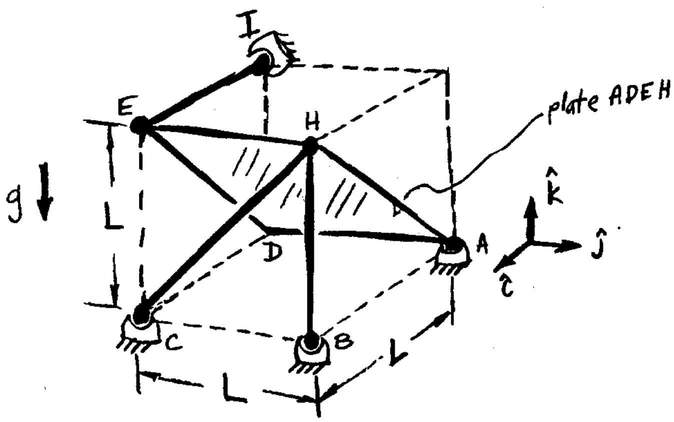
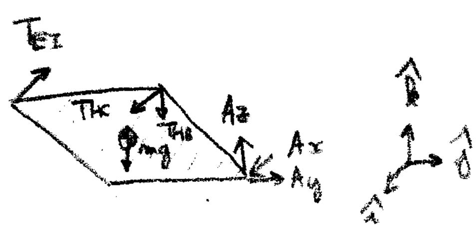
1) (20pt) Uniform plate ADEH with mass m is connected to the ground with a ball and socket joint at A. It is also held by three massless bars (IE, CH and BH) that have ball and socket joints at each end, one end at the rigid ground (at I, C and B) and one end on the plate (at E and H).

In terms of some or all of m, g, and L find the reaction at A (the force of the ground on the plate) and the three bar tensions T_{IE}, T_{CH} and T_{BH}.



FBD of the plate:
(10 pts)



$\Sigma M_{/AH} = 0 \Rightarrow \frac{\sqrt{2}L}{4} mg - \frac{\sqrt{2}}{2} L T_{EI} = 0$

$\Rightarrow T_{EI} = Mg/2$

$\Sigma M_{/HB} = 0 \Rightarrow T_{EI} L - A_y L = 0 \Rightarrow A_y = \frac{mg}{2}$

$\Sigma M_{/A\hat{k}} = 0 \Rightarrow T_{EI} L - \frac{\sqrt{2}}{2} L T_{HC} = 0 \Rightarrow T_{HC} = \frac{\sqrt{2}}{2} mg$

$\Sigma M_{/AC} = 0 \Rightarrow \frac{\sqrt{2}}{4} T_{HB} L + \frac{\sqrt{2}}{2} T_{EI} L + \frac{\sqrt{2}}{4} T_{HB} L = 0$

$\Rightarrow T_{HB} = -T_{EI} \Rightarrow T_{HB} = -\frac{mg}{2}$

(4 pts for the method)

$(\Sigma F) \cdot \hat{\imath} \Rightarrow A_x = T_{EI}$

$(\Sigma F) \cdot \hat{j} \Rightarrow A_y = \frac{\sqrt{2}}{2} T_{HC}$

$(\Sigma F) \cdot \hat{k} \Rightarrow A_z = mg + T_{HB} + \frac{\sqrt{2}}{2} T_{HC}$

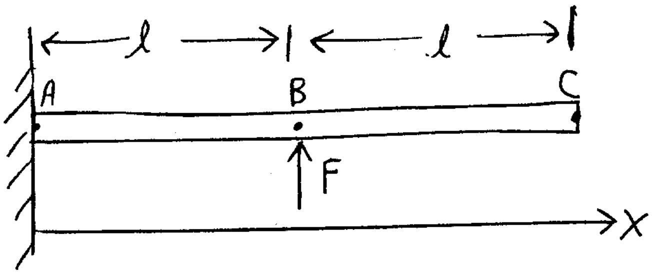
(1 pt each)

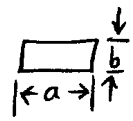
$\Rightarrow$ In summary

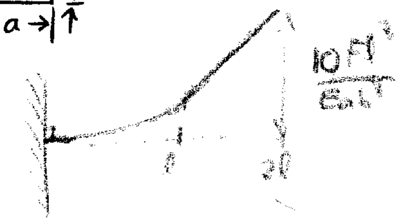
$$\boxed{\begin{cases} A_x = mg/2 \\ A_y = mg/2 \\ A_z = mg \end{cases} \qquad \begin{cases} T_{IE} = mg/2 \\ T_{CH} = \frac{\sqrt{2}}{2} mg \\ T_{BH} = -mg/2 \end{cases}}$$

2) (20 pt) The uniform solid linear elastic beam ABC has a rectangular cross section with sides a and b. It has moduli E, G, and ν. A vertical load F is applied halfway along its total length of 2ℓ. Answer the questions below in terms of some or all of a, b, E, G, ν, F and ℓ.

a) What is the maximum tensile stress and where does it occur?
b) What is the deflection at C?

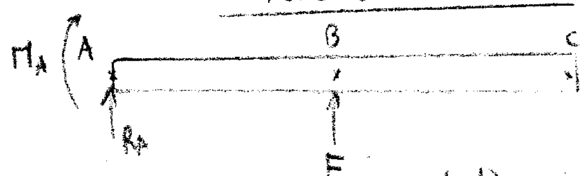






Partial credit if you
i) Find the reactions at A.
ii) Find $V(x)$ and draw a shear force diagram.
iii) Find $M(x)$ and draw a bending moment diagram.
iv) Find $u'(x)$.
v) Find $u(x)$ and neatly draw the deflected shape.
Though you don't need to do these if have got the correct answers to (a) and (b) above by reasoning that does not depend on them.

Reactions at A.

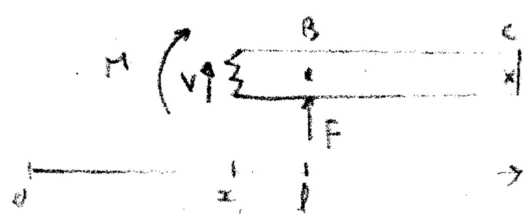


(2 pts)

$\Sigma F = 0 \Rightarrow R_A = -F$ (1 pt)
$\Sigma M_{/A} = 0 \Rightarrow M_A = F\ell$ (1 pt)

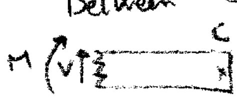
Shear Force & bending moment

→ Between A & B:

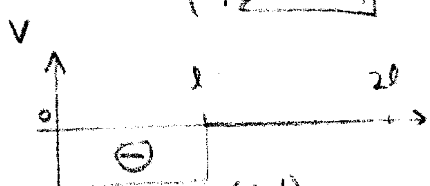


$V = -F$ (1 pt)
$M = F(\ell - x)$ (1 pt)

Between B & C



→ $V = 0$ (1 pt)
$M = 0$ (1 pt)



(1 pt)

(1 pt)

$u'(x) \Rightarrow$ Between AB:

$$u_1'' = \frac{M}{EI} = \frac{F}{EI}(l-x) \Rightarrow u_1' = \frac{F}{EI}\left(lx - \frac{x^2}{2}\right) + C_1$$

Now $u_1'(0) = 0 \Rightarrow C_1 = 0$

$$u_1' = \frac{F}{EI}\left(lx - \frac{x^2}{2}\right) \qquad \underline{(1 \text{ pt})}$$

Between BC:

$$u_2'' = 0 \Rightarrow u_2' = C_2$$

Now $u_1'(l) = u_2'(l) \Rightarrow C_2 = \frac{F}{EI}\frac{l^2}{2}$

$$u_2' = \frac{Fl^2}{2EI} \qquad \underline{(1 \text{ pt})}$$

$u(x) \Rightarrow$ Between AB:

$$u_1 = \frac{F}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_3$$

Now $u_1(0) = 0 \Rightarrow C_3 = 0$

$$u_1 = \frac{F}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \qquad \underline{(2 \text{ pts})}$$

Between BC:

$$u_2 = \frac{Fl^2}{2EI}x + C_4$$

Now $u_1(l) = u_2(l) \Rightarrow \frac{F}{EI}\left(\frac{l^3}{2} - \frac{l^3}{6}\right) - \frac{Fl^3}{2EI} = C_4$

$$C_4 = -\frac{Fl^3}{EI6}$$

$$\Rightarrow u_2 = \frac{Fl^2}{2EI}\left(x - \frac{l}{3}\right) \qquad \underline{(2 \text{ pts})}$$

(a) $\sigma = \frac{My}{I} \Rightarrow$ Maximum tensile stress at A $= +\frac{Fl \times b/2}{I} = \frac{Flb/2}{ab^3/12}$

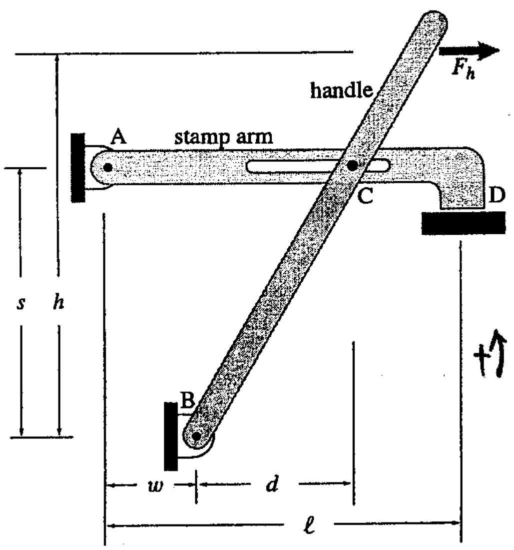
$$\boxed{\sigma_m = \frac{6Fl}{ab^2}} \qquad \underline{(2 \text{ pts})}$$

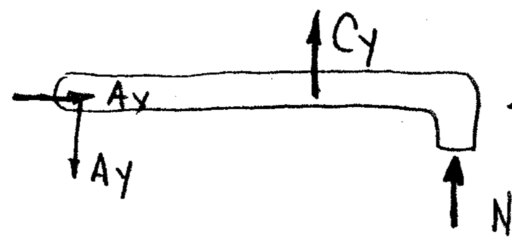
(b) $u_2(2l) = \frac{Fl^2}{2EI}\left(2l - \frac{l}{3}\right) = \frac{Fl^2}{2EI}\left(\frac{6l}{3} - \frac{l}{3}\right) = \frac{Fl^2}{2EI}\frac{5l}{3} = \frac{5Fl^3}{6EI}$

$$= \frac{5Fl^3}{6Eab^3/12} = \frac{10Fl^3}{Eab^3} \qquad \boxed{u_2(2l) = \frac{10Fl^3}{Eab^3}} \qquad \underline{(2 \text{ pts})}$$

3) (20 pt) Stamp machine: Pulling on the handle causes the stamp arm to press down at D. Neglect gravity and assume that the hinges at A and B, as well as the roller at C, are frictionless.

Find the force N that the stamp machine causes on the support at D in terms of some or all of F_h, w, d, ℓ, h, and s.

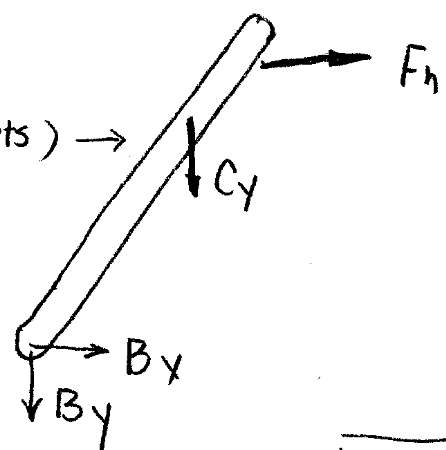




← (5 pts)

$\circlearrowleft + \sum M_A = 0:$ $\quad N l + C_y(w+d) = 0$ (4 pts)

$\Rightarrow \quad N = -\dfrac{C_y(w+d)}{l}$



(5 pts) →

$\circlearrowleft + \sum M_B = 0:$

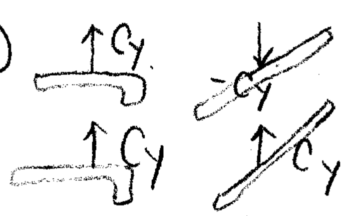
$F_h(h) + C_y(d) = 0$ (4 pts)

$\Rightarrow \quad C_y = -\dfrac{F_h(h)}{d}$ (2)

$\Rightarrow \boxed{N = \dfrac{h F_h (w+d)}{d l}}$ (2 pts)

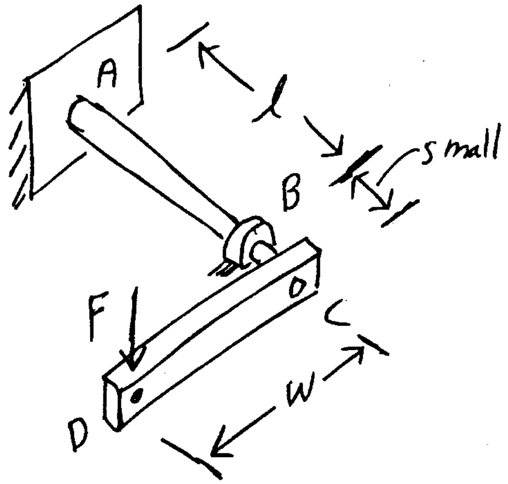
Note : ① if Forget B_x, B_y, A_x, A_y (−3 pts)

② C_x appeared and w/o saying $C_x = 0$ (−3 pts)

③  (−3 pts)

4) (20 pt) **Torsion bar.** The uniform solid round linear elastic bar AB has radius r and length ℓ. It has elastic moduli E, G, and ν. Right next to the bearing at B the rod is attached to a very stiff (modeled as rigid) bar CD with length w. You can neglect the distance BC. You can assume that the total rotation of the shaft end at C is small ($\ll 1$).

In terms of some or all of ℓ, r, w, E, G, ν, and F find the deflection of the point D.

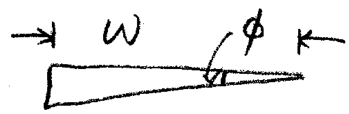


$$\phi_B = \frac{T L}{J G} \qquad (\phi_A = 0)$$

$$= \frac{F w L}{\frac{1}{2}\pi r^4 G}$$

$$= \frac{2 F w L}{\pi r^4 G} \qquad \text{(total of 15 pts)}$$

$$\delta_D = \phi w \qquad \text{(5 pts)}$$



$$= \frac{2 F w^2 L}{\pi r^4 G}$$

or $\delta_D = w \tan\phi$

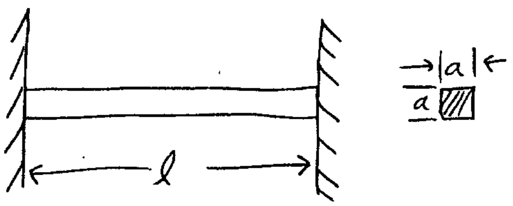
$= w \sin\phi$

Note: $J = \frac{1}{2}\pi r^4$ (2 pts)

$T = F w$ (2 pts)

5) (20 pt) A uniform linear elastic bar with a solid square cross section and length ℓ is welded between two rigid walls at room temperature in a stress-free state. The bar has thermal expansion coefficient α.

a) What is the compression in the bar when the temperature is raised ΔT? Answer in terms of some or all of $E, G, \nu, \alpha, \Delta T, \ell$, and a.
b) What temperature rise ΔT will cause the bar to buckle? Answer in terms of some or all of E, G, ν, α, ℓ, and a.



a) $\delta = \partial \Delta t L + \frac{PL}{EA} = 0$

$\Rightarrow P = -EA\partial\Delta t$

$= -Ea^2\partial\Delta t$

Note: $\delta = 0$ (4 pts)

$\delta_P = \frac{PL}{EA}$ (3 pts)

$\delta_{\Delta T} = \partial \Delta T L$ (2 pts)

$P = -Ea^2\partial\Delta t$ (2 pts)

b) $P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} = P$

$L_{eff} = \frac{1}{2}L$

$I = \frac{1}{12}a^4$

$\Rightarrow \Delta T = -\frac{\pi^2 a^2}{3L^2\partial}$

$P_{cr} = P$ (5 pts)

$L_{eff} = \frac{1}{2}L$ (2 pts)

$I = \frac{1}{12}a^4$ (2 pts)