with "Solutions"

TAM 202 Prelim 2, Nov 5, 2002

Problem Statements

(Exam directions and student solutions are in a separate booklet.)

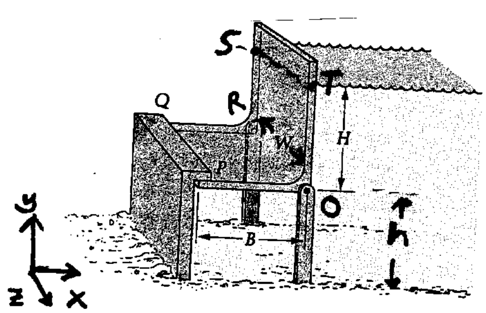
1. (33 pts). The gate OPQRST, held by hinges at O and R, closes an opening of width W in a water (mass density ρ) channel of initial depth h. With the gate open (not shown) the water flows like a waterfall over the lip QP. Once the gate is manually held closed for a short time, the water rises and the gate is then held closed by the water pressure on the bottom of surface OPQR (configuration shown below). Neglect the gate's weight and atmospheric pressure. When closed, a vertical line load f_{QP} (force per unit length) acts from the lip QP on to the gate.

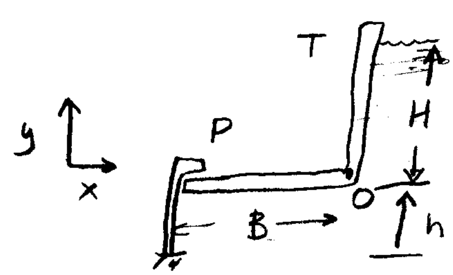
(a; 6 pts). Draw a side view of gate OPQRST (showing POT). Show the fluid pressure distributions on the vertical and horizontal surfaces. Magnitudes of the fluid pressures should be labeled in terms of some or all of the given parameters ρ, g, W, B, h and H.

(b; 6 pts). Draw a free-body diagram showing all forces on the gate. Each fluid pressure distribution should be represented by an equivalent force (one force on OPQR and one force on OTSR). The force magnitudes and locations should be clearly indicated.

(c; 13 pts). What are the reaction forces at hinges O and R, and the line load (force per length) along the seal QP? Answer in terms of some or all of ρ, g, W, B, h and H.

(d; 8 pts). Eventually when the water gets high enough above O, water pressure on surface OTSR pushes the gate open once more, rotating the gate counterclockwise about OR. Determine the value of H at which the gate will just be pushed open this way.





2. (33 pts). The sketch (a) shows the box-beam hanger *as built* in the Hyatt Regency hotel in Kansas City; this structure failed in 1981, killing 114 guests.
The sketch (b) shows the intended design. The sketch (c) is a detail of rod, nuts, and washers at L, R, and S. The portion of the third floor walkway that is carried by the hanger rod weighs $W = 3000$ kN. The portion of the second floor walkway carried by the hanger rod extending down also weighs $W = 3000$ kN.

Consider the hanger rods to be made of steel (E = 200 Gpa) and all other parts of the structure to be aluminum (E = 70 Gpa). The box beam walls are 15 mm thick. Approximate the hexagonal nut as a cylinder. To keep the arithmetic simpler, use the following slightly inconsistent dimensions:

	Hanger rods	nut	washer
Diameter	50 mm	62.5 mm	100 mm
Circumference	150 mm	200 mm	300 mm
Area enclosed by perimeter	2,000 mm^2	3,000 mm^2	8,000 mm^2

Some of the answers may not be tidy. Reduce them the best you can without doing long multiplication or long division.

(a; 8 pts). See Fig. (a). Using free-body diagrams, find (i) the force on the washer L from the box beam, (ii) the tension in the upper rod U, and (iii) the force on the washer R from the box beam.

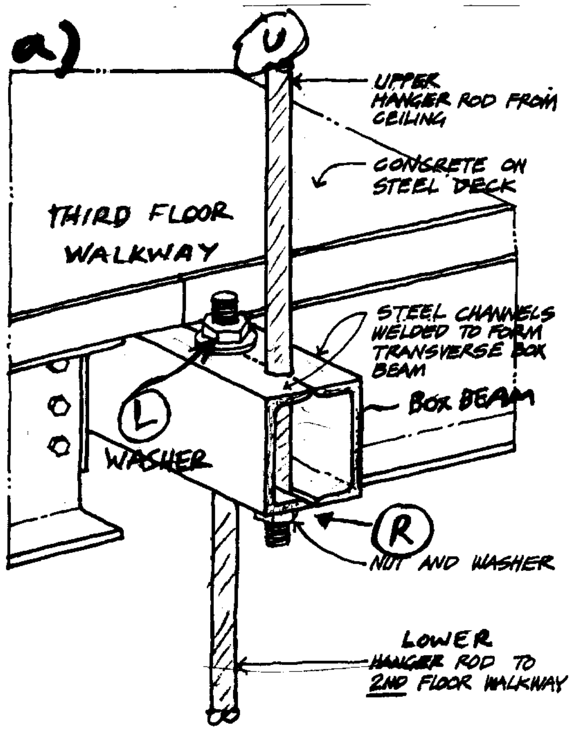
(b; 5 pts). What is the *average tension stress* σ in the upper rod U?

(c; 5 pts). What is the *average bearing (normal) stress* between the washer R and the box beam?

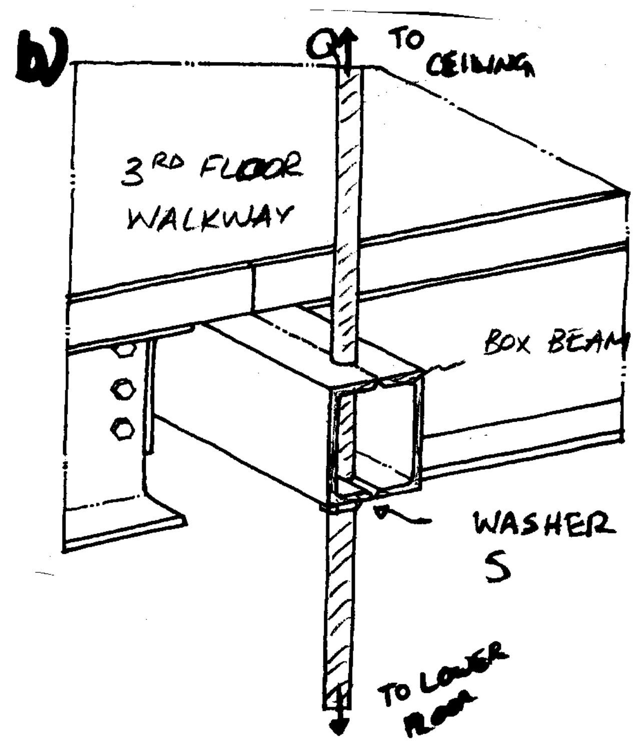
(d; 7 pts). What is the *average shear stress* in the box beam directly under the washer R's outer edge (this is a short cylindrical surface)?

(e; 4 pts). Now consider the intended design in Fig. (b). Find (i) the tension in the upper rod Q, and (ii) the force of the box beam on the washer S.

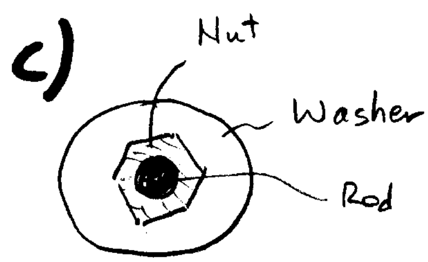
(f; 4 pts). In one short and precise phrase or sentence explain the main reason why it was such a big mistake to replace the design (b) with the superficially similar construction (a).



As built (parts a-d)



As designed (parts e-f)



Cross-section of Washer-Nut Rod

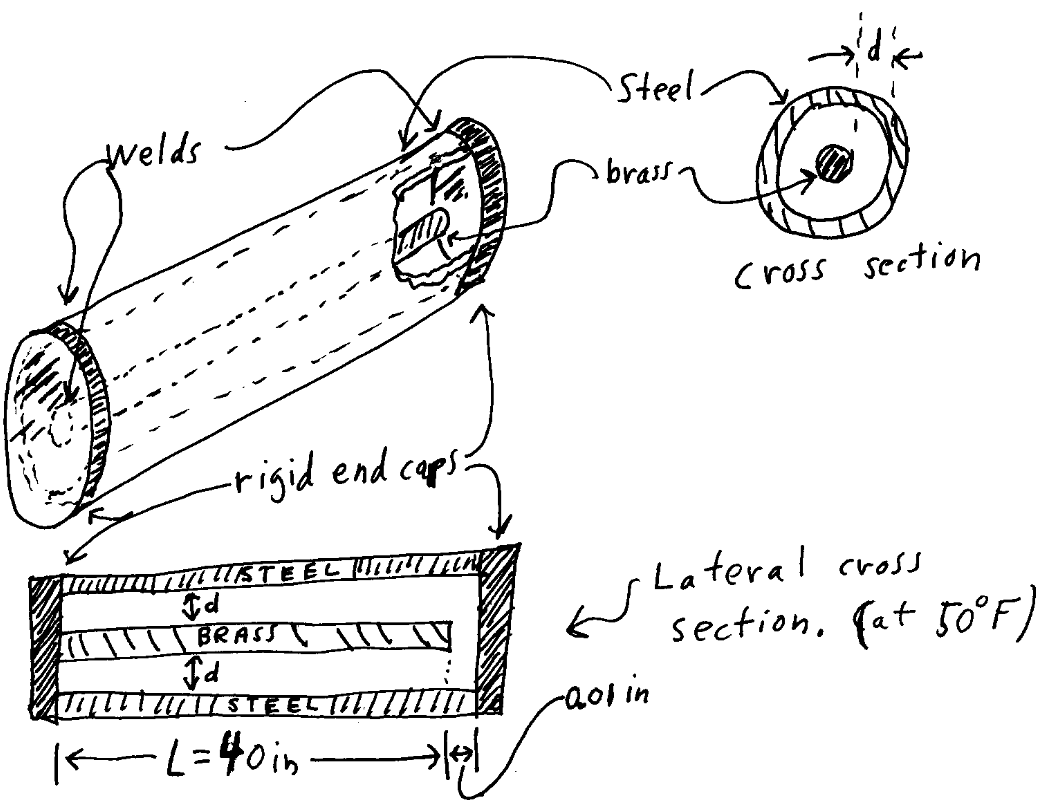
3. (34 pts). A brass rod ($E_b = 15 \times 10^6$ psi; $\nu_b = 0.3$; cross-sectional area $A_b = 2$ in^2) is enclosed by a slightly longer, hollow steel tube ($E_s = 30 \times 10^6$ psi; $\nu_s = 0.3$; cross-sectional area $A_s = 1$ in^2). Rigid flat plates are welded to each end of the steel tube and to one end of the brass tube. The end plates are not held in place by any external forces (except in part b below). Approximate the coefficients of thermal expansion in the steel and the brass as $5 \times 10^{-6}/°F$ and $10 \times 10^{-6}/°F$, respectively. When the temperature is 50°F the brass rod has length L = 40", and the steel sleeve is 0.01" longer than that.

The temperature of the assembly is raised 100°F (to 150°F) and both the brass and steel expand, the brass expanding more. As the heating proceeds, the brass closes the gap and presses against the end plate.

Some of the answers may not be tidy. Reduce them the best you can without doing long multiplication or long division.

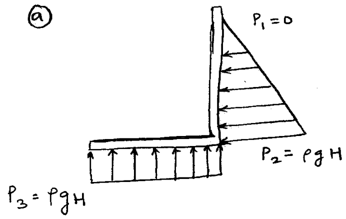
(a; 28 pts). At the raised temperature what are the stresses in the brass and steel?

(b; 6 pts). If, in its heated state, the end plates of the structure had equal and opposite compressive loads applied, would this cause an increase or decrease in the gap *d* (as measured far from the end plates)? Why?

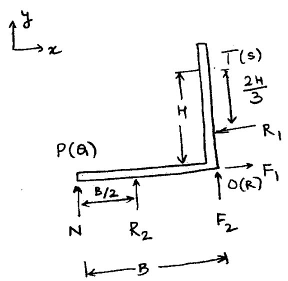


P.1) (by Tian Tang)

(a)



(b)



$R_1 = \bar{P}_{OTSR} \cdot A_{OTSR}$

$= \frac{0 + \rho g H}{2} \cdot HW$

$\boxed{R_1 = \frac{1}{2} \rho g H^2 W}$ — ①

$R_2 = \bar{P}_{OPQR} \cdot A_{OPQR}$

$R_2 = \rho g H (WB)$

$\Rightarrow \boxed{R_2 = \rho g HWB}$ — ②

(c) use FBD above

$\Sigma M_{/OR} = 0 \Rightarrow R_1 \cdot \frac{H}{3} - R_2 \frac{B}{2} - N \cdot B = 0$ — ③

sub. ① and ② into ③ $\Rightarrow N = \frac{\rho g HW}{6B} (H^2 - 3B^2)$ — ④

* Note that N is total force of the line load along QP so the line load along QP will be

$\bar{N} = \frac{N}{W} \Rightarrow \boxed{\bar{N} = \frac{\rho g H}{6B} (H^2 - 3B^2)}$ — ⑤

$\Sigma F_x = 0 \Rightarrow F_1 - R_1 = 0 \Rightarrow \boxed{F_1 = \frac{1}{2} \rho g H^2 W}$ — ⑥

$\Sigma F_y = 0 \Rightarrow N + R_2 + F_2 = 0 \Rightarrow F_2 = -(N + R_2)$

$\boxed{F_2 = -\frac{\rho g HW}{6B} (H^2 + 3B^2)}$ — ⑦

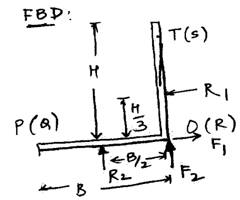
* Note that there are two hinges O and R and forces F_1 & F_2 are sum of the forces at O and R and because of symmetry forces at O & R will be half of F_1 and F_2.

$\boxed{F_{Rx} = F_{Ox} = \frac{F_1}{2} = \frac{1}{4} \rho g H^2 W}$ — ⑧

$\boxed{F_{Ry} = F_{Oy} = \frac{F_2}{2} = -\frac{\rho g H W}{12B} (H^2 + 3B^2)}$ — ⑨

(d) when the gate is just pushed open, the normal force along QP is equals to zero.

FBD:



$\Sigma M_{/OR} = 0$

$\Rightarrow R_1 \cdot \frac{H}{3} - R_2 \cdot \frac{B}{2} = 0$ — ⑩

sub ① & ② into ⑩

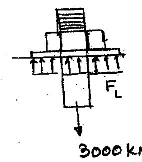
$\frac{1}{2} \rho g H^2 W \cdot \frac{H}{3} = \rho g HWB \frac{B}{2}$

$\Rightarrow \boxed{H = \sqrt{3} B}$ — ⑪

Solution by Pankaj and Vijay.

2 (a)

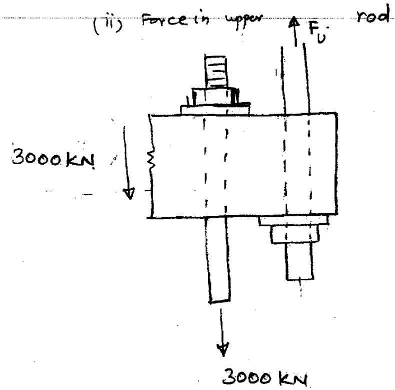
(i) Force on the washer L.



$\Sigma F_y = 0 \Rightarrow F_L - 3000\text{ kN} = 0$

$\Rightarrow \boxed{F_L = 3000\text{ kN}}$

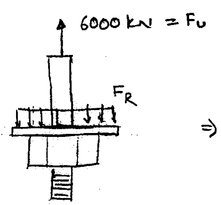
(ii) Force in upper rod.



$\Sigma F_y = 0 \quad F_U - 3000\text{ kN} - 3000\text{ kN} = 0$

$\Rightarrow \boxed{F_U = 6000\text{ kN}}$

(iii) Force on washer at R.



$\Sigma F_y = 0$

$\Rightarrow 6000\text{ kN} - F_R = 0$

$\Rightarrow \boxed{F_R = 6000\text{ kN}}$

(b) Avg. tension stress in the upper rod.

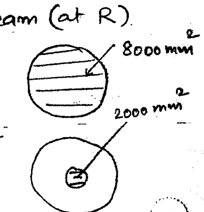
$\sigma_T = \dfrac{F_U}{A_r}$

A_r = cross-section area of rod.
$= 2000\text{ mm}^2$

$= \dfrac{6000\text{ kN}}{2000\text{ mm}^2}$

$\boxed{\sigma_T = 3\text{ GPa}}$

(c) Avg. bearing stress between washer and box beam (at R).



$\sigma_b = \dfrac{F_R}{A_b}.$

A_b = bearing area.
$= 8000\text{ mm}^2 - 2000\text{ mm}^2$
$= 6000\text{ mm}^2$.

$\Rightarrow \sigma_b = \dfrac{6000\text{ kN}}{6000\text{ mm}^2}$

$\Rightarrow \boxed{\sigma_b = 1\text{ GPa}}$

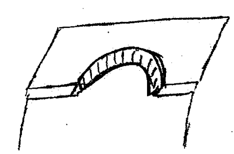
(d) Shear stress in the box beam

$F_s = 6000\text{ kN}$.
A_s = Area in shear.

$\tau = \dfrac{F_s}{A_s}$

$\tau = \dfrac{6000\text{ kN}}{4500\text{ mm}^2}$

$\boxed{\tau = 1.33\text{ GPa}}$

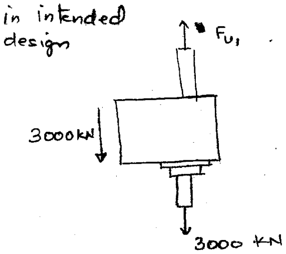


$A_s = c * t = (300\text{ mm})(15\text{ mm}) = 4500\text{ mm}^2$

c = circumference of washer = 300 mm

t = thickness of box beam = 15 mm.

(e) F_{U1} in intended design



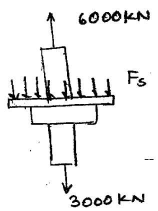
$\Sigma F_y = 0 \Rightarrow F_{U1} - 3000\,kN - 3000\,kN = 0$

$\Rightarrow \boxed{F_{U1} = 6000\,kN}$

(same as built design.)

(f)

(a)



$\Sigma F_y = 0$

$6000\,kN - 3000\,kN + F_s = 0$

$\Rightarrow \boxed{F_s = 3000\,kN}$

(half the built design.)

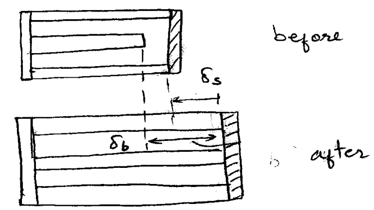
(b) the load carried by washar at R (buit design a) is twice as much as by S (~~intended~~ design).

(by Prof. Ruina)

(3). Sign convention
* tension is positive
* lengthening is positive

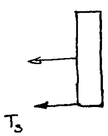
Geometry

$\boxed{\delta_b = \delta_s + \underbrace{.01\,in}_{\delta}}$ — (1)



FBD of end plate (1D)

ok that this turns out negative



$\Sigma F_x = 0 \quad \boxed{T_b = -T_s}$ — (2)

Material Properties

$$\boxed{\begin{aligned} \delta_s &= \frac{T_s \ell}{E_s A_s} + \alpha_s \ell \Delta T \quad (3a) \\ \delta_b &= \frac{T_b \ell}{E_b A_b} + \alpha_b \ell \Delta T \quad (3b) \end{aligned}}$$

(use $\ell_s = \ell_b = \ell$)

sub (3) into (1) too get one equation in on unknow i.e. T_s ($T_b = -T_s$)

$$\frac{-T_s \ell}{E_b A_b} + \alpha_b \ell \Delta T = \frac{T_s \ell}{E_s A_s} + \alpha_s \ell \Delta T + \delta$$

$$T_s \ell \left[\frac{1}{E_s A_s} + \frac{1}{E_b A_b} \right] = (\alpha_b - \alpha_s) \ell \Delta T - \delta$$

$$T_s = \frac{(\alpha_b - \alpha_s)\,\ell\,\Delta T - \delta}{\ell\left[\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right]}$$

$$\sigma_s = \frac{T_s}{A_s} = \frac{(\alpha_b - \alpha_s)\,\ell\,\Delta T - \delta}{\ell\left[\frac{1}{E_s} + \frac{1}{E_b (A_b/A_s)}\right]}$$

$$\sigma_s = \frac{(\alpha_b - \alpha_s)\quad \Delta T - \delta/\ell}{\left[1 + \frac{1}{(E_b A_b / E_s A_s)}\right]}$$

$$\sigma_s = \frac{(5 * 10^{-6} /{}^\circ F)*(100\,{}^\circ F) - (.01/40)}{\left(1 + \frac{1}{1}\right)} * 30 * 10^6\ lb/in^2$$

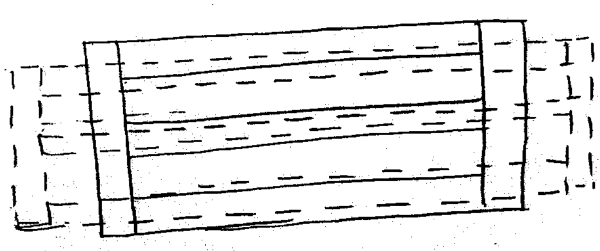
$$= \frac{5*10^{-4} - 0.00025}{2} * 30*10^6 \quad lb/in^2$$

$$= \frac{2.5}{2} * 30 * 10^2\ lb/in^2 = \boxed{3750\ lb/in^2}$$

$$\sigma_b = \frac{T_b}{A_b} = -\frac{T_s}{A_s} \cdot \frac{A_s}{A_b} = -\sigma_s \frac{A_s}{A_b}$$

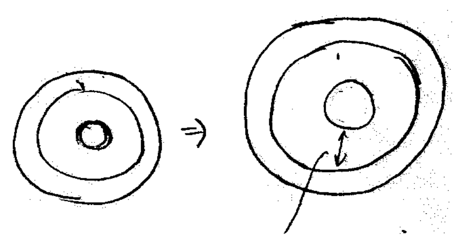
$$\sigma_b = -(3750\,lb/in^2)\ \frac{1}{2} \Rightarrow \boxed{\sigma_b = -1875\ \ lb/in^2}$$

(b)



uniform poisson expansion $\nu_b = \nu_s$

$\Rightarrow$ whole and gap expand.



gap expands in same proportion to metal.