Three identical steel balls, each of mass \( m \), are placed in the cylindrical ring which rests on a horizontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force \( P \) exerted by the ring on each of the three lower balls.

**Solution:**

Top View:

Angles \( \angle OAB \) and \( \angle OBA \) are 90°.

**FBD for the ball A (typical).** Note, assume no contact between ball A, B, C.

Geometry:

- \( C \) is the centre of the upper ball.
- Length of \( AB = R_B = 2R = R_C \).

From \( \triangle AOB, \ \overline{AO} = \frac{R}{\cos 30°} = \frac{2R}{\sqrt{3}} \).

From \( \triangle AOC, \ \overline{OC} = \sqrt{R_C^2 - OA^2} = R \sqrt{2^2 - \left(\frac{2}{\sqrt{3}}\right)^2} \).

Equilibrium implies:

\[ 2F_z = 0 \]

So:

\[ 2F_z = 3R \cos \theta - mg = 0 \]

or:

\[ 3R \left( \frac{\overline{OC}}{R^2} \right) - mg = 0 \]

[\( \cos \theta = \frac{\overline{OC}}{R} \) from \( \triangle OAC \)]

or:

\[ R = \frac{mg}{xy} - \frac{2R}{\sqrt{3}} \]

Equilibrium for ball (see FBD), ball A:

\[ 2F_x = 0 \Rightarrow P - R \sin \theta = 0 \]

or:

\[ P = R \sin \theta = \frac{mg}{\sqrt{x}} = \frac{2R}{\sqrt{3}} \]

So:

\[ P = \frac{mg}{2R} \]

(continued)
The semicylindrical shell of mass \( m \) and radius \( r \) is rolled through an angle \( \theta \) by the force \( P \) which remains tangent to its periphery at A as shown. If \( P \) is slowly increased, plot the tilt angle \( \theta \) as a function of \( P \) up to the point of slipping. Determine the tilt angle \( \theta_{\text{max}} \) and the corresponding value \( P_{\text{max}} \) for which slipping occurs. The coefficient of static friction is 0.30.

Ans. \( \theta_{\text{max}} = 59.9^\circ \)

\( P_{\text{max}} = 0.296mg \)

\( \mu_s = \mu_c = \mu \)

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Solution:

FBD for the Shell:

\[ \sum F_x = 0 \Rightarrow P \sin \theta - F = 0 \]  
\[ \sum F_y = 0 \Rightarrow N - mg + P \cos \theta = 0 \]  
\[ \sum M_B = 0 \Rightarrow mgd \sin \theta - P(BC) = 0 \]  
\[ BC = BD + BC = r + y \sin \theta \]

At the start of slipping \( F = \mu N \)  

\( N = \frac{P}{\sin \theta} \)

Relation between \( P \) & \( \theta \) can be derived from (3):

\[ P = \frac{2 \sin \theta \cdot mg}{\pi(1 + \sin \theta)} \]  

Substitute \( F = \mu N \) in (1), we get:

\[ P \sin \theta = \mu N \Rightarrow N = \frac{P \sin \theta}{\mu} \]  

Substitute \( N = \frac{P \sin \theta}{\mu} \) in (2) to find relation between \( P \) and \( mg \):

\[ \frac{P \sin \theta - mg + P \cos \theta = 0 \Rightarrow P = \frac{mg}{\left(\frac{\mu \cos \theta}{\sin \theta} + \sin \theta\right)} \]  

Comparing (5) & (7):

\[ \frac{\mu}{\mu_{\text{max}} + \sin \theta} = \frac{2 \sin \theta}{\pi(1 + \sin \theta)} \]  

Solve (8) numerically to obtain \( \theta_{\text{max}} = 59.9^\circ \)

\( \mu_{\text{max}} = 0.245 \) mg using (5).

(theta=0:0.1:(59.9*pi/180); 
\( P=2.*\sin(theta)/(pi.*((1+\sin(theta)))); 
\) plot(P,theta) 
\( xlabel('P/mg') 
\) ylabel('theta in radians') 
grid on)