Q1. This is a unit conversion problem, people can do that I guess.

Q2. Basic information you need to understand:

- Motor curve
- Motor gives a torque (moment) = \( M \)
- \( \omega \) is angular velocity = \( \omega \)
- \( M \) and \( \omega \) of a motor are not independent
- If you expect high speed \( \omega \), \( M \) will be small
  - If you expect high torque \( M \), \( \omega \) will be small
- Ideally, this is expressed as
  \[ M = M_0 - C \omega \]
  - Moment \( M = M_0 \) if \( \omega = 0 \) → stalling, Max. Torque
  - Where \( M = 0 \) → \( M_0 - C \omega_0 = 0 \) → no load rpm
  \[ \omega_0 = \frac{M_0}{C} \]

- Muscles, like of the biker, in this problem behave similar to motor → they give more force (torque) at low velocity → slow force at high speed.
gears

- They are used to step up or (more commonly) step down the speed.

\[
\omega_2 = \frac{R_1}{R_2} \omega_1
\]

\[
\begin{align*}
\text{output} & \quad \text{input} \\
\omega_2 & \quad \omega_1 \quad \frac{R_1}{R_2} \quad \frac{\text{step}}{R_2 \neq R_1} \\
\end{align*}
\]

- Hint: if confused between \(\frac{R_1}{R_2}\) or \(\frac{R_2}{R_1}\), just remember smaller gear is faster always.

- Or simply \(\omega_1 R_1 = \omega_2 R_2\)

\[
\begin{align*}
\omega_2 & = \omega_3 \quad \rightarrow \text{on same shaft} \\
\omega_1 R_1 & = \omega_2 R_2 \\
\omega_3 R_3 & = \omega_4 R_4 \quad \rightarrow \omega_4 = \frac{R_1 R_3}{R_2 R_4} \omega_1 \\
\end{align*}
\]

- Note: in all of above, radius of gear can be replaced with number of teeth it has, because for meshing teeth have same thickness and hence more radius, means more circumference, means more number of teeth.
2) If speed is stepped up (or stepped down), torque is stepped down (or up) by same factor. So that
\[ \text{power in} = \text{power out} \]
\[ (M_{\text{in}}, \omega_{\text{in}}) = (M_{\text{out}}, \omega_{\text{out}}) \]

3) Therefore you should picture gear box as told in class:

\[ M_{\text{out}} = M_{\text{in}} \times (\text{same factor}) \]
\[ \frac{\omega_{\text{out}}}{\omega_{\text{in}}} = \text{(some factor)} \]

4) In class for easiness we define gear ratio 'G' as:

\[ \frac{\omega_{\text{out}}}{\omega_{\text{in}}} = G \]
\[ \frac{M_{\text{out}}}{M_{\text{in}}} = G \]

\[ \text{motor} \]

\[ \text{velocity and, friction of wheel} \]

\[ V_{\text{out}} = \frac{W_{\text{in}}}{G} \]
\[ F_{\text{out}} = G \times M_{\text{in}} \]

\[ \text{power in} = M_{\text{in}} \times \omega_{\text{in}} = \text{power out} = F_{\text{out}} \times V_{\text{out}} \]

6) \[ W_{\text{in}}, M_{\text{in}} \rightarrow \text{motor speed & torque} \]
\[ V_{\text{out}}, F_{\text{out}} \rightarrow \text{speed of wheel (hence bike) & force of friction which drives the bike} \]
Q2a  Find 'G' for a real bike.

\[ \omega_2 \cdot r_2 = \omega_{in} \cdot r_1 \]

\[ \omega_2 \] is also angular speed of wheel

\[ V_{out} = \text{velocity of wheel} = \omega_2 \cdot r_3 \]

\[ V_{out} = r_3 \cdot \frac{r_1}{r_2} \cdot \omega_{in} = \frac{\omega_{in}}{G_1} = \frac{\omega_{in}}{\left( \frac{r_2}{r_1} \cdot r_3 \right)} \]

\[ G_1 = \frac{r_2}{r_1} \frac{1}{r_3} \]

\[ = \frac{N_2}{r_1 \cdot r_3} \]

\[ \text{Now plug in your numbers} \]
(Q2.b) - fastest the rider turns crank = \( \omega \) = no load rpm

- peak power of the rider = 0.5 Hp.

\[
\text{Power} = M \omega = (M_0 - C \omega^2) \omega = M_0 \omega - C \omega^2
\]

- power of rider as function of angular velocity

\[
P = 0 \quad \text{when} \quad \omega = 0
\]

\[
P = 0 \quad \text{when} \quad \omega = \omega_f
\]

\[
p = M_0 \left( \frac{M_0}{C} \right) - C \left( \frac{M_0}{C} \right)^2 = 0
\]

at peak, slope = \( \frac{dP}{d\omega} = 0 \)

\[
\frac{d}{d\omega} (M_0 \omega - C \omega^2) = 0
\]

\[
M_0 - 2C \omega = 0 \quad \text{at peak}
\]

\[
\omega_{\text{at peak}} = \frac{M_0}{2C} = \frac{1}{2} \omega_f
\]

\[
M_{\text{at peak}} = M_0 - C \omega_{\text{at peak}} = M_0 - C \frac{M_0}{2C} = \frac{1}{2} M_0
\]

\[
\text{Power at peak} = \omega_{\text{peak}} \times M_{\text{peak}} = \frac{1}{4} M_0 \omega_f
\]

\[
\therefore \quad 0.5 \text{ hp} \times 746 \text{ W} = \frac{1}{4} M_0 \times 180 \text{ rev/min} \times 2\pi \text{ rad} \times \frac{1}{60} \text{ min}
\]
\[ M_0 = \frac{746 \times 0.5 \times 4 \times 60}{180 \times 2\pi} \]

\[ M_0 = 79.153 \text{ Nm} \]

\[ w_f = \frac{M_0}{C} \quad \Rightarrow \quad C = \frac{M_0}{w_f} = \frac{79.153}{\left(\frac{180 \times 2\pi}{60}\right)} \text{ Nms}^2/\text{rad} \]

\[ C = 4.199 \text{ Nms}^2/\text{rad} \]

---

2c) By Calculus

1) Assuming very little (negligible) force on front wheel, because its light compared to (rider + bike) mass and needs very little force to rotate.

(you'll understand more in .203)

2) Only force \( \vec{w}_a \) (external to bike + rider) = \( \vec{F} \) on rear wheel.

3) By Newton's law

\[ F = ma = m \frac{dv}{dt} \]

\[ m = 150 \text{ lbm} = 68.038 \text{ kg} \]

When you pedal, you rotate the rear wheel, if there was no friction it would slip due to friction \( \vec{F}_f \) that opposes this slipping, that the bike moves forward, the wheel rolls.
From before:

\[ F = \frac{GM}{r} \text{ (membrane by bike)} \]

\[ v = \frac{\omega}{G} \text{ (angular speed by bike at crank)} \]

\[ \Rightarrow \quad GM = m \cdot \frac{\omega}{G} = \frac{m}{6} \]

And:

\[ M = M_0 - c \omega = M_0 - c (Gv) \]

\[ F = \frac{md\omega}{dt} = G (M_0 - Gc V) \]

\[ \frac{md\omega}{dt} = GM_0 - G^2 c^2 V \]

But also:

\[ C = M_0 + \frac{M_0}{\omega} \text{ from page 6, } \omega = \frac{4P_{\text{peak}}}{M_0} \text{ from page 5} \]

\[ C = \frac{M_0^2}{4P_{\text{peak}}} \]

\[ \frac{md\omega}{dt} = GM_0 - \frac{G^2 M_0^2}{4P_{\text{peak}}} V \]

New from this differential equation, we can integrate to get \( V(t) \) \( x(t) \) and all good things of life.

Note: you can also use power generated by motor = change of K.E. of bike:

\[ Mw = \frac{d}{dt}(\frac{1}{2}mv^2) \]

This gives the same above differential equation.
\[
\frac{dv}{dt} = \frac{GM_0}{m} - \frac{(GM_0)^2}{\frac{1}{A} - \frac{p_{\text{peak}} m}{B}} \\
V = A - B V
\]

\[
\int \frac{dv}{A - B V} = \int dt \\
\ln \left( \frac{A - B V}{B} \right) = t + C \\
\text{at } t = 0, V = 0 \\
C = \ln \left( \frac{A}{B} \right)
\]

\[
-t \ln \left( \frac{A - B V}{B} \right) = t - \ln A \\
t = \ln A - \ln \left( \frac{A - B V}{B} \right) = \ln \left( \frac{A}{A - B V} \right) = \ln \left( \frac{1}{A - B V} \right)
\]

\[
t = -\ln \left( \frac{1 - B/AV}{B} \right)
\]

\[
\left(1 - \frac{B}{A} V\right) = e^{-Bt}
\]

\[
V = \frac{A}{B} (1 - e^{-Bt})
\]

\[
V = \frac{q_{\text{peak}}}{G M_0} \left(1 - e^{-\frac{(GM_0)^2}{q_{\text{peak}} m}} \cdot t\right)
\]

\[
\text{new } m \text{ can plot this} \\
\text{using } p_{\text{peak}} = 0.5 \times 746 \text{ W} \\
M_0 = 79.153 \text{ Nm} \\
C_t = \text{whatever you had in 2a} \\
\left(\frac{1}{m}\right) \\
m = 68.038 \text{ kg}
\]
\[ v = \frac{dx}{dt} = \frac{y P_{\text{peak}}}{GM_0} \left( 1 - e^{-\frac{(GM_0)^2}{4ymP_{\text{peak}}} t} \right) \]

\[ Sdx = \frac{y P_{\text{peak}}}{GM_0} \left[ \int \left( 1 - e^{-\frac{(GM_0)^2}{4ymP_{\text{peak}}} t} \right) dt \right] \]

\[ x = \frac{y P_{\text{peak}}}{GM_0} \left[ t + e^{-\frac{(GM_0)^2}{4ymP_{\text{peak}}} t} \right] + C \]

\[ x = 0 \quad \text{at} \quad t = 0 \]

\[ 0 = \frac{y P_{\text{peak}}}{GM_0} \left( \frac{4ymP_{\text{peak}}}{(GM_0)^2} \right) + C \]

\[ x = \frac{y P_{\text{peak}}}{GM_0} \left[ t + \frac{-\frac{(GM_0)^2}{4ymP_{\text{peak}}} t}{\frac{(GM_0)^2}{4ymP_{\text{peak}}}} - 1 \right] \]

Plot this using MATLAB.

It looks like a line.

If you are plotting over a small time range, it might look like a line.
\[ t = -\ln \left( 1 - \frac{G M_0}{4 \rho_{\text{peak}} V} \right) \]
\[ \frac{(G M_0)^2}{4 \rho_{\text{peak}} m} \]

plug in \( V = 20 \text{ mph} = \frac{20 \times 1609.3 \text{ m}}{3600 \text{ sec}} = 8.94 \)

get \( t \)

your answer depends on \( G \)

\[ \text{Q2c iv} \]

use above \( t \) and plug it in the distance relation on page 9.

\[ \text{Q2c} \]

\text{using MATLAB}

\text{idea:}

1. we need to find \( x \) and \( V \) for all times.
2. we know their rate of change, in terms of each other.
\[ \dot{x} = V \]
\[ \dot{V} = a = F/m = \frac{G M_0}{m} - \frac{G^2 M_0^2}{4 m \rho_{\text{peak}}} V \]

\text{conceptualize visualize}

1. define a vector \( z \) of two numbers, let first be \( x \) and second be \( V \)
\[ z(1) = x ; \quad z(2) = V ; \]
\[ z = [ ; ] \]
define a function which takes in two numbers in form of a vector \( \mathbf{Z} \) and return the rate of change of both numbers in a vector (let’s say \( \mathbf{Z}_{\text{dot}} \))

```matlab
function \( \mathbf{Z}_{\text{dot}} = \text{myfun}(t, \mathbf{Z}) \)

% a number \( t \) and a vector \( \mathbf{Z} \) is given to this function
% we need to find \( \mathbf{Z}_{\text{dot}} \) just using that info.
% to make it more intuitive extract and rename the 4 numbers from \( \mathbf{Z} \).
\( x = \mathbf{Z}(1); \)
\( v = \mathbf{Z}(2); \)

% to find their rate of change we need \( G, m, M_0, P_k \).
\( G = \ldots \); % put it here
\( m = 68.038; \)
\( M_0 = 79.153 \Omega; \)
\( P_k = 0.5 \times 746; \)

% define rate of change in terms of some new variables
\( x_{\text{dot}} = v; \) % \( v \) is defined above.
\( v_{\text{dot}} = G*\frac{M_0}{m} - (G*\frac{M_0}{m})^2/(4*m*P_k) = v \);
% reback these into a vector which tells rate of change of numbers in \( \mathbf{Z} \), that what we want
\( \mathbf{Z}_{\text{dot}} = [x_{\text{dot}}; v_{\text{dot}}]; \)

end.
```
new write a main function which uses this info

to solve ode we need
1) a function which defines it (myfun)
2) initial values of the two numbers (zzero)
3) time you want to find solution for.
   call it tsplan.

```
fuction ode.

tspan = [0 100]; % put a big number like 100 hopefully
zzero = [0; 0]; % v will reach 20 mph during that
                % otherwise increase it.

[t zarray] = ode45 (@myfun, tspan, zzero);
% this solves the ode ie finds the
% values of both number in Z for
% various times between 0 and 100
% and stores them in [t zarray]
% which is
%        [0 0 0] % t=0  Z = [0; 0];
%                100 100 100 % t=100  Z = [some; some; some];
%                  2array

% extract the x data and v data from zarray. These are vectors
% containing x and v values at times in t.

x = zarray(:, 1); % all rows and 1st column of zarray
v = zarray(:, 2); % 2nd...

plot (t, v); % for part 2c i
plot (t, x); % " " 2c ii

% for parts 2c iii and 2c iv you can manually zoom
% in these plots and get time where v = 8.94889 m/s and
% then find x for this time by zoom in on x plot
% otherwise we can interpolate as follows.
% vdata contains velocity only for selected times specified in 't'
% chosen by ode 45 bet 0 and 100 automatically.
```
// it probably won't have a number. 8.94, so we
// can find a number just below 8.94 and just above it
// and interpolate to get \( x \) for 8.9408

\[ i = 0; \]
\[ n = \text{length}(t); \quad \% \text{the size of } t, v, x \text{ vectors.} \]

\[ \text{while } (v(i) < 8.9408) \]
\[ i = i+1; \]
\[ \text{end.} \]

\[ /\text{ at the end of while } i = \text{number such that } v(i) \text{ is just} \]
\[ /\text{ greater than 8.9408.} \]

\[ \text{display ('t for } v = 20 \text{ mph is ')} \]
\[ T = t(i-1) + \left( \frac{t(i) - t(i-1)}{v(i) - v(i-1)} \right) \times (8.9408 - v(i-1)) \]
\[ \text{display ('the distance at that time')} \]
\[ x = x(i-1) + \left( \frac{x(i) - x(i-1)}{v(i) - v(i-1)} \right) \times (8.9408 - v(i-1)) \]
\[ \text{end} \]

\[ \text{without all the comments code is still small} \]

\[ \text{function code:} \]
\[ [t zarray] = \text{ode45 ('myfun', [0 10], [0;0]);} \]
\[ x = \text{zarray}(:,1); \quad v = \text{zarray}(:,2) \]
\[ \text{plot (t,v); plot (t,x);} \]
\[ T = \text{copy from above}; \quad x = \text{copy from above}; \]
\[ \text{end} \]

\[ \text{function } zdet = \text{myfun}(t,z) \]
\[ zdet = [z(2); (G * 79.15318038 - (\text{z(1) 79.153}^2) / (\text{z(2) 68.038}^2 * 5.746) - \text{z(2)})]; \]
\[ \text{end.} \]
Define a function which takes in two numbers, the vector \( z \) and time \( t \), and returns the rate of change of these, at that particular \( t \), called \( \text{myrhs} \).

```matlab
function zdot = myrhs(t, z)

% a time number and a vector is given to this
% lets extract x and v from z, just to make it intuitive
x = z(1);
v = z(2);

% find their rate of change
xdot = x^2 * v;
vdot = x * v^2;
```