\[3.65\text{ continued}\]

\[\begin{align*}
\sum & \text{M}_{x} = 0 \quad \text{[the key equation!]} \\
\Rightarrow & \sum \text{M}_{A} \cdot \text{any vector in } \overrightarrow{AC} \text{ dir.} = 0 \\
\Rightarrow & (\sum \text{M}_{A}) \cdot \hat{\text{i}} = 0
\end{align*}\]

\[\begin{align*}
\sum \text{M}_{x} &= 0 \quad \text{(3 eqs)} \\
\sum \text{M}_{y} &= 0 \quad \text{(3 eqs)}
\end{align*}\]

in 5 unknowns: \(T_1, T_2, R_{x}, R_{y}, R_{z}\).  
For this problem one of these equations is just 0=0. That is, none of the forces contribute to \(\sum \text{M}_{z}\) (in moments about the axis through \(A\)).

You could solve the 5 eqns in 5 unknowns and get \(T_1\), \(T_2\), \(R_{x}\), \(R_{y}\), \(R_{z}\).

Although you had 5 eqns in 5 unknowns, if you were alert you might have noticed that one of them \(\sum \text{M}_{z} = 0\), had just one unknown, namely \(T_1\). So you could solve it without solving the other 4 eqns.

That one eqn is equivalent to \(\sum \text{M}_{z} = 0\), the solution presented,

\[T_1 = 208 \text{ kN} \approx 4.9 \text{ kN}\]
Neglect weight. No comment on friction at A, B, C. So leave it in & see if we can get an answer.

\[ \Sigma M_{BC} = 0 \quad (1) \]

\[ \Sigma \text{sum of moments about axis BC}. \]

\[ R_B \text{ & } R_C \text{ go through axis & don't contribute. Likewise for } R_{AX} \text{ & } R_{AY}. \]

So eqn (1) is one eqn in one unknown, (1) => \[ 500 \times L(\hat{k}) + 1100 \times (R_A \hat{k}) \]

\[ = 0 \]

Slide \( L \hat{k} \) down to x-y plane and all vectors are \( \perp \).

\[ R_A = L(1 + \sin 30^\circ) = (L v + L \sin 30^\circ) \]

\[ R_A = \frac{W + 825}{3.5} \]

\[ = \frac{35 + 125}{3.25} \]

\[ R_A = \frac{200}{3.25} = \frac{8L}{15} \approx 0.53L \]

\[ \Sigma M_{AE} = 0 \quad (2) \]

\[ R_C = R_B \text{ because they have the same lever arm } 30^\circ \text{ about axis AE (the y axis).} \]

\[ \Sigma F_z = 0 \quad (3 \text{ eqns}) \]

\[ \Sigma M_{C} = 0 \quad (3 \text{ eqns}) \]

6 unknowns: \( T_1, T_2, R_y, R_x, R_c \) (don't be thrown off because the book answer only gives \( R_c \)).

Approach 1: Try to be clever.

For example \( \Sigma M_{AB} = 0 \) gives \( R_c = \frac{2kN}{35} \).
and \[ S \mathbf{M}_{E} = 0 \Rightarrow \mathbf{R}_C = 0 \]

etc.

**Approach 2:** "Brute Force" (like end of lect) of text

Write out 6 eqs. in 6 unknowns, put in matrix form, and solve on computer.

Some geometry first:

\[
\hat{\lambda}_{EA} = \frac{-4 \hat{i} - 1.5 \hat{j} + 2.5 \hat{k}}{\sqrt{4^2 + 1.5^2 + 2.5^2}}
\]

\[
\hat{\lambda}_{HB} = \frac{-2.5 \hat{i} + 1.5 \hat{j} + 2.5 \hat{k}}{\sqrt{2.5^2 + 1.5^2 + 2.5^2}}
\]

\[S_{DH} = 2.5 \text{ m} \hat{j}\]

\[S_{DE} = 4 \text{ m} \hat{i}\]

\[S_{DC} = 2 \text{ m} \hat{i} - 7 \text{ m} \hat{k}\]

\[S_{PC} = -1 \text{ m} \hat{k}\]

**Force Balance**

\[0 = 2 F\]

\[\Rightarrow T_1 \hat{\lambda}_{EA} + T_2 \hat{\lambda}_{HB} + R_{Dy} \hat{j} + R_C \hat{k} = mg \hat{k} \quad (1)\]

We can take \( x, y, z \) components on dot w/ \( \hat{i}, \hat{j}, \hat{k} \) to get 3 eqs. in 6 unknowns. Note eqn (1) is re-arranged to put knowns on right & unknowns on left.

**Moment Balance**

\[0 = 8 \overline{M}_{10}\]

\[\Rightarrow S_{DE} \times (T_1 \hat{\lambda}_{EA}) + S_{DH} \times (T_2 \hat{\lambda}_{HB}) + S_{DC} \times R_C = S_{DC} \times (mg \hat{k}) \quad (2)\]

After carrying out cross products we can also break (2) into comps. to get 3 more eqs. for the same 6 unknowns. Fortunately the cross products are pretty sparse.

\[\Rightarrow 4 \hat{i} \times (\lambda_{EA} \hat{i} + \lambda_{EB} \hat{j} + \lambda_{EC} \hat{k}) T_1 + 2.5 \hat{i} \times (\lambda_{HB} \hat{i} + \lambda_{HB} \hat{j} + \lambda_{HC} \hat{k}) T_2 + R \times (R_{Cx} \hat{i} + R_{Cy} \hat{j} + R_{Cz} \hat{k}) = (2 \hat{i} - R) \times (mg \hat{k}) \quad (2)\]

\[\Rightarrow (4 \lambda_{EA} \hat{k} - 4 \lambda_{EB} \hat{j}) T_1 + (2.5 \lambda_{HB} \hat{k} - 2.5 \lambda_{HC} \hat{j}) T_2 + R_{Cx} \hat{i} + R_{Cy} \hat{j} = -2mg \hat{j} \quad (2)\]

Now break (1) into comps., break (2) into comps., & write out all 6 eqs. in an organized way.

\[\lambda_{EA} T_1 + \lambda_{HB} T_2 + R_{Cz} = 0 \quad (3)\]

\[\lambda_{EA} T_1 + \lambda_{HB} T_2 + R_{Cy} = 0 \quad (4)\]

\[\lambda_{EA} T_1 + \lambda_{HB} T_2 + R_{Cz} = mg \quad (5)\]

\[-2 \lambda_{EA} T_1 - 2.5 \lambda_{HB} T_2 - R_{Cx} = -2mg \quad (3)\]

\[4 \lambda_{EA} T_1 + 2.5 \lambda_{HB} T_2 = 0 \quad (6)\]

Now, eqs. 3-8 (the comps. of force & moment balance) are 6 eqs. in 6 unknowns. 

One could fudge through but lets continue in "Brute Force" style. Eqns. 3-8 can be written in matrix form as

\[
[A] [X] = [Y]
\]

with

\[A = \begin{bmatrix}
\lambda_{EA} & \lambda_{HB} & 1 & 0 & 0 & 0 \\
\lambda_{EA} & \lambda_{HB} & 0 & 1 & 0 & 1 \\
\lambda_{EA} & \lambda_{HB} & 0 & 0 & 1 & 0 \\
\lambda_{EA} & \lambda_{HB} & 0 & 0 & 0 & 0 \\
\lambda_{EA} & \lambda_{HB} & 0 & 0 & 0 & 0 \\
\lambda_{EA} & \lambda_{HB} & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[Y = \begin{bmatrix}
0 \\
0 \\
0 \\
-2mg \\
0 \\
0
\end{bmatrix}
\]

We can now feed this to a calculator or computer to get a soln.

The following soln. uses MATLAB. The key is the backslash (\) command. Given a matrix \( A \) & a column vector \( Y \)

\[X = A \backslash Y\]

finds the col. vector \( X \) that solves \( AX = Y \) (cautions in Math 144!)

\[x = \begin{bmatrix}
T_1 \\
T_2 \\
R_{Cx} \\
R_{Cy} \\
R_{Cz} \\
R_{Dy}
\end{bmatrix}, \quad [Y] = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]
Find all tensions.

\[ \sum F_y = 0 \Rightarrow T_B = mg. \]

FBD of joint B:

\[ \sum F = 0 \Rightarrow \]

1. \( T_{AB} (\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) + T_{BC} (-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) - mg \hat{j} = 0 \]

2. \( T_{AB} \sin(60^\circ) - T_{BC} \sin(30^\circ) = 0 \)

(1) & (2) \Rightarrow \begin{align*}
T_{AB} &= mg \\
T_{BC} &= -T_{AB} = -736N
\end{align*}

"Tension in BC is -736N, comp. is 736N."

\[ T_{BC} = -T_{AB} \approx -736N \]

% End of m file

\[
\begin{align*}
T_1 &= 346.84N \\
T_2 &= 430.58N \\
R_{XZ} &= 560.57N \\
R_{Y} &= 0N \\
R_z &= 525.54N \\
R_{Y} &= -63.06N \\
R_{X} &= 768.39N
\end{align*}
\]
4.7 (cont'd)

Find all "bar forces".

\[ L = 30 \text{kN} \]

Note:

4 \[ \sqrt{41} \]

FBD of structure

\[ F_y = 0 \] & symmetry

\[ A_y = E_y = 2L \]

Now we start at joint E & work our way down the structure.

4.1 (cont'd)

FBD of joint C:

\[ \sum F_y = (8E)^\uparrow = 0 \]

\[ T_{AC} + T_{BC} \cos 60^\circ = 0 \]

\[ T_{AC} = -T_{BC} \cos 60^\circ \]

\[ = (-736 \text{N}) \frac{\sqrt{3}}{2} \]

\[ T_{AC} \approx 637 \text{N} \]

"tension in AC is 637N"

4.7 (cont'd)

Joint E:

\[ \sum T_E = 2L \]

\[ \sum F_y = 0 \Rightarrow 2L + \frac{T_{DE}}{\sqrt{41}} = 0 \]

\[ T_{DE} = -\frac{\sqrt{41}}{2} \left( \frac{30 \text{KN}}{2} \right) \]

\[ = 96 \text{KN} \]

(Constraint by symmetry)

Tension in DE is 96 KN, compression is 96 KN.

Joint F:

\[ \sum T_F = 60 \text{KN} \]

\[ L = 30 \text{KN} \]

\[ \sum F_y = 0 \Rightarrow T_{DF} = 75 \text{KN} \]

\[ \sum F_x = 0 \Rightarrow T_{FE} = 75 \text{KN} \]

(Constraint by symmetry)

Joint D:

\[ \sum T_D = -15 \sqrt{41} \text{KN} \]

\[ \sum F_y = 0 \Rightarrow -T_{CD} + \frac{T_{DE}}{\sqrt{41}} = 0 \]

\[ T_{CD} = \frac{15 \sqrt{41} \text{KN}}{2} \]

\[ \approx -75 \text{KN} \]

(Constraint by symmetry)

Joint G:

\[ \sum T_G = 5 \frac{\sqrt{41}}{2} \text{KN} \]

\[ \approx 112.5 \text{KN} \]

(Constraint by symmetry)

Finally:

Joint G:

\[ \sum F_y = 0 \Rightarrow \frac{T_{CG}}{60 \text{KN} \times 2L} = 2L \]
That's all folks.