3.4-3 A circular tube of outer diameter $d_1 = 2.75$ in. and inner diameter $d_2 = 2.35$ in. is welded at the right-hand end to a fixed plate and at the left-hand end to a rigid end plate (see figure). A solid circular bar of diameter $d_3 = 1.60$ in. is inside of, and concentric with, the tube. The bar passes through a hole in the fixed plate and is welded to the end plate. The bar is 40 in. long and the tube is half as long as the bar. A torque $T = 10,000$ lb-in. acts at end $A$ of the bar. Also, both the bar and tube are made of an aluminum alloy with shear modulus of elasticity $G = 3.9 \times 10^6$ psi.

(a) Determine the maximum shear stresses in both the bar and tube.

(b) Determine the angle of twist (in degrees) at end $A$ of the bar.

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**Solution:**

**Bar:**

Torque $T = 10,000$ lb-in.

Maximum Shear Stresses

\[ \tau_{\text{bar}} = \frac{16 T}{\pi d_3^3} = \frac{16 \times (10,000 \text{ lb-in})}{\pi (1.60 \text{ in})^3} = 12,430 \text{ psi} \]

**Tube:**

\[ \tau_{\text{tube}} = \frac{T (d_3/2)}{(3\pi)^{1/2} L} = \frac{(10,000 \text{ lb-in}) (2.75 \text{ in})}{2.621 \text{ in}^{3/2}} = 5,250 \text{ psi} \]

(b) \[ \phi_A = \frac{d}{d_{\text{bar}}} + \frac{d}{d_{\text{tube}}} \]

\[ \phi_{\text{bar}} = \frac{T L_{\text{bar}}}{G IP_{\text{bar}}} = \frac{(10,000 \text{ lb-in}) (40 \text{ in})}{(3.9 \times 10^6 \text{ psi}) (10 \text{ in}^4)} = 0.1594 \text{ rad} \]

\[ \phi_{\text{tube}} = \frac{T L_{\text{tube}}}{G IP_{\text{tube}}} = \frac{(10,000 \text{ lb-in}) (20 \text{ in})}{(3.9 \times 10^6 \text{ psi}) (2.621 \text{ in}^4)} = 0.0198 \text{ rad} \]

\[ \phi_A = \phi_{\text{bar}} + \phi_{\text{tube}} = 0.1790 \text{ rad} = 10.3' \]

\[ \phi_A = 10.3' = 0.179 \text{ rad} \]

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3.4-10 The bar shown in the figure is tapered linearly from end $A$ to end $B$ and has a solid circular cross section. The diameter at the smaller end of the bar is $d_a = 25$ mm. The bar is made of steel with shear modulus of elasticity $G = 82$ GPa.

If the torque $T = 90$ N-m and the allowable rate of twist is 0.55 rad/m, what is the maximum allowable diameter $d_a$ at the larger end of the bar? (Hint: Use the results of Example 3-3.)

(Continued)
3.5-4 A solid circular bar of diameter $d = 50$ mm (see figure) is twisted in a testing machine until the applied torque reaches the value $T = 1300$ N·m. At this value of torque, a strain gage oriented at 45° to the axis of the bar gives a reading $\varepsilon = 331 \times 10^{-6}$.

Determine the shear modulus $G$ of the material.

From symmetry and equilibrium in vertical direction,

$$ R_B = R_C = \frac{q}{2} (L + 2b) $$

FBD of left half of beam:

$$ M = 0 \text{ (given)} $$

$$ \Sigma M = 0: (6k \times 0) + (10k \times 5ft) - R_A \times 20ft = 0 $$

$$ \Rightarrow R_A = 7.0K $$

FBD of segment $AC$

$$ \Sigma F_y = 0: V = 7k - 6k = 1.0k $$

$$ \Sigma M = 0: (6k \times 5ft) + M - (7k \times 10ft) = 0 $$

$$ \Rightarrow M = 40k-ft $$

4.3-4 The beam $ABCD$ shown in the figure has overhangs at each end and carries a uniform load of intensity $q$. For what ratio $b/L$, will the bending moment at the midpoint of the beam be zero?

4.3-1 Determine the shear force $V$ and bending moment $M$ at the midpoint $C$ of the simple beam $AB$ shown in the figure.

$$ \theta = \frac{\delta}{L} \Rightarrow \theta = \frac{1.9445}{3} $$

$$ \Rightarrow \theta = \frac{180}{72} \Rightarrow \theta = 0.0362 $$

$$ \Rightarrow \phi = \frac{1.9445}{3} $$

$$ \Rightarrow \phi = \frac{180}{72} $$

$$ \Rightarrow \phi = 0.25N/m $$

4.3-3 The beam $ABCD$ shown in the figure has overhangs at each end and carries a uniform load of intensity $q$. For what ratio $b/L$, will the bending moment at the midpoint of the beam be zero?

Solution:

$$ R_B = R_C = \frac{q}{2} (L + 2b) $$

From symmetry and equilibrium in vertical direction,

$$ R_B = R_C = \frac{q}{2} (L + 2b) $$

FBD of left half of beam:

$$ M = 0 \text{ (given)} $$

$$ \Sigma M = 0: (6k \times 0) + (10k \times 5ft) - R_A \times 20ft = 0 $$

$$ \Rightarrow R_A = 7.0K $$

FBD of segment $AC$

$$ \Sigma F_y = 0: V = 7k - 6k = 1.0k $$

$$ \Sigma M = 0: (6k \times 5ft) + M - (7k \times 10ft) = 0 $$

$$ \Rightarrow M = 40k-ft $$

4.3-4 The beam $ABCD$ shown in the figure has overhangs at each end and carries a uniform load of intensity $q$. For what ratio $b/L$, will the bending moment at the midpoint of the beam be zero?