^2 \ell \underline{e}_r]$

$\Rightarrow -\ell mg \sin\theta = \ell^2 m \ddot{\theta}$

$\Rightarrow \ddot{\theta} = -(g/\ell)\sin\theta$

$\Rightarrow$ 2) $\dot{\omega} = -(g/\ell)\sin\theta$ 1) $\dot{\theta} = \omega$ — 2 first order ODEs

```
m= 3; l = 2; g = 10;
theta0 = -pi;
omega0 = -.001;
z0 =[theta0  omega0]';
tspan= [0:.001:10]; % long enough, lots of pts.
[t z]= ode23('pendrhs', tspan, z0);
omegamax = max(z(:,2));
Tmax = m*(g + l*omegamax^2)
```

solve ODEs for long enough time

max $\dot{\omega}$ will be near bottom

calculate tension

pendrhs.m

```
function zdot = pendrhs (t,z)
m=3; l=2; g=10
theta= z(1); omega= z(2);
thetadot= omega;
omegadot= -(g/l)*sin(theta);
zdot=[thetadot  omegadot]';
```

pend ODEs.

[This gives an answer of 149.7, close to 5·3·10]