4.5.9 Beam ABCD is simply supported at B and C and has overhangs at each end (see figure). The span length is L and each overhang has length \( \frac{L}{3} \). A uniform load of intensity \( q \) acts along the entire length of the beam.

Draw the shearing force and bending moment diagrams for this beam.

**PROB. 4.5.9**

**Solution**

Span length = distance between B and C = L.

Length of each overhang = \( \frac{L}{3} \).

**FBD for beam ABCD.**

By symmetry: \( R_B = R_C = \frac{38qL}{16} \)

In general detail:

\[ M_B = 0 = -q\left(L + \frac{L}{3} + \frac{L}{3}\right) + R_B + R_C \]

or \( R_B + R_C = \frac{5L}{3}q \)

To find other equations to solve for reactions, do moment balance about B.

(Anticlockwise moments (+))

\[ EM_B = 0 \]

\[ + q\left(\frac{L}{2}\right)R_C + R_C\left(\frac{L}{2}\right) - q\left(\frac{L}{3} + \frac{L}{3}\right)\left(\frac{L}{2}\right) = 0 \]

\[ 2 \]

\[ \text{Note: the moment due to a distributed load with constant magnitude is } \frac{qL^2}{2} \] where \( q \) is the magnitude and \( L \) is the distance from the point about which moment is taken.

To draw shear force diagram, start from either end and take cuts! (ie Method 1: FBD’s)

\[ M = \frac{qL^2}{2} \]

\[ V = qL \]

**Cut A-A.**

\[ M = -qL^2 \]

Using (1) and (2) Draw shear force diagram.

**Cut B-B.**

\[ \frac{qL^2}{2} \]

Use (2) and (3) to draw bending moment diagram. Not to scale.
Method 2:

Use the equations: $\frac{dv}{dx} = -q$, $\frac{dm}{dx} = v$

Integrate, find out $v$ and $M$ in terms of $x$ and $v$ in terms of $x$.

In this problem $q = q_o$ (a constant).

\[
\frac{dv}{dx} = -q, \quad 0 \leq x \leq \frac{L}{3}
\]

Integrate, $v = v(0) - \int_0^x q \, dx = v(0) - qx$

To calculate $v(0)$, use $v = 0$ at $x = 0$, so $v_0 = 0$.

So $v = -qx$, $0 \leq x \leq \frac{L}{3}$.

Similarly

$\frac{dm}{dx} = v \Rightarrow M = \int_0^x v \, dx + M_o$.

Use $v = -qx \Rightarrow M = M(0) + \int_0^x (-qx) \, dx$ or $M = M(0) - qx^2$.

To calculate $M(0)$ use $M = 0$ at $x = 0$.

So $M = -\frac{qL^2}{2}$, $0 \leq x \leq \frac{L}{3}$.

Now $V(\frac{L}{3}) = \frac{-qL^2 + 5qL}{6}$.

$V(\frac{L}{3})$, jump in $V$ due to reaction at B (KB).

Similarly $V = \frac{-qL^2 - qL}{2}$.

So $V = V(\frac{L}{3}) + q(L - L/3)$

$V = \frac{qL}{2} + \frac{qL}{6} - qx$

or $V = \frac{5qL}{6} - qx$.

Integrate for $M$

$M = M(\frac{L}{3}) + \int_{\frac{L}{3}}^x (\frac{5qL}{6} - qx) \, dx$

or $M = M(\frac{L}{3}) + \frac{5qL}{6}(x - \frac{L}{3}) - \frac{qL^3}{2}(x - \frac{L}{3})^2$

$M(\frac{L}{3}) = \frac{-qL^2}{18}$.

So $M = \frac{-qL^2 + 5qL}{6}(x - \frac{L}{3}) - \frac{qL^3}{2}(x - \frac{L}{3})^2$

or $M = \frac{-qL^2 + 5qL}{6}x - \frac{qL^3}{18}$.
4.5-10 Draw the shear-force and bending-moment diagrams for a cantilever beam AB supporting a linearly varying load of maximum intensity $q_0$ (see figure).

**Solution**

**FBD for the Cantilever Beam**

$\begin{align*}
\Sigma F_Y &= 0 \Rightarrow V_B = -q(x) = (q_0 + \frac{q_0}{L}x) \frac{L}{2} = 0 \\
\Sigma M &= 0 \Rightarrow M_B = \left. \frac{1}{2} (q_0 + \frac{q_0}{L}x) \frac{L^2}{2} \right|_0^L = \frac{q_0 L^2}{2} \frac{L}{2} = \frac{q_0 L^3}{6} \\
\end{align*}$

Now to draw shear force & bending moment diagram take a cut $(n-n)$ & look at $V$ & $M$.

**Method 1**

FBD for Section A $(n-n)$

$\begin{align*}
\Sigma F_Y &= 0 \Rightarrow V + \frac{1}{2} \frac{q_0 x}{L} \frac{x}{2} = 0 \\
\frac{q_0}{L} x = 0 \\
V &= -\frac{q_0 x^2}{2L} \\
\end{align*}$

$\begin{align*}
\Sigma M &= 0 \Rightarrow M = \frac{1}{2} \frac{q_0 x^2}{L} \frac{x}{2} - 0 \\
M &= \frac{-q_0 x^2}{6L} \\
\end{align*}$

**Method 2**

Use $\frac{dV}{dx} = -q$ & $\frac{dM}{dx} = V$.

Here $q$ is a function of $x$.

$\begin{align*}
q &= \frac{q_0 x}{L} \\
\frac{dV}{dx} &= -\frac{q_0 x}{L} \\
V &= V(0) - \frac{q_0 x^2}{2L} \\
V(0) &= 0 \Rightarrow V = \frac{q_0 x^2}{2L} \\
\end{align*}$

So $V = \frac{-q_0 x^2}{2L}$.

$m = m(0) + \int V \, dx$

So $m = m(0) - \int \frac{q_0 x^2}{2L} dx$

$m(0) = 0 \Rightarrow m = -\frac{q_0 x^3}{6L}$

$m(0) = 0 \Rightarrow m = -\frac{q_0 x^3}{6L}$

Use (1) & (3) to plot Bending moment & Shear force diagrams.
5.4-2 A copper wire having diameter $d = 3$ mm is bent into a circle and held with the ends just touching (see figure). If the maximum permissible strain in the copper is $e_{max} = 0.004$, what is the shortest length $L$ of wire that can be used?

**Solution:**

\[ e = \frac{d}{2} = \frac{3 \times 10^{-3}}{2} = \frac{3 \times 10^{-3}}{2} = 0.004 \]

Now

\[ e_{max} = e \cdot \frac{d}{2} = \frac{e d}{2} \]

or

\[ L_{min} = \frac{\pi d}{e_{max}} = \frac{\pi (3 \times 10^{-3})}{0.004} \]

or

\[ L_{min} = 2.36 \text{ m} \]

**PROB. 5.4-2**

5.5-4 A simply supported beam AB with span length $L = 3.75$ m carries a uniform load of intensity $q = 6.4$ kN/m (see figure).

(a) Calculate the maximum bending stress $\sigma_{max}$ due to the load $q$ if the beam has a rectangular cross section with width $b = 150$ mm and height $h = 300$ mm.

(b) Does the stress increase or decrease if the thickness of the beam is increased?

**Solution:**

The maximum bending stress occurs where the bending moment is maximum.

So

\[ \sigma_{max} = \frac{M_{max}}{I} \]

\[ 2Fy = 0 \]

\[ M_{max} = \frac{qL^2}{2} = 0 \]

\[ \frac{RA}{L} = \frac{qL}{2} \]

\[ \frac{RB}{L} = \frac{qL}{2} \]

**PROB. 5.5-4**
5.5.6 A freight-car axle $AB$ is loaded as shown in the figure, with the forces $P$ representing the car loads (transmitted through the axle boxes) and the forces $R$ representing the rail loads (transmitted through the wheels). The diameter of the axle is $d = 80$ mm, the wheel gauge is $L = 1.45$ m, and the distance between the forces $P$ and $R$ is $b = 200$ mm.

Calculate the maximum bending stress $\sigma_{max}$ in the axle if $P = 46.5$ kN.

**Solution:** $R = P = 46.5$ kN.

The bending moment diagram looks like this: