Your Name: ______________________

T&AM 203 Final Exam

Tuesday Dec 12, 2000 3:00 — 5:30 PM

Draft March 20, 2007

5 problems, 100 points, and 150 minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. Ask for extra scrap paper if you need it.

b) Full credit if

- →free body diagrams← are drawn whenever linear or angular momentum balance is used;
- correct vector notation is used, when appropriate;

↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
± all signs and directions are well defined with sketches and/or words;

→| reasonable justification, enough to distinguish an informed answer from a guess, is given;
you clearly state any reasonable assumptions if a problem seems *poorly defined*;

- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);

□ your answers are $\boxed{\text{boxed}}$ in; and

≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ `= 18`" instead of, say, "`theta7dot = 18`". Pick generic (not special) numerical values for constants not defined in the problem statement.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/20
Problem	2:	/20
Problem	3:	/20
Problem	4:	/20
Problem	5:	/20
TOTAL:		**/100**

1)(20 pts) Spring mass.

a) (5 pts) Find the equation of motion, a differential equation, for the variable x in the system above. Your differential equation can contain x, its time derivatives, m, c, k, and ℓ_0 (Please read item (b) on the cover page.)

b) (5 pts) Assume $c = 0$, $x(t = 0) = d$, and $\dot{x}(t = 0) = 0$. What is $\dot{x}$ at time t (answer in terms of some or all of m, k, ℓ_0, d, and t.

c) (5 pts) Assume relatively large c ($c^2 > 4km$), $x(t = 0) = d$, and $\dot{x}(t = 0) = 0$. Find $x(t)$ (or write code that would find $x(t)$).

d) (5 pts) Whether or not you have succeeded at part (c) above, make a clear plot of x *vs* t for the conditions in part (c) above.

(work for problem 1, cont'd.)

2)(20 pts) Car on a ramp. A junior level engineering design course asks students to build a cart (mass $= m_c$) that rolls down a ramp with angle θ. A small weight (mass $m_w \ll m_c$) is placed on top of the cart on a surface tipped with respect to the cart (angle ϕ). Assume the small mass does not slide. Assume massless wheels with frictionless bearings

a) (5 pts) Find the acceleration of the cart. Answer in terms of some or all of $m_c, g, \hat{\mathbf{i}}, \theta$ and $\hat{\mathbf{j}}$. (In accordance with the directions on the front cover you may use other convenient coordinates if you like.).

b) (10 pts) What coefficient of friction μ is required (the smallest that will work) to keep the small mass from sliding as the cart rolls down the slope? Answer in terms of some or all of m_c, m_w, g, θ, and ϕ.

c) (5 pts) What angle ϕ will allow a small mass to ride on the cart with the smallest coefficient of friction? Answer in terms of some or all of m_c, m_w, g, and θ. (You get full credit for a correct answer to this question even if the answer to (b) is incorrect. Conversely, an answer based on incorrect work in part (b) is incorrect.)

(Work for problem 2, cont'd.)

3)(20 pts) A swinging disk. A uniform disk of mass m and radius R is hinged at one end and swings in its plane from a hinge on its circumference.

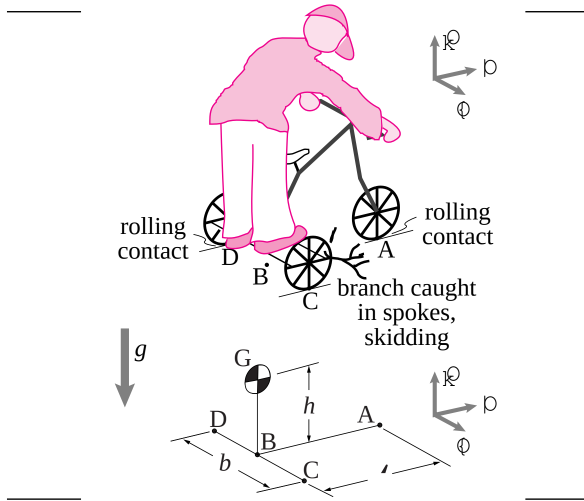
a) (10 pts) Find a differential equation that describes its motion. Describe the motion with an angle θ that is zero when the disk is hanging straight down. (Your equation should have in it some or all of θ, its time derivatives, m, g, and R.

b) (5 pts) What is the period t_p of small oscillations? Answer in terms of some or all of m, R and g.

c) (5 pts) If instead the disk was swinging in the perpendicular direction (with its center moving perpendicular to the plane of the disk) would the frequency of oscillation be higher, lower, or the same? (Correct guess earns one point.)

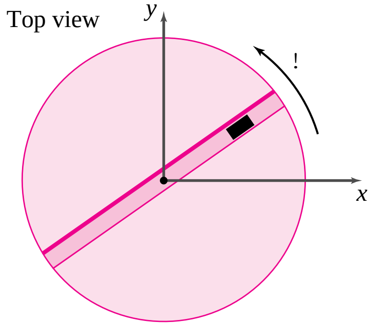
(work for problem 3, cont'd.)

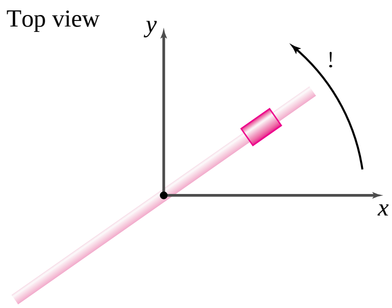
4)(20 pts) Speeding tricycle gets a branch caught in the right rear wheel. A scared-stiff tricyclist riding on level ground gets a branch stuck in the right rear wheel so the wheel skids with friction coefficient μ. Assume that the center of mass of the tricycle-person system is directly above the rear axle. Assume that the left rear wheel and the front wheel have negligible mass, good bearings, and have sufficient friction that they roll in the $\hat{\mathbf{j}}$ direction without slip, thus constraining the overall motion of the tricycle. Dimensions are shown in the lower sketch. *Find the acceleration of the tricycle* (in terms of some or all of ℓ, h, b, m, $[I^{cm}]$, μ, g, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$).
[Hint: check your answer against special cases for which you might guess the answer, such as when $\mu = 0$ or when $h = 0$.]



(Work for problem 4, cont'd.)

5)(20 pts) Mass on a lightly greased slotted turntable or spinning uniform rod. Assume that the rod/turntable in the figure is massless and also free to rotate. Assume that at $t = 0$, the angular velocity of the rod/turntable is $1\,\text{rad/s}$, that the radius of the bead is one meter, and that the radial velocity of the bead, dR/dt, is zero. The bead is free to slide on the rod. Where is the bead at $t = 5\,\text{sec}$?





(Work for problem 5, cont'd.)