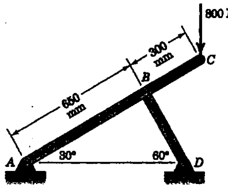
TAM 202 Solns. HW due OCT 1, 2002

4.65

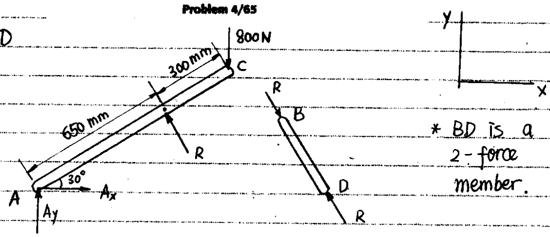
4/65 Determine the magnitudes of all pin reactions for the frame loaded as shown.

Ans. $A = 512$ N, $B = D = 1013$ N



Problem 4/65

① FBD



* BD is a 2-force member.

② $\Sigma M_{/A} = 0 = R \cdot 650\text{mm} - 800\text{N} \cdot (650+300)\text{mm} \cdot \cos 30°$

$\Rightarrow \boxed{R = 1013 \text{ N}}$

$\Sigma F_x = 0 = A_x - R\sin 30°$

$\Rightarrow A_x = 506$ N

$\Sigma F_y = 0 = A_y + R\cos 30° - 800\text{N}$

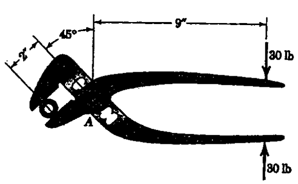
$\Rightarrow A_y = -77$ N

$\boxed{A = \sqrt{A_x^2 + A_y^2} = 512 \text{ N}}$

4.67

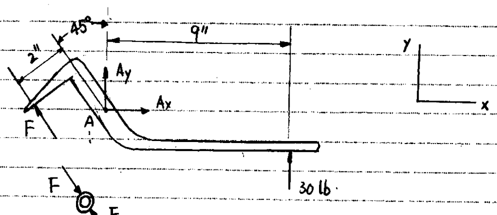
4/67 Compute the force supported by the pin at A for the slip-joint pliers under a grip of 30 lb.

Ans. $A = 157.6$ lb



Problem 4/67

① FBD



(Continued)

② $\Sigma M_{/A} = 0 = 30\text{lb} \cdot 9\text{in} - F \cdot 2\text{in}$

$\Rightarrow F = 135$ lb

$\Sigma F_x = 0 = A_x - F\sin 45°$

$\Rightarrow A_x = 95.5$ lb

$\Sigma F_y = 0 = A_y + F\cos 45° + 30\text{lb}$

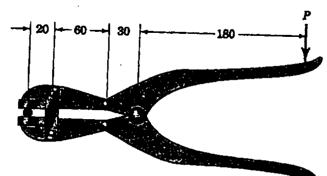
$\Rightarrow A_y = -125.5$ lb

$\boxed{A = \sqrt{A_x^2 + A_y^2} = 157.6 \text{ lb}}$

4.81

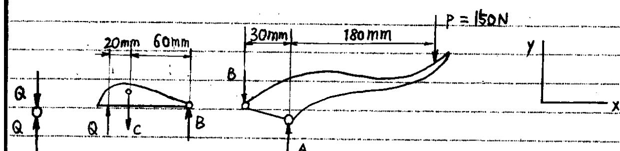
4/81 A small bolt cutter operated by hand for cutting small bolts and rods is shown in the sketch. For a hand grip $P = 150$ N, determine the force Q developed by each jaw on the rod to be cut.

Ans. $Q = 2.7$ kN



Problem 4/81

① FBD



Note that the force at A does not have x component because of symmetry. Thus forces at B and C don't have x components either.

② Equilibrium of the handle

$\Sigma M_{/A} = 0 = B \cdot 30\text{mm} - P \cdot 180\text{mm}$

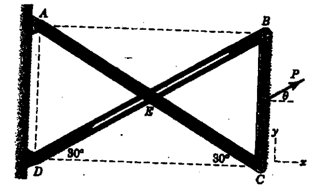
$\Rightarrow \boxed{B = 900 \text{ N}}$

③ Equilibrium of the jaw

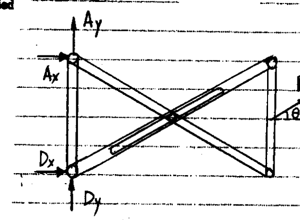
$\Sigma M_{/C} = 0 = B \cdot 60\text{mm} - Q \cdot 20\text{mm}$

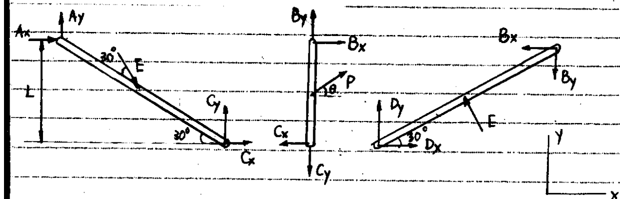
$\Rightarrow \boxed{Q = 2700 \text{ N}}$

4/91 Determine the x- and y-components of all forces acting on each member of the loaded frame for the conditions (a) $\theta = 0$ and (b) $\theta = 30°$. Force P is applied to the midpoint of member BC.



① FBD





② Equilibrium of AC

$\Sigma F_x = 0 = A_x + C_x + E\cos 60°$ (1)

$\Sigma F_y = 0 = A_y + C_y - E\sin 60°$ (2)

$\Sigma M_{/C} = 0 = E[\sin 60° \cdot L\cos 30° - \cos 60° \cdot L\sin 30°] - A_x \cdot L - A_y \cdot 2L\cos 30°$ (3)

③ Equilibrium of BC

$\Sigma F_x = 0 = B_x - C_x + P\cos\theta$ (4)

$\Sigma F_y = 0 = B_y - C_y + P\sin\theta$ (5)

$\Sigma M_{/C} = 0 = -B_x \cdot L - P\cos\theta \cdot \frac{L}{2}$ (6)

④ Equilibrium of DB

$\Sigma F_x = 0 = -B_x + D_x - E\cos 60°$ (7)

$\Sigma F_y = 0 = -B_y + D_y + E\sin 60°$ (8)

$\Sigma M_{/D} = 0 = B_x \cdot L - B_y \cdot 2L\cos 30° + E[\cos 60° \cdot L\sin 30° + \sin 60° \cdot L\cos 30°]$ (9)

Here we have 9 equations and 9 unknowns (A_x, A_y, B_x, B_y, C_x, C_y, D_x, D_y, E). We can solve for these 9 unknowns either by hand or by using computer. (Continued)

⑤ Solve the 9 equations by hand.

(6) $\Rightarrow$ $B_x = -\frac{P}{2}\cos\theta$ (10)

Sub (10) into (4) $\Rightarrow$ $C_x = \frac{P}{2}\cos\theta$ (11)

Sub (10) into (7) $\Rightarrow$ $D_x = \frac{1}{2}E - \frac{P}{2}\cos\theta$ (12)

Sub (10) into (9) $\Rightarrow$ $B_y = \frac{1}{\sqrt{3}}[E - \frac{P}{2}\cos\theta]$ (13)

Sub (11) into (1) $\Rightarrow$ $A_x = -\frac{1}{2}E - \frac{P}{2}\cos\theta$ (14)

Sub (13) into (5) $\Rightarrow$ $C_y = \frac{1}{\sqrt{3}}[E - \frac{P}{2}\cos\theta] + P\sin\theta$ (15)

Sub (13) into (8) $\Rightarrow$ $D_y = -\frac{1}{2\sqrt{3}}[E + P\cos\theta]$ (16)

Sub (15) into (2) $\Rightarrow$ $A_y = \frac{1}{2\sqrt{3}}[E + P\cos\theta] - P\sin\theta$ (17)

Sub (14) & (17) into (3)

$$\Rightarrow E[\tfrac{\sqrt{3}}{2}\cdot\tfrac{\sqrt{3}}{2} - \tfrac{1}{2}\cdot\tfrac{1}{2}] - [-\tfrac{1}{2}E - \tfrac{P}{2}\cos\theta]$$

$$-\{\tfrac{1}{2\sqrt{3}}[E + P\cos\theta] - P\sin\theta\}\cdot 2\cdot\tfrac{\sqrt{3}}{2} = 0$$

$\Rightarrow$ $E = -2\sqrt{3}\,P\sin\theta$ (18)

Sub (18) into (12) $\Rightarrow$ $D_x = -[\sqrt{3}\sin\theta + \frac{1}{2}\cos\theta]P$

Sub (18) into (13) $\Rightarrow$ $B_y = -[2\sin\theta + \frac{\sqrt{3}}{6}\cos\theta]P$

Sub (18) into (14) $\Rightarrow$ $A_x = [\sqrt{3}\sin\theta - \frac{1}{2}\cos\theta]P$

Sub (18) into (15) $\Rightarrow$ $C_y = -[\sin\theta + \frac{\sqrt{3}}{6}\cos\theta]P$

Sub (18) into (16) $\Rightarrow$ $D_y = [\sin\theta - \frac{\sqrt{3}}{6}\cos\theta]P$

Sub (18) into (17) $\Rightarrow$ $A_y = -[2\sin\theta - \frac{\sqrt{3}}{6}\cos\theta]P$

Summary:

$$\boxed{\begin{aligned}
A_x &= [\sqrt{3}\sin\theta - \tfrac{1}{2}\cos\theta]P \\
A_y &= -[2\sin\theta - \tfrac{\sqrt{3}}{6}\cos\theta]P \\
B_x &= -\tfrac{P}{2}\cos\theta \\
B_y &= -[2\sin\theta + \tfrac{\sqrt{3}}{6}\cos\theta]P \\
C_x &= \tfrac{P}{2}\cos\theta \\
C_y &= -[\sin\theta + \tfrac{\sqrt{3}}{6}\cos\theta]P
\end{aligned}}$$ (a)

(Continued)

$$\boxed{\begin{aligned}
D_x &= -[\sqrt{3}\sin\theta + \tfrac{1}{2}\cos\theta]P \\
D_y &= [\sin\theta - \tfrac{\sqrt{3}}{6}\cos\theta]P \\
E &= -2\sqrt{3}\,P\sin\theta
\end{aligned}}$$

Now if $\theta = 0°$, $\sin\theta = 0$, $\cos\theta = 1$, then

$$\boxed{\begin{aligned}
A_x &= -\tfrac{1}{2}P = -0.5P \\
A_y &= \tfrac{\sqrt{3}}{6}P = 0.289P \\
B_x &= -\tfrac{1}{2}P = -0.5P \\
B_y &= -\tfrac{\sqrt{3}}{6}P = -0.289P \\
C_x &= \tfrac{P}{2} = 0.5P \\
C_y &= -\tfrac{\sqrt{3}}{6}P = -0.289P \\
D_x &= -\tfrac{1}{2}P = -0.5P \\
D_y &= -\tfrac{\sqrt{3}}{6}P = -0.289P \\
E &= 0
\end{aligned}}$$ (b)

If $\theta = 30°$, $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, then

$$\boxed{\begin{aligned}
A_x &= \tfrac{\sqrt{3}}{4}P = 0.433P \\
A_y &= -\tfrac{3}{4}P = -0.75P \\
B_x &= -\tfrac{\sqrt{3}}{4}P = -0.433P \\
B_y &= -\tfrac{5}{4}P = -1.25P \\
C_x &= \tfrac{\sqrt{3}}{4}P = 0.433P \\
C_y &= -\tfrac{3}{4}P = -0.75P \\
D_x &= -\tfrac{3\sqrt{3}}{4}P = -1.299P \\
D_y &= \tfrac{1}{4}P = 0.25P \\
E &= -\sqrt{3}P = -1.732P
\end{aligned}}$$ (c)

(Continued)

⑥ Solve the 9 equations again using MATLAB

First, write down eqn (1) – (9) into matrix form:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -\frac{\sqrt{3}}{2} \\
-1 & -\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & \frac{\sqrt{3}}{2} \\
0 & 0 & 1 & -\sqrt{3} & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -P\cos\theta \\ -P\sin\theta \\ P\cos\theta/2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this set of linear equations in two ways: using MATLAB Symbolic Toolbox to get symbolic solution, or using numerical calculation to get numerical solution for a given θ.

(a) Method 1: Symbolic Calculation

```
% Meriam and Kraige 4.91;
% Solution by Tian Tang 09/25/02;
% Solve Ax=b using symbolic toolbox;
% See Pratap "Getting Started with Matlab 5" Chapter 8.

% Set up coefficient matrix A;
A=[  1      0        0     0         1    0    0    0    1/2
     0      1        0     0         0    1    0    0    -sqrt(3)/2
     -1     -sqrt(3) 0     0         0    0    0    0    1/2
     0      0        1     0         -1   0    0    0    0
     0      0        0     1         0    -1   0    0    0
     0      0        -1    0         0    0    0    0    0
     0      0        -1    0         0    0    1    0    -1/2
     0      0        0     -1        0    0    0    1    sqrt(3)/2
     0      0        1     -sqrt(3)  0    0    0    0    1     ];

% Set up right hand side b;
syms theta P;     % symbolic variables;
b=[ 0;   0;   0;    -cos(theta);    -sin(theta);    cos(theta)/2;    0;    0;    0;]*P;

% find the solution x=[Ax Ay Bx By Cx Cy Dx Dy E]';
x=A\b     % solve symbolic equations.
```

(Continued)

Output of the program on P9 8 is

```
x =
[    -1/2*P*cos(theta)+3^(1/2)*P*sin(theta)]
[  1/6*3^(1/2)*P*cos(theta)-2*P*sin(theta)]
[                         -1/2*P*cos(theta)]
[ -1/6*3^(1/2)*P*cos(theta)-2*P*sin(theta)]
[                          1/2*P*cos(theta)]
[   -1/6*3^(1/2)*P*cos(theta)-P*sin(theta)]
[   -3^(1/2)*P*sin(theta)-1/2*P*cos(theta)]
[  -1/6*3^(1/2)*P*cos(theta)+P*sin(theta)]
[                    -2*3^(1/2)*P*sin(theta)]
```

i.e.

$$\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\cos\theta + \sqrt{3}\sin\theta \\ \frac{\sqrt{3}}{6}\cos\theta - 2\sin\theta \\ -\frac{1}{2}\cos\theta \\ -\frac{\sqrt{3}}{6}\cos\theta - 2\sin\theta \\ \frac{1}{2}\cos\theta \\ -\frac{\sqrt{3}}{6}\cos\theta - \sin\theta \\ -\frac{1}{2}\cos\theta - \sqrt{3}\sin\theta \\ -\frac{\sqrt{3}}{6}\cos\theta + \sin\theta \\ -2\sqrt{3}\sin\theta \end{bmatrix} P$$

If $\theta = 0°$, we'll get again

$$\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{6} \\ -\frac{1}{2} \\ -\frac{\sqrt{3}}{6} \\ \frac{1}{2} \\ -\frac{\sqrt{3}}{6} \\ -\frac{1}{2} \\ -\frac{\sqrt{3}}{6} \\ 0 \end{bmatrix} P = \begin{bmatrix} -0.5 \\ 0.289 \\ -0.5 \\ -0.289 \\ 0.5 \\ -0.289 \\ -0.5 \\ -0.289 \\ 0 \end{bmatrix} P$$

If $\theta = 30°$,

$$\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{4} \\ -\frac{3}{4} \\ -\frac{\sqrt{3}}{4} \\ -\frac{5}{4} \\ \frac{\sqrt{3}}{4} \\ -\frac{3}{4} \\ -\frac{3\sqrt{3}}{4} \\ \frac{1}{4} \\ -\sqrt{3} \end{bmatrix} P = \begin{bmatrix} 0.433 \\ -0.75 \\ -0.433 \\ -1.25 \\ 0.433 \\ -0.75 \\ -1.299 \\ 0.25 \\ -1.732 \end{bmatrix} P$$

(Continued)

(b) Method 2 : Numerical Calculation

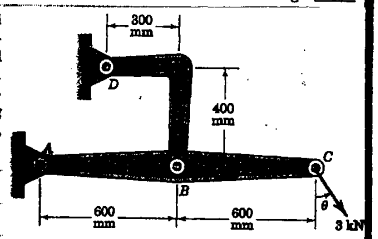
```
% Meriam and Kraige 4.91;
% Solution by Tian Tang 09/25/02;
% Solve Ax=b using numerical calculation, not symbolic calculation.

% Set up coefficient matrix A;
A=[  1    0        0    0         1    0    0    0    1/2
     0    1        0    0         0    1    0    0    -sqrt(3)/2
    -1    -sqrt(3) 0    0         0    0    0    0    1/2
     0    0        1    0        -1    0    0    0    0
     0    0        0    1         0   -1    0    0    0
     0    0       -1    0         0    0    0    0    0
     0    0       -1    0         0    0    1    0    -1/2
     0    0        0   -1         0    0    0    1    sqrt(3)/2
     0    0        1    -sqrt(3)  0    0    0    0    1     ];

% Set up right hand side b;
% Set P=1, and the solution to the problem should be
% P times the solution obtained from this program;
P=1;
% Given theta;
theta=0;
b=[ 0;  0;  0;  -cos(theta);  -sin(theta);  cos(theta)/2;  0;  0;  0]*P;

% find the solution x=[Ax Ay Bx By Cx Cy Dx Dy E]';
x=A\b;
format short;   % use format short to get x with less digits;
x     % show x on the screen.
```

Output of the program above

$\theta = 0$	$\theta = \frac{\pi}{6}$
x =	x =
-0.5000	0.4330
0.2887	-0.7500
-0.5000	-0.4330
-0.2887	-1.2500
0.5000	0.4330
-0.2887	-0.7500
-0.5000	-1.2990
-0.2887	0.2500
0	-1.7321

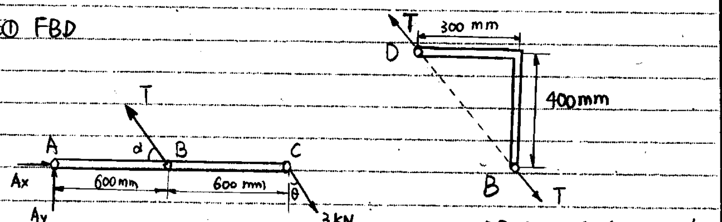
The structural members support the 3-kN load which may be applied at any angle θ from essentially $-90°$ to $+90°$. The pin at A must be designed to support the maximum force transmitted to it. Plot the force F_A at A as a function of θ and determine its maximum value and the corresponding angle θ.

Ans. $F_{A_{max}} = 6$ kN at $\theta = -26.6°$



Problem 4/143

① FBD



* DB is a 2-force member

$\tan\alpha = \frac{400}{300} = \frac{4}{3}$ $\cos\alpha = \frac{3}{5}$ $\sin\alpha = \frac{4}{5}$

② $\Sigma M_{/A} = 0 = T\frac{4}{5}\cdot 600\,mm - 3KN\cdot\cos\theta\cdot 1200\,mm$

$\Rightarrow T = \frac{15}{2}\cos\theta$ KN

$\Sigma F_x = 0 = A_x + 3KN\cdot\sin\theta - T\frac{3}{5}$

$\Rightarrow A_x = \frac{9}{2}\cos\theta - 3\sin\theta$ KN

$\Sigma F_y = 0 = A_y + T\frac{4}{5} - 3KN\cdot\cos\theta$

$\Rightarrow A_y = -3\cos\theta$ KN

③ $F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{[(\frac{9}{2}\cos\theta - 3\sin\theta)KN]^2 + (-3\cos\theta\, KN)^2}$

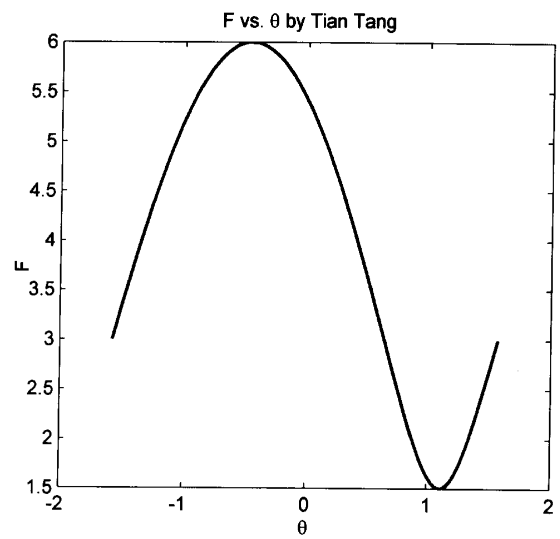
$= \frac{3}{2}\sqrt{9\cos^2\theta - 12\cos\theta\sin\theta + 4}$ KN $-90° \le \theta \le 90°$

④ Now we can use MATLAB to plot F_A vs. θ and find out the maximum value of F_A and the corresponding θ.

(Continued)


```
% Meriam and Kraige 4.143;
% Solution by Tian Tang 09/25/02;
% Plot F as a function of theta;
% Find the maximum value of F and corresponding theta.

theta=linspace(-pi/2,pi/2,100); % define the variable theta;     (100 points)
F=3/2*(9*cos(theta).^2-12*cos(theta).*sin(theta)+4).^(1/2); % find corresponding F;
plot(theta,F);   % plot F vs.theta;
xlabel('\theta');    % label x axis with theta;
ylabel('F');     % label y axis with F;
[y,i]=max(F);     % find maximum value of F and corresponding index of theta;
theta0=theta(i) % find corresponding theta;
theta0=theta0*180/pi      % find theta which has unit of degrees;
Fmax=y    % maximum value of F.
```

Plot of F vs. θ



Output of Fmax and corresponding θ

```
theta0 =
    -0.4601
theta0 =
  -26.3636
Fmax =
    6.0000
```