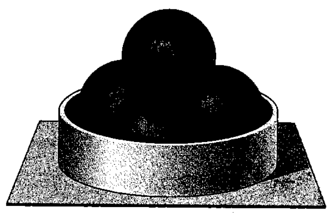
TAM 202, HW 8 Solutions, Prepared by Peeyush Bhargava

3/108 Three identical steel balls, each of mass m, are placed in the cylindrical ring which rests on a horizontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force P exerted by the ring on each of the three lower balls.



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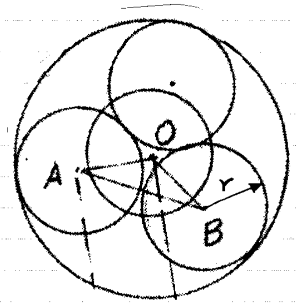
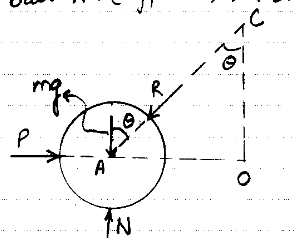
Solution:

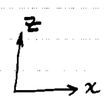
Top View →

Angles OAB and ABO are 30°.



FBD for the ball A (typical). Note, assume no contact between ball A, B, D

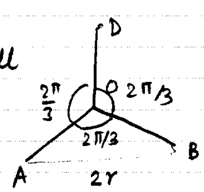




Geometry

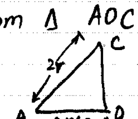
C is the centre of the upper ball

Length of AB = $\bar{AB}$ = $2r$ = $\bar{AC}$



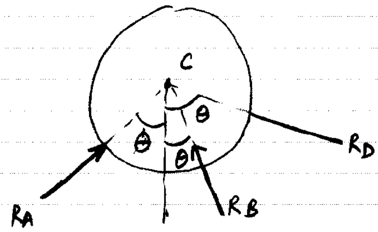
From Δ AOB, $\bar{AO} = \dfrac{r}{\cos 30°} = \dfrac{2r}{\sqrt{3}}$.

From Δ AOC $\bar{OC} = \sqrt{AC^2 - OA^2} = r\sqrt{2^2 - (2/\sqrt{3})^2}$



or $\bar{OC} = 2r\sqrt{\dfrac{2}{3}}$, (continued)

FBD for the upper (top) ball



Equilibrium implies. $\Sigma F_z = 0$.

so $\Sigma F_z = 3R\cos\theta - mg = 0$

or $3R\left(\dfrac{\bar{OC}}{\bar{AC}}\right) - mg = 0$ $\left[\cos\theta = \dfrac{\bar{OC}}{\bar{AC}} \text{ from } \Delta\ OAC\right]$

or $R = \dfrac{mg \cdot 2r}{2r\sqrt{\frac{2}{3}}} \cdot \dfrac{1}{3} = \dfrac{mg}{\sqrt{6}}$

Equilibrium for ball (see FBD), ball A

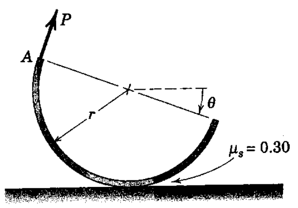
$\Sigma F_x = 0 \Rightarrow P - R\sin\theta = 0$.

or $P = R\sin\theta = \dfrac{mg}{\sqrt{6}} \dfrac{2r/\sqrt{3}}{2r} = \dfrac{mg}{3\sqrt{2}}$

So $\boxed{P = \dfrac{mg}{3\sqrt{2}}}$ lb.

***6/131** The semicylindrical shell of mass m and radius r is rolled through an angle θ by the force P which remains tangent to its periphery at A as shown. If P is slowly increased, plot the tilt angle θ as a function of P up to the point of slipping. Determine the tilt angle θ_{max} and the corresponding value P_{max} for which slipping occurs. The coefficient of static friction is 0.30.

Ans. $\theta_{max} = 59.9°$
$P_{max} = 0.295mg$



$(\mu_s = \mu_k = \mu)$

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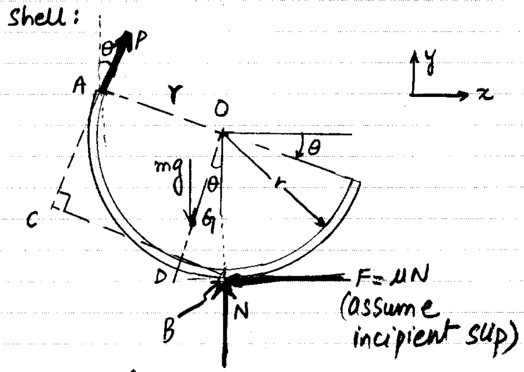
Solution :

FBD for the shell :



$\overline{OG} = d = \frac{2r}{\pi}$; (See Table D/4, Pg 285 Meriam & Kraige)
$\mu = 0.3$

Equilibrium :

$\Sigma F_x = 0 \Rightarrow P\sin\theta - F = 0$ (1)

$\Sigma F_y = 0 \Rightarrow N - mg + P\cos\theta = 0$ (2)

$\Sigma M_B = 0 \Rightarrow mg\,d\sin\theta - P(\overline{BC})$ (3)

$\overline{BC} = \overline{BD} + \overline{BC} = r + r\sin\theta.$

At the start of Slipping $F = \mu N$. (4)

(continued)

Relation between P & θ can be derived from (3).

$$P = \frac{2\sin\theta * (mg)}{\pi(1+\sin\theta)} \quad (5)$$

Substitute $F = \mu N$ in (1), we get

$$P\sin\theta = \mu N \Rightarrow N = \frac{P\sin\theta}{\mu}. \quad (6)$$

Substitute $N = \frac{P}{\mu}\sin\theta$ in (2) to find relation between P and mg.

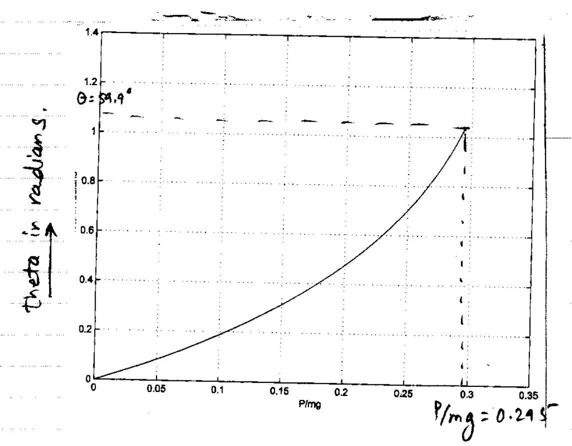
$$\frac{P\sin\theta}{\mu} - mg + P\cos\theta = 0 \Rightarrow \boxed{P = \frac{\mu mg}{(\mu\cos\theta + \sin\theta)}} \quad (7)$$

Comparing (5) & (7)

$$\frac{\mu}{(\mu\cos\theta + \sin\theta)} = \frac{2\sin\theta}{\pi(1+\sin\theta)} \quad (8)$$

Solve (8) numerically to obtain $\theta_{max} = 59.9°$.

& corresponding $P_{max} = 0.295\,mg$ using (5).



Matlab Program.

```
theta=0:0.1:(59.9*pi/180);
P=2.*sin(theta)./(pi.*(1+sin(theta)));
plot(P,theta)
xlabel('P/mg')
ylabel('theta(in radians')
grid on
```