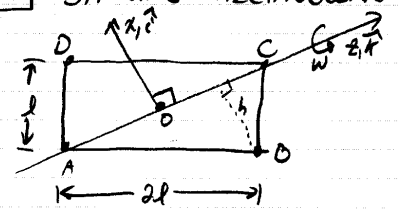
TAM 203 – Fall 2000
HW #10 Solutions

8.17 Spinning Rectangular Plate



a) Find velocity, acceleration of point B.

Say $\underline{r}_{B/O} = x_B\hat{i} + z_B\hat{k}$, $\underline{\omega} = \omega\hat{k}$

$$\underline{v}_{B/O} = \underline{\omega}\times\underline{r}_{B/O} = \omega\hat{k}\times(x_B\hat{i}+z_B\hat{k}) = \omega x_B\hat{j}$$

Need to find x_B:

Area of Rectangle $= 2l^2$

Area of Triangle AOC $= \frac{1}{2}bh$

b = distance from A to C $= \sqrt{4l^2+l^2} = \sqrt{5}l$

Area of Triangle $= \frac{1}{2}$ Area of Rectangle $= l^2$

So $\frac{1}{2}\sqrt{5}lh = l^2 \Rightarrow h = 2l/\sqrt{5}$

$h = -x_B$, so $x_B = -2l/\sqrt{5}$

$$\boxed{\underline{v}_{B/O} = -\frac{2\omega l}{\sqrt{5}}\hat{j} = -2\sqrt{5}\omega l\hat{j}}$$

$$\underline{a}_{B/O} = \underbrace{\dot{\underline{\omega}}}_{\underline{0}}\times\underline{r}_{B/O} + \underline{\omega}\times(\underline{\omega}\times\underline{r}_{B/O}) \rightarrow \text{circular motion}$$

$$= \omega\hat{k}\times\left(-\frac{2\omega l}{\sqrt{5}}\hat{j}\right) = -\frac{2\omega^2 l}{\sqrt{5}}(\hat{k}\times\hat{j})$$

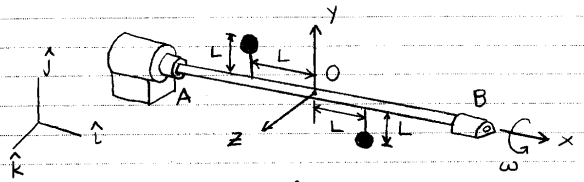
$$\boxed{\underline{a}_{B/O} = \frac{2\omega^2 l}{\sqrt{5}}\hat{i} = 10\sqrt{5}\,l\hat{i}}$$

b) If plate is in yz plane, then can say all $\hat{i}$ components are now $\hat{j}$ components, and $\hat{j}$ components are now $-\hat{i}$ components, meaning

$$\boxed{\underline{v}_{B/O} = \frac{2\omega l}{\sqrt{5}}\hat{i}} \qquad \boxed{\underline{a}_{B/O} = \frac{2\omega^2 l}{\sqrt{5}}\hat{j}}$$

8.22

The two mass system shown has angular velocity $\underline{\omega} = \omega\hat{i}$. At the instant shown, masses are in the xy-plane.



a) What are $\underline{L}$ & $\dot{\underline{L}}$?

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m(-L)+m(L)}{m+m} = 0$$

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m(L)+m(-L)}{m+m} = 0$$

$\Rightarrow$ CM on the axis of rotation

$\Rightarrow \underline{v}_{CM} = \underline{0}$, $\underline{a}_{CM} = \underline{0}$

$$\therefore \boxed{\dot{\underline{L}} = \underline{0},\ \underline{L} = \underline{0}}$$

b) What are $\underline{H}_O$ & $\dot{\underline{H}}_O$?

$$\underline{H}_O = \sum_{i=1}^{2}\underline{r}_{i/O}\times m_i\underline{v}_i \quad \text{where } \underline{v}_i = \underline{\omega}\times\underline{r}_{i/O}$$

$$= \underline{r}_{1/O}\times m(\underline{\omega}\times\underline{r}_{1/O}) + \underline{r}_{2/O}\times m(\underline{\omega}\times\underline{r}_{2/O})$$

$$= L(-\hat{i}+\hat{j})\times m[\omega\hat{i}\times L(-\hat{i}+\hat{j})] + L(\hat{i}-\hat{j})\times m[\omega\hat{i}\times L(\hat{i}-\hat{j})]$$

$$= L(-\hat{i}+\hat{j})\times m\omega L\hat{k} + L(\hat{i}-\hat{j})\times -m\omega L\hat{k}$$

$$= m\omega L^2(\hat{j}+\hat{i}) + m\omega L^2(\hat{j}+\hat{i})$$

$$\therefore \boxed{\underline{H}_O = 2m\omega L^2(\hat{i}+\hat{j})}$$

$$\dot{\underline{H}}_O = \sum_{i=1}^{2}\underline{r}_{i/O}\times m_i\underline{a}_i \quad \text{where } \underline{a}_i = \underline{\omega}\times(\underline{\omega}\times\underline{r}_{i/O}) = \underline{\omega}\times\underline{v}_i$$

$$= \underline{r}_{1/O}\times m(\underline{\omega}\times\underline{v}_1) + \underline{r}_{2/O}\times m(\underline{\omega}\times\underline{v}_2)$$

$$= L(-\hat{i}+\hat{j})\times m[\omega\hat{i}\times m\omega L\hat{k}] + L(\hat{i}-\hat{j})\times m[\omega\hat{i}\times -m\omega L\hat{k}]$$

$$= L(-\hat{i}+\hat{j})\times -m\omega^2 L\hat{j} + L(\hat{i}-\hat{j})\times m\omega^2 L\hat{j}$$

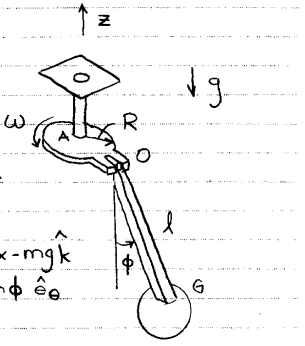
$$= m\omega^2L^2\hat{k} + m\omega^2L^2\hat{k}$$

$$\therefore \boxed{\dot{\underline{H}}_O = 2m\omega^2L^2\hat{k}}$$

8.25

A turntable rotates @ constant rate, ω, about the z-axis. At the edge swings a pendulum which makes a constant angle ϕ w/ the z-axis. The pendulum consists of a pt. mass connected to a massless rod.

a) Find an equation for ϕ.



$\underline{AMB}_O: \sum\underline{M}_O = \dot{\underline{H}}_O$

$$\sum\underline{M}_O = \underline{M} + \underline{r}_{G/O}\times -mg\hat{k}$$

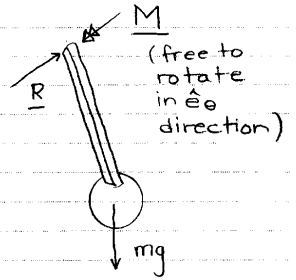
$$= M_r\hat{e}_R + M_z\hat{k} + l(\sin\phi\hat{e}_R - \cos\phi\hat{k})\times -mg\hat{k}$$

$$= M_r\hat{e}_R + M_z\hat{k} + mgl\sin\phi\,\hat{e}_\theta$$

$$\dot{\underline{H}}_O = \underline{r}_{G/O}\times m\underline{a}$$

where $\underline{a}_G = \underline{\omega}\times(\underline{\omega}\times\underline{r}_{G/A})$

FBD



$$\Rightarrow \underline{a}_G = \omega\hat{k}\times[\omega\hat{k}\times(r\hat{e}_R - l\cos\phi\,\hat{k})]$$

$$= \omega\hat{k}\times\omega r\hat{e}_\theta$$

$$= -\omega^2 r\hat{e}_R$$

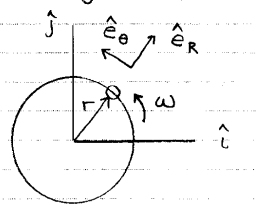
$$\Rightarrow \dot{\underline{H}}_O = l(\sin\phi\hat{e}_R - \cos\phi\hat{k})\times -m\omega^2 r\hat{e}_R$$

$$= ml\omega^2 r\cos\phi\,\hat{e}_\theta$$

$\underline{AMB}_O\cdot\hat{e}_\theta$

$$\Rightarrow \cancel{m}gl\sin\phi = \cancel{m}\omega^2 l r\cos\phi$$

Top view of pt. mass trajectory



$$g\sin\phi = \omega^2 r\cos\phi = \omega^2(R+l\sin\phi)\cos\phi$$

$$\therefore \boxed{\tan\phi = \frac{\omega^2}{g}(R+l\sin\phi)}$$

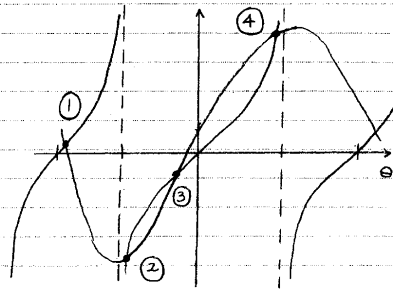
where $\underline{r = R + l\sin\phi}$

b) Find all possible angles for steady circular motion.

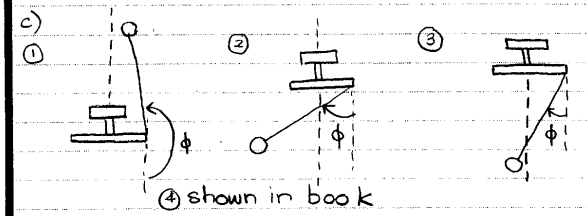
get $\tan\phi = \frac{4}{5}(1+5\sin\phi)$ $\left\{\begin{array}{l}\omega = 2\ \text{rad/s},\ R = 2\text{m}\\ g = 10\text{m/s}^2,\ l = 10\text{m}\end{array}\right.$

(continued)

Plots of $\tan\phi$ and $\frac{4}{5}(1+5\sin\phi)$ show 4 intersections $\Rightarrow$ 4 possible angles for steady motion.

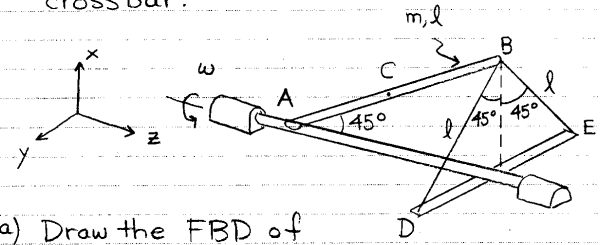


c)



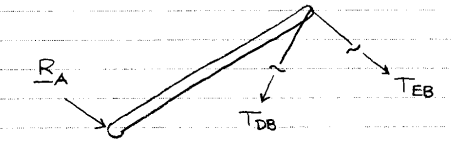
④ shown in book

8.43 A rod is attached to a shaft which spins @ constant rate ω about the z-axis. There is a ball and socket joint @ A and B is held by 2 strings connected to a crossbar.



a) Draw the FBD of the rod.

FBD



b) Find T_{DB}.

i) LMB: $\Sigma\underline{F} = m\underline{a}_c$

where $\underline{a}_c = \underline{\omega}\times(\underline{\omega}\times\underline{r}_{C/A})$

$= \omega\hat{k}\times[\omega\hat{k}\times\frac{\ell}{2\sqrt{2}}(\hat{i}+\hat{k})]$

$= \omega\hat{k}\times\frac{\omega\ell}{2\sqrt{2}}\hat{j}$

(continued)

$\Rightarrow \underline{a}_c = -\frac{\omega^2\ell}{2\sqrt{2}}\hat{i}$

$\Sigma\underline{F} = \underline{R}_A + T_{DB}\hat{\lambda}_{DB} + T_{EB}\hat{\lambda}_{EB}$

where $\hat{\lambda}_{DB} = \frac{\underline{r}_{D/B}}{|\underline{r}_{D/B}|} = \frac{\ell\hat{j}-\ell\hat{i}}{\ell\sqrt{2}} = \frac{1}{\sqrt{2}}(\hat{j}-\hat{i})$

$\hat{\lambda}_{EB} = \frac{\underline{r}_{E/B}}{|\underline{r}_{E/B}|} = \frac{-\ell\hat{j}-\ell\hat{i}}{\ell\sqrt{2}} = \frac{1}{\sqrt{2}}(-\hat{j}-\hat{i})$

- Using LMB, we get 3 equations for 5 unknowns ($\underline{R}_A$, T_{DB}, T_{EB}). So, take AMB about, say, pt. A:

AMB/A: $\Sigma\underline{M}_{/A} = \dot{\underline{H}}_{/A}$

$\Sigma\underline{M}_{/A} = \underline{r}_{B/A}\times T_{DB}\hat{\lambda}_{DB} + \underline{r}_{B/A}\times T_{EB}\hat{\lambda}_{EB}$

$= \frac{\ell}{\sqrt{2}}(\hat{k}+\hat{i})\times\frac{1}{\sqrt{2}}T_{DB}(\hat{j}-\hat{i}) + \frac{\ell}{\sqrt{2}}(\hat{k}+\hat{i})\times\frac{1}{\sqrt{2}}T_{EB}(-\hat{j}-\hat{i})$

$= \frac{1}{2}T_{DB}\ell(-\hat{i}-\hat{j}+\hat{k}) + \frac{1}{2}T_{EB}\ell(\hat{i}-\hat{j}-\hat{k})$

$\dot{\underline{H}}_{/A} = \int \underline{r}_{/A}\times\underline{a}\,dm$

$= \int\left[\frac{1}{\sqrt{2}}s(\hat{k}+\hat{i})\times\underbrace{-\frac{1}{\sqrt{2}}s\omega^2\hat{i}}\right]\frac{m}{\ell}ds$

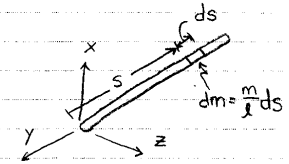
similar to $\underline{a}_c$ above w/ $\frac{\ell}{2}\mapsto s$

$= \left(\int_0^\ell -\frac{1}{2}\omega^2\frac{m}{\ell}s^2\,ds\right)\hat{j}$

$= -\frac{m\omega^2}{\ell}\int_0^\ell s^2\,ds\,\hat{j}$

$= -\frac{m\omega^2}{\ell}(\frac{1}{3}\ell^3)\hat{j}$

$= -\frac{1}{6}m\omega^2\ell^2\hat{j}$



AMB/A · $\hat{i}$ $\Rightarrow -\frac{1}{2}T_{DB}\ell + \frac{1}{2}T_{EB}\ell = 0$ (1)
(& AMB/A · $\hat{k}$)

AMB/A · $\hat{j}$ $\Rightarrow -\frac{1}{2}T_{DB}\ell - \frac{1}{2}T_{DB}\ell = -\frac{1}{6}m\omega^2\ell^2$

(1) $\Rightarrow T_{DB} = T_{EB}$

(2) $\Rightarrow -T_{DB}\ell = -\frac{1}{6}m\omega^2\ell^2$

$\therefore \boxed{T_{DB} = \frac{1}{6}m\omega^2\ell}$

(continued)

ii) AMB/axis AE: $\{\Sigma\underline{M}_{/A} = \dot{\underline{H}}_{/A}\}\cdot\hat{\lambda}_{EA}$

where $\hat{\lambda}_{EA} = \frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

$\Sigma\underline{M}_{/A}\cdot\hat{\lambda}_{EA} = \frac{1}{2}T_{DB}\frac{\ell}{\sqrt{2}}(1+1) + \frac{1}{2}T_{EB}\frac{\ell}{\sqrt{2}}(1-1)$

$= T_{DB}\frac{\ell}{\sqrt{2}}$

$\dot{\underline{H}}_{/A}\cdot\hat{\lambda}_{EA} = \frac{1}{\sqrt{2}}\frac{1}{6}m\omega^2\ell$

$\Rightarrow \frac{\ell}{\sqrt{2}}T_{DB} = \frac{1}{\sqrt{2}}\frac{1}{6}m\omega^2\ell$ · same as above using just one equation

$\therefore \boxed{T_{DB} = \frac{1}{6}m\omega^2\ell}$ ←

c) Find $\underline{R}_A$ by using LMB in b) i):

$\underline{R}_A = -T_{DB}\hat{\lambda}_{DB} - T_{EB}\hat{\lambda}_{EB} + m\underline{a}_c$

$= -\frac{1}{6}m\omega^2\ell\frac{1}{\sqrt{2}}(\hat{j}-\hat{i}) - \underbrace{\frac{1}{6}m\omega^2\ell\frac{1}{\sqrt{2}}}(-\hat{j}-\hat{i})$ from (1)

$-\frac{m}{2\sqrt{2}}\omega^2\ell\hat{i}$

$\Rightarrow \boxed{\underline{R}_A = -\frac{\sqrt{2}}{12}m\omega^2\ell\hat{i}}$