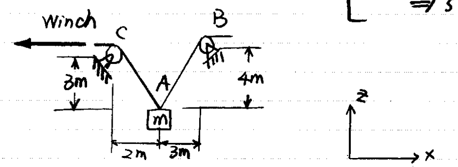
1). 4.23

Given: $m = 3\,kg$, $g = 10\,m/s^2$, $v = 4\,m/s$ (constant) in $\hat{k}$ direction

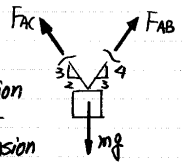
Find: tension in the string AB.

[note: constant v $\Rightarrow$ statics]

(b).



Soln:



Tension in the string is a function of time, because at different time t, the direction of the tension is different.

At this moment,

$$\vec{F}_{AB} = F\hat{\lambda}_{AB} = F_{AB}\frac{(3\hat{i} + 4\hat{k})}{5}$$

$$\vec{F}_{AC} = F\hat{\lambda}_{AC} = F_{AC}\frac{(-2\hat{i} + 3\hat{k})}{\sqrt{13}}$$

$\Sigma\vec{F} = \vec{0}$ $\quad(\vec{a} = \vec{0})$

$\Rightarrow$ $\vec{F}_{AB} + \vec{F}_{AC} - mg\,\hat{k} = \vec{0}$

$\Rightarrow$ $(F_{AB}\frac{3}{5} - F_{AC}\frac{2}{\sqrt{13}})\,\hat{i} + (\frac{4}{5}F_{AB} + \frac{3}{\sqrt{13}}F_{AC} - mg)\,\hat{k} = \vec{0}$ (1)

(1) $\cdot\,\hat{i} = \frac{3}{5}F_{AB} - \frac{2}{\sqrt{13}}F_{AC} = 0$

$\Rightarrow$ $F_{AC} = \frac{3}{10}\sqrt{13}\,F_{AB}$

Alt. method: (1) $\cdot\,\underline{j}\times\underline{\lambda}_{AB}$ $\Rightarrow$ one eqn. for just F_{AC}

(1) $\cdot\,\hat{k} = \frac{4}{5}F_{AB} + \frac{3}{\sqrt{13}}F_{AC} - mg = 0$

plugging F_{AC} into (1) $\cdot\,\hat{k}$

$\Rightarrow$ $\frac{4}{5}F_{AB} + \frac{3}{\sqrt{13}}\frac{3}{10}\sqrt{13}\,F_{AB} = mg$

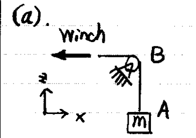
$\Rightarrow$ $F_{AB} = \frac{10}{17}\,mg$

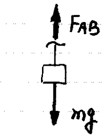
$= \frac{10}{17}\,3\,(10)$ N

$\boxed{= 17.6 \text{ N}}$

Cont. 4.23.

(a).





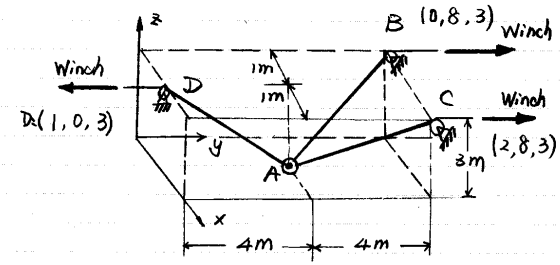
F.B.D

Soln:

$\Sigma\vec{F} = F_{AB}\,\hat{k} - mg\,\hat{k} = \vec{0}$

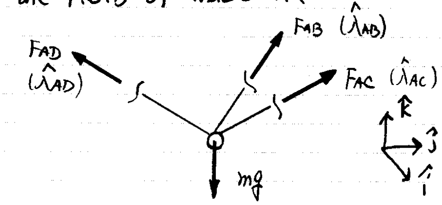
$\Rightarrow$ $F_{AB} = mg = \underline{30 \quad N}$

(c)



Soln:

Draw the F.B.D of mass A:



$\Sigma\vec{F} = \vec{0}$ $\quad(\vec{a} = \vec{0})$

- Finding $\vec{F}_{AB}$, $\vec{F}_{AC}$ and $\vec{F}_{AD}$

$\vec{r}_A = \hat{i} + 4\hat{j}$; $\vec{r}_B = 8\hat{j} + 3\hat{k}$

$\vec{r}_C = 2\hat{i} + 8\hat{j} + 3\hat{k}$; $\vec{r}_D = \hat{i} + 3\hat{k}$

$$\hat{\lambda}_{AB} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = \frac{8\hat{j} + 3\hat{k} - (\hat{i} + 4\hat{j})}{\sqrt{1 + 4^2 + 3^2}}$$

$$= \frac{1}{\sqrt{26}}(-\hat{i} + 4\hat{j} + 3\hat{k})$$

$$\hat{\lambda}_{AC} = \frac{\vec{r}_C - \vec{r}_A}{|\vec{r}_C - \vec{r}_A|} = \frac{\hat{i} + 4\hat{j} + 3\hat{k}}{\sqrt{26}}$$

Cont. 4.23 (c)

$$\hat{\lambda}_{AD} = \frac{\vec{r}_D - \vec{r}_A}{|\vec{r}_D - \vec{r}_A|} = \frac{-4\hat{j} + 3\hat{k}}{5}$$

- Finding F_{AB} by using $\Sigma\vec{F} = \vec{0}$

$\Sigma\vec{F} = 0 = \vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} - mg\,\hat{k}$

$\Rightarrow$

$F_{AB}\frac{1}{\sqrt{26}}(-\hat{i} + 4\hat{j} + 3\hat{k}) + F_{AC}\frac{1}{\sqrt{26}}(\hat{i} + 4\hat{j} + 3\hat{k}) +$

$F_{AD}\frac{1}{5}(-4\hat{j} + 3\hat{k}) - mg(\hat{k}) = \vec{0}$

$\Rightarrow$

$$\boxed{\frac{1}{\sqrt{26}}(F_{AC} - F_{AB})\,\hat{i} + \left(\frac{1}{\sqrt{26}}(4F_{AC} + 4F_{AB}) - \frac{4}{5}F_{AD}\right)\hat{j} + \left(\frac{3}{\sqrt{26}}(F_{AB} + F_{AC}) + \frac{3}{5}F_{AD} - mg\right)\hat{k} = \vec{0}} \quad (2)$$

3 eqs. (2a, 2b, 2c)
3 unknowns

(2) $\cdot\,\hat{i} = 0$

$\Rightarrow$ $F_{AC} = F_{AB}$ (2a)

(2) $\cdot\,\hat{j} = 0$

$\Rightarrow$ $\frac{1}{\sqrt{26}}(4F_{AC} + 4F_{AB}) - \frac{4}{5}F_{AD} = 0$ (2b)

plugging $F_{AC} = F_{AB}$ into (2) $\cdot\,\hat{j} = 0$

$\Rightarrow$ $F_{AD} = \frac{10}{\sqrt{26}}F_{AB}$

(2) $\cdot\,\hat{k} = 0$

$\Rightarrow$ $\frac{3}{\sqrt{26}}(F_{AB} + F_{AC}) + \frac{3}{5}F_{AD} - mg = 0$ (2c)

plugging $F_{AC} = F_{AB}$, $F_{AD} = \frac{10}{\sqrt{26}}F_{AB}$ into (2) $\cdot\,\hat{k}$

$\Rightarrow$ $\frac{6}{\sqrt{26}}F_{AB} + \frac{3}{5}\frac{10}{\sqrt{26}}F_{AB} = mg$

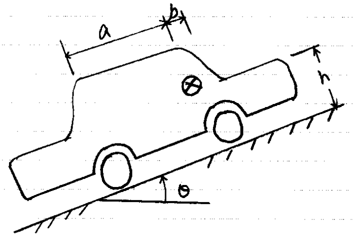
$\Rightarrow$ $F_{AB} = \frac{\sqrt{26}}{12}\,mg$

$= \frac{\sqrt{26}}{12}\,30$ N

$\boxed{= 12.75 \text{ N}}$

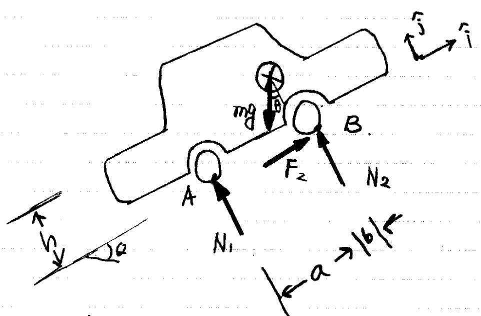
Alt. method: $\{②\}\cdot\,\underline{r}_{AC}\times\underline{r}_{AD}$ $\Rightarrow$ one eqn. for one unknown F_{AB}

2. Find the μ_{min} for a front wheel drive car to drive steadily uphill.



Soln:

Draw the free body diagram of the car



For steady motion:

$$\begin{cases} |F_2| \le \mu N_2 \\ \Sigma \vec{F} = \vec{0} \\ \Sigma \vec{M} = \vec{0} \end{cases}$$

- $\Sigma \vec{F} = \vec{0}$

$\Rightarrow$

$$N_1 \hat{j} + N_2 \hat{j} + mg(-\sin\theta\,\hat{i} - \cos\theta\,\hat{j}) + F_2 \hat{i} = \vec{0} \quad (*)$$

$(*) \cdot \hat{i} = 0$

$\Rightarrow$

$$mg\sin\theta = F_2$$

- $\Sigma \vec{M}_A = \vec{0}$

$\Rightarrow$

$$(a+b)\hat{i} \times N_2 \hat{j} + (a\hat{i} + h\hat{j}) \times (-mg\cos\theta\,\hat{j} - mg\sin\theta\,\hat{i}) = \vec{0}$$

$\Rightarrow$

$$(a+b)N_2 - amg\cos\theta + hmg\sin\theta = 0$$

$\Rightarrow$

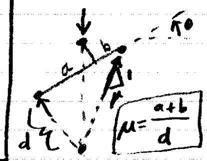
$$N_2 = mg\,\frac{(a\cos\theta - h\sin\theta)}{a+b}$$

- $F_2 < \mu N_2 \Rightarrow \mu > \frac{F_2}{N_2}$

$$\Rightarrow \mu > \frac{mg\sin\theta\,(a+b)}{mg(a\cos\theta - h\sin\theta)}$$

$$\Rightarrow \boxed{\mu_{min} = \frac{(a+b)\sin\theta}{a\cos\theta - h\sin\theta}}$$

2) Alt. method: $F_2 = \mu N$ at highest slope. 3-force body

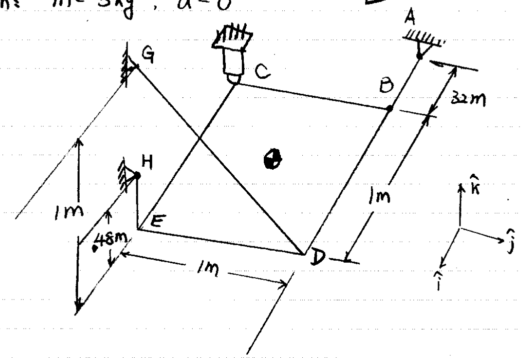


$$\boxed{\mu = \frac{a+b}{d}}$$

$$d = a\cot\theta - h$$

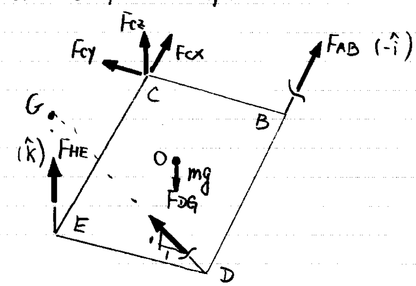
3). 4.25

Given: $m = 5kg$, $\vec{a} = \vec{0}$



Find:

a). Draw a FBD of the shelf



b). Yes. Take the moment about axis CD, then all the reactions but F_{HE} will have zero moment about it.

$$\Sigma M_{\vec{CD}} = F_{HE}\,d = 0 \Rightarrow F_{HE} = 0$$

if there's more, it will be given in d)

c). Write down the equation for force equilibrium

$$\Sigma \vec{F} = \vec{0}$$

Cont. 4.25 (c)

$$\Sigma \vec{F} = -mg\hat{k} + \vec{F}_{AB} + \vec{F}_{DG} + \vec{F}_{HE} + \vec{F}_C$$

$$\begin{cases} \vec{F}_{AB} = F_{AB}(-\hat{i}) \\ \vec{F}_{DG} = F_{DG}(\vec{r}_G - \vec{r}_D)/|\vec{r}_G - \vec{r}_D| \\ \quad = F_{DG}\,\frac{-\hat{j}+\hat{k}}{\sqrt{2}} \\ \quad = \frac{1}{\sqrt{2}} F_{DG}(-\hat{j}+\hat{k}) \\ \vec{F}_{HE} = F_{HE}\,\hat{k} \end{cases}$$

$$\Rightarrow \boxed{\Sigma \vec{F} = (-F_{AB} - F_{cx})\hat{i} + (-\tfrac{1}{\sqrt{2}}F_{DG} - F_{cy})\hat{j} + (\tfrac{1}{\sqrt{2}}F_{DG} + F_{HE} + F_{cz} - mg)\hat{k}}$$

d). Write down $\Sigma \vec{M}_{cm} = \vec{0}$

let cm = O point

$\Rightarrow$

$$\Sigma \vec{M}_{cm} = \Sigma \vec{M}_O = \vec{r}_{OB} \times \vec{F}_{AB} + \vec{r}_{OD} \times \vec{F}_{DG} + \vec{r}_{OE} \times \vec{F}_{EH} + \vec{r}_{OC} \times \vec{F}_C$$

$$\begin{cases} \vec{r}_{OB} = -0.5\hat{i} + 0.5\hat{j} \\ \vec{r}_{OC} = -0.5\hat{i} - 0.5\hat{j} \\ \vec{r}_{OD} = 0.5\hat{i} + 0.5\hat{j} \\ \vec{r}_{OE} = 0.5\hat{i} - 0.5\hat{j} \end{cases}$$

$\Rightarrow$

$$\Sigma \vec{M}_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.5 & 0.5 & 0 \\ -F_{AB} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0.5 & 0.5 & 0 \\ 0 & -\frac{1}{\sqrt{2}}F_{DG} & \frac{1}{\sqrt{2}}F_{DG} \end{vmatrix} + \begin{vmatrix} i & \hat{j} & \hat{k} \\ 0.5 & -0.5 & 0 \\ 0 & 0 & F_{EH} \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.5 & -0.5 & 0 \\ -F_{cx} & -F_{cy} & F_{cz} \end{vmatrix}$$

$$= (0.5F_{AB}\hat{k}) + \left(0.5\tfrac{1}{\sqrt{2}}F_{DG}\hat{i} - 0.5\tfrac{1}{\sqrt{2}}F_{DG}\hat{j} - \tfrac{0.5}{\sqrt{2}}F_{DG}\hat{k}\right) + (-0.5F_{EH}\hat{i} - 0.5F_{EH}\hat{j}) + (-0.5F_{cz}\hat{i} + 0.5F_{cz}\hat{j} + (0.5F_{cy} - 0.5F_{cx})\hat{k})$$

$$= \boxed{0.5(\tfrac{1}{\sqrt{2}}F_{DG} - F_{EH} - F_{cz})\hat{i} + 0.5(-\tfrac{1}{\sqrt{2}}F_{DG} - F_{EH} + F_{cz})\hat{j} + 0.5(F_{AB} - \tfrac{1}{\sqrt{2}}F_{DG} + F_{cy} - F_{cx})\hat{k}}$$

e). turn c and d into 6 eqns in 6 unknows.

- Three eqns from $\Sigma\vec{F}=\vec{0}$

$$\Sigma F_x = \hat{i}\cdot\Sigma\vec{F}=0$$

$$\Rightarrow \quad F_{AB} = -F_{cx} \qquad (1)$$

$$\Sigma F_y = \hat{j}\cdot\Sigma\vec{F}=0$$

$$\Rightarrow \quad \tfrac{1}{\sqrt{2}} F_{DG} = -F_{cy} \qquad (2)$$

$$\Sigma F_z = \hat{k}\cdot\Sigma\vec{F}=0$$

$$\Rightarrow \quad \tfrac{1}{\sqrt{2}} F_{DG} + F_{HE} + F_{cz} - mg = 0 \qquad (3)$$

- Three eqns from $\Sigma\vec{M}_0=\vec{0}$

$$\Sigma M_x = \hat{i}\cdot\Sigma\vec{M}_0=0$$

$$\Rightarrow \quad \tfrac{1}{\sqrt{2}} F_{DG} - F_{EH} - F_{cz} = 0 \qquad (4)$$

$$\Sigma M_y = \hat{j}\cdot\Sigma\vec{M}_0=0$$

$$\Rightarrow \quad \tfrac{1}{\sqrt{2}} F_{DG} + F_{EH} - F_{cz} = 0 \qquad (5)$$

$$\Sigma M_z = \hat{k}\cdot\Sigma\vec{M}_0=0$$

$$\Rightarrow \quad F_{AB} - \tfrac{1}{\sqrt{2}} F_{DG} + F_{cy} - F_{cx} = 0 \qquad (6)$$

- The six eqns are:

$$\boxed{\begin{cases} F_{AB} = -F_{cx} & (1) \\ F_{DG} = -\sqrt{2}F_{cy} & (2) \\ \tfrac{1}{\sqrt{2}} F_{DG} + F_{HE} + F_{cz} - mg = 0 & (3) \\ \tfrac{1}{\sqrt{2}} F_{DG} - F_{EH} - F_{cz} = 0 & (4) \\ \tfrac{1}{\sqrt{2}} F_{DG} + F_{EH} - F_{cz} = 0 & (5) \\ F_{AB} - \tfrac{1}{\sqrt{2}} F_{DG} + F_{cy} - F_{cx} = 0 & (6) \end{cases}}$$

f). Solve this eqn by hand

$(3)+(4) = \sqrt{2}F_{DG} - mg = 0 \Rightarrow F_{DG} = \frac{1}{\sqrt{2}}mg$

$(4)-(5) = -2F_{EH} = 0 \quad \Rightarrow \quad F_{EH} = 0$

put F_{EH}, F_{DG} back into (4)

$\Rightarrow \frac{mg}{2} = F_{cz}$

Cont. f)

put $F_{DG} = \frac{1}{\sqrt{2}}mg$ into (2)

$\Rightarrow$

$F_{cy} = -\frac{1}{\sqrt{2}}F_{DG} = -\frac{1}{2}mg$

put $F_{AB} = -F_{cx}$ (1) and F_{cy}, F_{DG} into (6)

$\Rightarrow$

$-\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}mg - \frac{1}{2}mg - 2F_{cx} = 0$

$\Rightarrow$

$F_{cx} = -\frac{1}{2}mg$

$F_{AB} = \frac{1}{2}mg$

$\Rightarrow$ soln is

$$\boxed{\begin{aligned} F_{AB} &= \tfrac{1}{2}mg = 25 \text{ N} \\ F_{HE} &= 0 \\ F_{DG} &= \tfrac{1}{\sqrt{2}}mg = 35.4 \text{ N} \\ F_{cx} &= -\tfrac{1}{2}mg = -25 \text{ N} \\ F_{cy} &= -\tfrac{1}{2}mg = -25 \text{ N} \\ F_{cz} &= \tfrac{1}{2}mg = 25 \text{ N} \end{aligned}}$$

---

g). Solve T_{EH} by one eqn

Already did in b):

$\Sigma M_{CD} = dF_{EH} = 0$ d: distance betwee F_{EH} and line CD

$\Rightarrow F_{EH} = 0$

---

h). How many of the reactions can be found from one eqn. w/out knowing others?

Soln:

- Finding F_{EH}:

$\Sigma M_{CD} = 0 \Rightarrow F_{EH} = 0$

- Finding F_{DG}:

$\Sigma M_{CE} = 0$

$\Rightarrow$

$1\,(F_{DG}\frac{1}{\sqrt{2}}) - mg\,(0.5) = 0$

$\Rightarrow$

$F_{DG} = \frac{1}{\sqrt{2}}mg$

- Finding F_{AB}

$\Sigma M_{CG} = 0$

$\Sigma \hat{\lambda}_{CG}\cdot\vec{r}_i\times\vec{F}_i = 0$

$\Rightarrow$

$\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})\cdot\left[(\hat{j}\times F_{AB}(-\hat{i})) + (0.5\hat{i}+0.5\hat{j})\times mg(-k)\right]$

$= \frac{1}{\sqrt{2}}(\hat{i}+\hat{k})\cdot\left[F_{AB}\hat{k} + 0.5\,mg\,\hat{j} - 0.5\,mg\,\hat{i}\right]$

$= -\frac{0.5}{\sqrt{2}}mg + F_{AB}/\sqrt{2}$

$= 0$

$\Rightarrow F_{AB} = \frac{1}{2}mg$

- Finding F_{cz}

$\Sigma M_{ED} = 0$ ($\vec{F}_{AB}$, $\vec{F}_{DG}$, $\vec{F}_{HE}$, F_{cx}, F_{cy} don't have moment on this line)

$\Rightarrow$

$F_{cz}\,(1) - mg\,(0.5) = 0$

$\Rightarrow$

$F_{cz} = \frac{1}{2}mg$

- Find F_{cy}

$\Sigma M_{GG'} = 0$ GG' is on i direction

then

$\vec{F}_{cz}$, $\vec{F}_{HE}$, $\vec{F}_{DG}$ will pass through GG' and

$\vec{F}_{AB}$, $\vec{F}_{cx}$ are parallel to GG'

$\Rightarrow$ only $-mg\,(\hat{k})$ and F_{cy} will be left.

$F_{cy}\,(1\,m) - mg\,(0.5)\,m = 0$

$\Rightarrow$

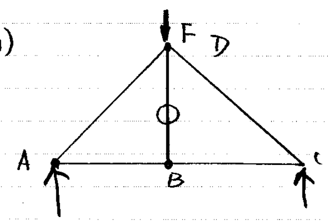
$F_{cy} = \frac{1}{2}mg$

All of these five forces should be the same as those of in e).

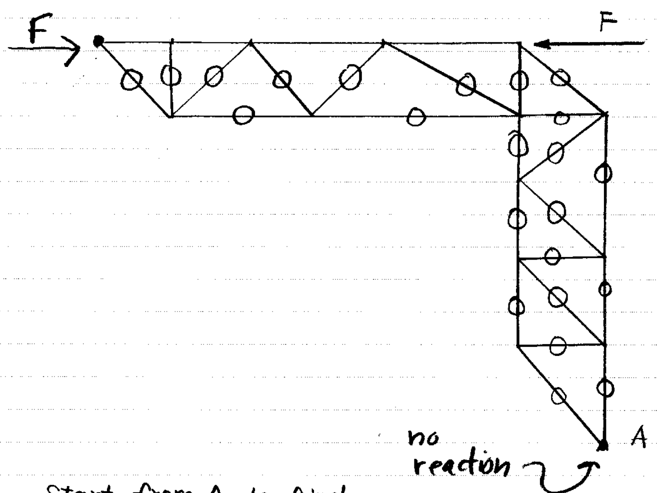
(No-one has yet found 1 eq. for the one unknown F_{cx})

4). Find the zero-force members

(a)

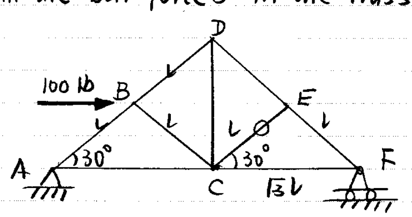


(b)



Start from A to find.

5) Find all the bar forces in the truss



Soln:

We can directly tell $F_{CE} = 0$

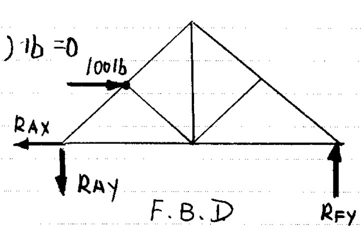
- Finding reactions

$\Sigma M_A = 0$

$\Rightarrow R_F \cdot 2\sqrt{3}L - 100(\frac{1}{2}L)\ \text{lb} = 0$

$\Rightarrow$

$R_F = \frac{25}{\sqrt{3}}$ lb



$\Sigma F_x = 0$

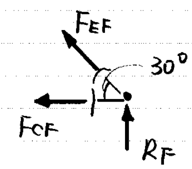
$\Rightarrow R_{Ax} = 100$ lb

$\Sigma F_y = 0$

$\Rightarrow R_{Ay} = R_{Fy} = \frac{25}{\sqrt{3}}$ lb

- Solving by method of joints

For point F:



$\Sigma F_y = 0$

$\Rightarrow F_{EF} \sin 30° + R_{Fy} = 0$

$\Rightarrow$

$F_{EF} = -2R_{Fy}$

$= -\frac{50}{\sqrt{3}}$ lb

$\Sigma F_x = 0$

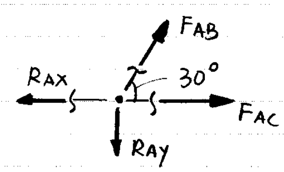
$\Rightarrow F_{CF} + F_{EF}\cos 30° = 0$

$\Rightarrow F_{CF} = -\frac{\sqrt{3}}{2} F_{EF}$

$= -\frac{\sqrt{3}}{2}(-\frac{50}{\sqrt{3}})$ lb

$= 25$ lb

For point A:



$\Sigma F_y = 0$

$\Rightarrow F_{AB} \sin 30° - R_{Ay} = 0$

$\Rightarrow$

$F_{AB} = \frac{25}{\sqrt{3}} \cdot 2$ lb $= \frac{50}{\sqrt{3}}$ lb

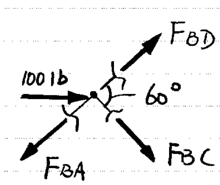
$\Sigma F_x = 0$

$\Rightarrow F_{AC} + F_{AB}\cos 30° - R_{Ax} = 0$

$\Rightarrow F_{AC} = 100 - \frac{50}{\sqrt{3}} \frac{\sqrt{3}}{2}$ lb

$= 75$ lb

For point B:



$\Sigma F_x = 0$

$\Rightarrow$

$F_{BC}\cos 30° + F_{BD}\cos 30°$

$+ 100 - F_{BA}\cos 30° = 0$

$\Rightarrow$

$F_{BC} + F_{BD} = F_{BA} - 100\frac{2}{\sqrt{3}}$ lb

$= \frac{50}{\sqrt{3}} - \frac{200}{\sqrt{3}}$ lb

$= \frac{-150}{\sqrt{3}}$ lb (1)

Cont. 5)

$\Sigma F_y = 0$

$\Rightarrow F_{BD}\sin 30° - F_{BC}\sin 30° - F_{BA}\sin 30° = 0$

$\Rightarrow$

$F_{BD} - F_{BC} = \frac{50}{\sqrt{3}}$ lb (2)

Solving for (1) and (2):

$F_{BD} = -\frac{50}{\sqrt{3}}$ lb

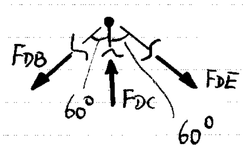
$F_{BC} = -\frac{100}{\sqrt{3}}$ lb

For point D:

$\Sigma F_y = 0$

$\Rightarrow F_{DC} = 2F_{DE}\sin 30°$

$= F_{DE}$

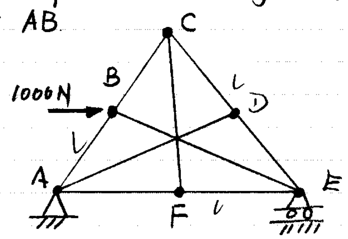


$F_{DE} = F_{EF} = -\frac{50}{\sqrt{3}}$ lb

$\Rightarrow F_{DC} = \frac{50}{\sqrt{3}}$ lb

F_{AC} =	75	lb
F_{AB} =	$50/\sqrt{3}$	lb
F_{CB} =	$-100/\sqrt{3}$	lb
F_{CF} =	25	lb
F_{CE} =	0	
F_{DC} =	$50/\sqrt{3}$	lb
F_{DE} =	$-50/\sqrt{3}$	lb
F_{EF} =	$-50/\sqrt{3}$	lb
F_{DB} =	$-50/\sqrt{3}$	lb

6). For the equivalent triangle, find the tension in bar AB.



Soln:

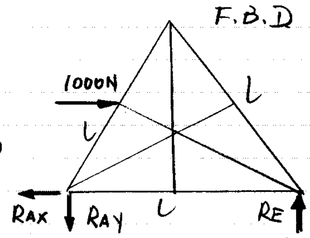
· Finding reactions

$\Sigma M_A = 0$

$\Rightarrow R_E(L) - 1000\,N\left(\frac{\sqrt{3}}{4}L\right) = 0$

$\Rightarrow R_E = 250\sqrt{3}$ N

F.B.D



$\Sigma F_y = 0$

$\Rightarrow R_{Ay} = R_E = 250\sqrt{3}$ N

$\Sigma F_x = 0$

$\Rightarrow R_{Ax} = 1000$ N

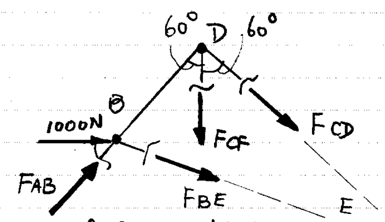
- Finding zero force member

  bar CF and bar AD

- Method of section:

  cut AB, BE, CF and CD

F.B.D.



$F_{CF} = 0$ because it's zero force member

$\Sigma M_E = 0$

$\Rightarrow$

$F_{AB}|\vec{r}_{BE}| + 1000\,N\left(\frac{1}{2}|\vec{r}_{CF}|\right) = 0$

$|\vec{r}_{BE}| = \frac{\sqrt{3}}{2}L$

$|\vec{r}_{CF}| = \frac{\sqrt{3}}{4}L = \frac{1}{2}|\vec{r}_{BE}|$

$\Rightarrow$

$\boxed{F_{AB} = -500 \text{ N}}$

We actually don't need to solve for reactions