Your Name: ____________________

T&AM 203 Prelim 1

Tuesday Sept 26, 2000 7:30 — 9:00$^+$ PM

Draft September 26, 2000

3 problems, 100 points, and 90$^+$ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. Two pages of formulas, from the front of the text, and a blank page for tentative scrap work are provided at the back. Ask for extra scrap paper if you need it.

b) Full credit if

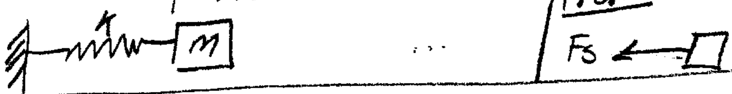
- →free body diagrams← are drawn whenever linear or angular momentum balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- ⊣ reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`".

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/40
Problem	2:	/30
Problem	3:	/30
TOTAL:		**/100**

1) (40 pts)

1a) (15 pts) A mass m is connected to a spring k and launched from its static equilibrium position at a speed of v_0. It then oscillates back and forth repeatedly crossing the equilibrium. How much time passes from release until the mass moves through the equilibrium position for the second time? (Answer in terms of some or all of m, k, and v_0.) [Neglect gravity and friction.] $\rightarrow$ X=0 AT EQUILIBRIUM



$F_s = kx$

$\underline{L}: \; m\ddot{x} = -kx \Rightarrow \ddot{x} + \frac{k}{m}x = 0$ I.C.'s $x(0)=0$, $\dot{x}(0) = V_0$

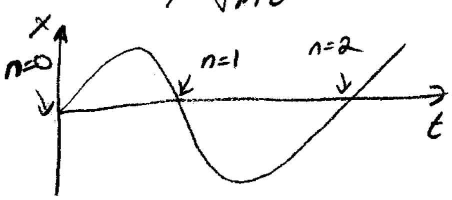
GENERAL SOLUTION $\Rightarrow x(t) = A\cos\left(\sqrt{\frac{k}{m}}t\right) + B\sin\left(\sqrt{\frac{k}{m}}t\right)$

$\dot{x}(t) = -\sqrt{\frac{k}{m}}A\sin\left(\sqrt{\frac{k}{m}}t\right) + \sqrt{\frac{k}{m}}B\cos\left(\sqrt{\frac{k}{m}}t\right)$

$\left.\begin{aligned} x(0) &= A = 0 \Rightarrow A = 0 \\ \dot{x}(0) &= \sqrt{\frac{k}{m}}B = V_0 \Rightarrow B = V_0\sqrt{\frac{m}{k}} \end{aligned}\right\} \Rightarrow x(t) = V_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right)$

NEED TO FIND WHEN $x(t) = 0 \Rightarrow V_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right) = 0 \Rightarrow \sin\left(\sqrt{\frac{k}{m}}t\right) = 0$

$\Rightarrow \sqrt{\frac{k}{m}}t = n\pi$

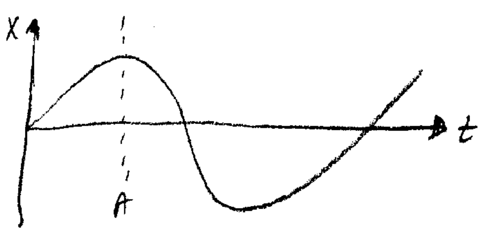


$\sqrt{\frac{k}{m}}t = 2\pi$, $\boxed{t = 2\pi\sqrt{\frac{m}{k}}}$

1b) (10 pts)

For the mass above, how far does the mass move from the launch position before it first reverses its velocity?

FROM 1a), $x(t) = V_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right)$



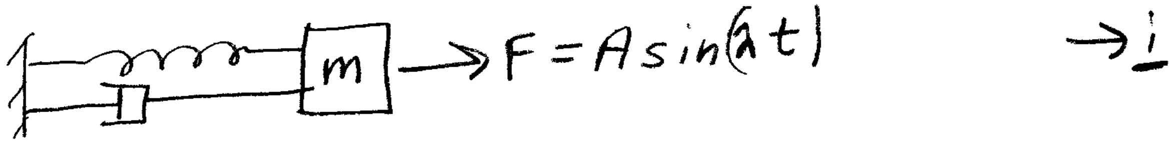
REVERSES VELOCITY AT $t = A$, ITS POSITION CORRESPONDS TO THE <u>AMPLITUDE</u> OF THE MOTION

THE AMPLITUDE OF $x(t)$ IS $V_0\sqrt{m/k}$,

SO $\boxed{d = V_0\sqrt{\frac{m}{k}}}$

1c) (15 pts)

A mass $m = 1\,\text{kg}$ is held in place by a spring $k = 1\,\text{N/m}$ and dashpot $c = 1\,\text{N/(m/s)}$. An oscillating force is applied of $F = A\sin(\lambda t)$, with $A = 1\,\text{N}$ and $\lambda = 1/\text{s}$. After any initial transients have died down, how far does the mass go back and forth (the distance from one extreme to the other)?



→ x

relaxed position of spring

FBD:

LMB: $\{\sum \underline{F} = \underline{\dot{L}}\}$

$$-T_s - T_D + F = m\ddot{x}$$

$\{\}\cdot\underline{i} \Rightarrow$

$$-kx - c\dot{x} + A\sin\lambda t = m\ddot{x}$$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = A\sin\lambda t$$

$$\Rightarrow \ddot{x} + \dot{x} + x = \sin(t) \quad \text{[in consistant m, kg, s units]}$$

Guess steady soln. of form: $x = B\cos t + C\sin t$

why? Because it works

$$\overset{\bullet\bullet}{(B\cos t + C\sin t)} + \overset{\bullet}{(B\cos t + C\sin t)} + (B\cos t + C\sin t) = \sin t$$

$$(-B\cos t - C\sin t) + (-B\sin t + C\cos t) + (B\cos t + C\sin t) = \sin t$$

Collect sine & cosine terms [eg, $\int_0^{2\pi}\{\ \}\cos(t)\,dt$ plucks out cosine]

$$-B + C + B = 0 \quad \text{(cos terms)}$$

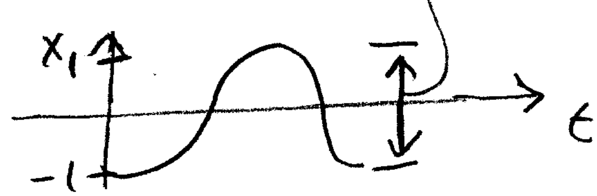
$$\Rightarrow C = 0$$

$$-C - B + C = 1 \quad \text{(sin terms)}$$

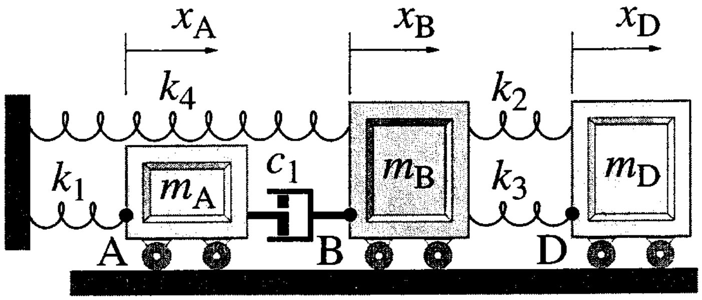
$$\Rightarrow B = -1$$

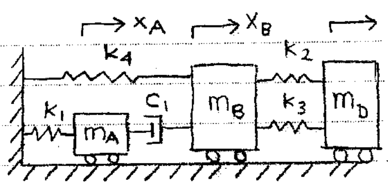
$$\Rightarrow x = -\cos(t)$$

$$\boxed{X_{dist} = 2\,\text{m}}$$

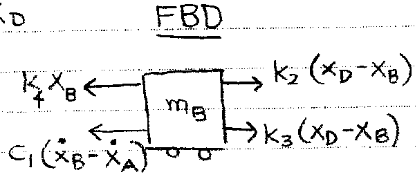


2)(30 pts) A system of three masses, four springs, and one damper are connected as shown. Assume that all the springs are relaxed when $x_A = x_B = x_D = 0$. Given k_1, k_2, k_3, k_4, c_1, m_A, m_B, m_D, x_A, x_B, x_D, $\dot{x}_A$, $\dot{x}_B$, and $\dot{x}_D$, find the acceleration of mass B, $\underline{\mathbf{a}}_B = \ddot{x}_B\,\hat{\mathbf{i}}$.





FBD



LMB: $\{\sum \underline{F} = m_B \underline{a}_B\} \cdot \hat{i}$

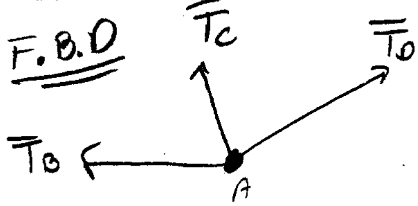
$$\Rightarrow -k_4 x_B - c_1(\dot{x}_B - \dot{x}_A) + (k_2+k_3)(x_D - x_B) = m_B \ddot{x}_B$$

$$\boxed{\begin{aligned} \therefore\ \ddot{x}_B &= \frac{1}{m_B}\big[c_1\dot{x}_A - c_1\dot{x}_B - (k_2+k_3+k_4)x_B \\ &\quad + (k_2+k_3)x_D\big] \\ \underline{a}_B &= \ddot{x}_B\,\hat{i} \end{aligned}}$$

3) (30 pts)

In three-dimensional space with no gravity a particle with $m = 3\,\text{kg}$ at A is pulled by three strings which pass through points B, C, and D respectively. The acceleration is known to be $\underline{\mathbf{a}} = (a\,\hat{\mathbf{i}})\,\text{m/s}^2$ where a is not yet known. The tension in AB is 4 N The position vectors of B, C, and D relative to A are given in the first few lines of code below. Complete the code to find a. The last line should read `a = ...` with a being assigned to the acceleration in the $\hat{\mathbf{i}}$ direction.

F.B.D



$\Sigma = \vec{T}_B + \vec{T}_C + \vec{T}_D = m\vec{a} = 3a\vec{i}$ m/s

$|\vec{T}_B| = T_B = 4N$

DEFINE UNIT VECTORS $\vec{T}_B = T_B\vec{\lambda}_B$, $\vec{T}_C = T_C\vec{\lambda}_C$, $\vec{T}_D = T_D\vec{\lambda}_D$

DEFINE $\vec{b}$ S.T. $m\vec{a} = a\vec{b}$ $\{\vec{b} = (m\hat{i})\}$

THEN $T_B\vec{\lambda}_B + T_C\vec{\lambda}_C + T_D\vec{\lambda}_D = a\vec{b}$ → NOW PUT ALL UNKNOWNS ON LEFT KNOWNS ON RIGHT

$\Rightarrow T_C\vec{\lambda}_C + T_D\vec{\lambda}_D - a\vec{b} = -T_B\vec{\lambda}_B$

$\Downarrow$

$$\left[\vec{\lambda}_C \,\middle|\, \vec{\lambda}_D \,\middle|\, -\vec{b}\right]\begin{bmatrix} T_C \\ T_D \\ a \end{bmatrix} = -T_B\left[\vec{\lambda}_B\right]$$

```
% a MATLAB script file to find 3 tensions
m   = 3;
a   = [ 1 2 3]';

rAB = [ 2 3 5]';
rAC = [-3 4 2]';
rAD = [ 1 1 1]';

uAB = rAB/norm(rAB); % norm gives vector magnitude
% You write the code below (however many lines you need).
% Don't copy any of the numbers above.
% Don't do any arithmetic on the side.
uAC= rAC/norm(rAC);
uAD= rAD/norm(rAD);
b= [m 0 0]';
Tb= 4;
r= -Tb*uAB;
A= [uAC uAD -b];
res = A\r;
a= res(3);
```