" Solutions "

ab TA name, and Lab Section day:

Your name: ANOOP GREWAL

Cornell ENGRD 1170 Quiz 1

No calculators, books or notes allowed.

October 15, 2009

50 minutes total. Your grade will be based on the best 2 answers. So you only need to work on 2 problems. No notes. No calculators. No text messaging.

Directions. To ease your TA's grading and to maximize your score, please:

- Draw **Free body diagrams** whenever you use balance of forces or balance of moments.
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and box in your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "θ_7 = 18" instead of, say, "`theta7= 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- □ Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

1) ______ /80

2) ______ /80

3) ______ /80

TOTAL ______ /160

1a) Given d and X you want to know the area under the curve $e^{-(x/d)^2}$ in the interval $0 \le x \le X$. That is, you want to know

$$F(X) = \int_0^X f(x)\,dx \qquad \text{where} \qquad f(x) = e^{-(x/d)^2}$$

My guess is that you don't know a function $F(x)$ whose derivative is $f(x)$.

So you probably have to settle for an approximate answer using, say, Matlab.

Write the text of a matlab function

`myfun`

so that when you type

```
>> d = 3;   X = 5;   n = 100;
>> myfun(d, X, n)
```

at the command line you get an approximation for the integral

$$F(X) = \int_0^5 e^{-(x/3)^2}\,dx.$$

Make it so the approximation is better if n is bigger. You may use any method you like using any Matlab commands that work.

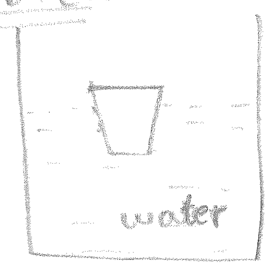
```
function area = myfun(d, X, n)
Xrange = linspace (0, X, n+1);   % divide 0-X interval
                                 % into n intervals
width = X/n;                     % width is distance bet
                                 %   2 points
Yrange = exp(-(Xrange / d).^2);  % function values
                                 % at all points
area = sum (Yrange) * width;     % in the Xrange
end
```

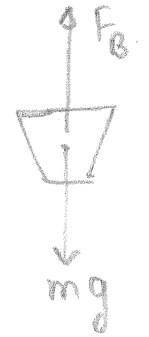
1b) What should be the name of the file where you put your function `myfun`?

myfun.m

2) Water has a density of 1 gram/cm^3. A weighted cork (mass m, volume V) floats in the water 90% submerged (with 10% of its volume above the water surface). Oil is poured on top, submerging the cork. Then the cork floats with half of its volume in the water and half in the oil. *What is the density of the oil?*

WITH WATER





CORK 90% submerged

F_B = buoyant force of water = $V_{submerge}$ × density water (weight)

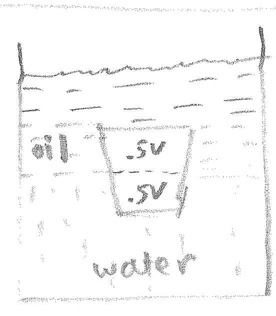
$= (0.9 \cdot V) \rho_w g$

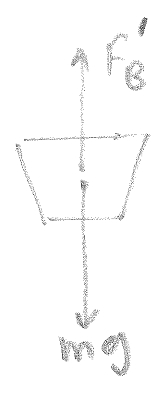
at equilibrium by LMB ← Linear Momentum Balance (Force Balance in statics)

$F_B = mg$

$$0.9 \cdot V \rho_w g = mg \quad \text{— (1)}$$

WITH WATER & OIL





F_B' = buoyant force of water and oil

= $V_{submerged\ in\ water}$ × density of water + $V_{in\ oil}$ × density of oil

$$= (0.5 V \rho_w + 0.5 V \rho_{oil}) g$$

at equilibrium by LMB

$F_B' = mg$

$0.5V(\rho_{water} + \rho_{oil})g = mg = .9V\rho_w g$ (from ①)

$\Rightarrow 0.5(\rho_w + \rho_{oil}) = 0.9\rho_w$

$0.5(1 + \rho_{oil}/\rho_w) = 0.9$

$.5\rho_{oil}/\rho_w = .9 - .5 = .4$

$\rho_w = 1 g/cm^3 \Rightarrow \boxed{\rho_{oil} = \frac{4}{5} g/cm^3}$

3) Balsa wood is used to make light-weight fragile models. High strength steel is used to make sturdy stiff buildings and machines.

Say you have two beams which have

- the same length and
- the same weight and
- the same load is applied in the middle (simply supported beam, as in the lab).

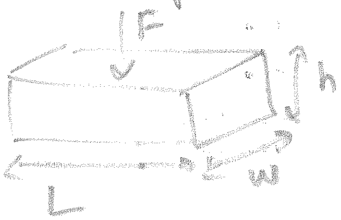
Both are solid (not hollow) long and narrow with square cross sections. One beam is made of balsa wood. One is made of high strength steel.

Use these numbers (in the range of real available materials).

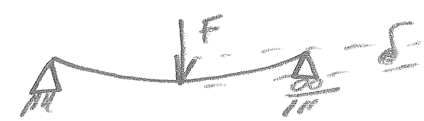
$$\begin{aligned}
\rho_b &= 0.16 \quad \text{gm/cm}^3 &&= \text{density of balsa wood.}\\
\rho_s &= 8 \quad \text{gm/cm}^3 &&= \text{density of high strength steel} = 50\,\rho_b.\\
E_b &= 4\cdot 10^9 \quad \text{N/m}^2 &&= \text{elastic (Young's) modulus of balsa wood.}\\
E_s &= 200\cdot 10^9 \quad \text{N/m}^2 &&= \text{elastic (Young's) modulus of high strength steel} = 50E_b.
\end{aligned}$$

How much bigger or smaller is the deflection in the beam made of balsa wood than the beam made of steel?

deflection = $\delta = \dfrac{\text{Force in middle} \times \text{length}^3}{4 \times \text{Elastic modulus} \times \text{width} \times \text{thickness}^3}$



$\delta = C\dfrac{F L^3}{E w h^3}$ (C = 1/4, but the answer does not depend on this)



$\delta_b = C\dfrac{F_b L_b^3}{E_b w_b h_b^3}$ b=balsa $\quad \delta_s = C\dfrac{F_s L_s^3}{E_s w_s h_s^3}$ —— (1) s=steel

- $F_b = F_s$, $L_b = L_s$
- $E_b = \frac{1}{50} E_b$
- $w_b = h_b$, $w_s = h_s$ (square crossections)

also weight of balsa = weight of steel beam

$\rho_b L_b w_b^2 = \rho_s L_s w_s^2$ $\quad (\rho_s = 50\,\rho_b)$

$\Rightarrow w_b^2 = 50\, w_s^2$ —— (2)

from (1)

$$\frac{\delta_b}{\delta_s} = \overbrace{\left(\frac{E_s}{E_b}\right)}^{50}\frac{w_s h_s^3}{w_b h_b^2} = 50\,\frac{w_s^4}{w_b^4} = 50 \times \frac{1}{50^2} = \frac{1}{50}$$

other cancell out for being equal

from (2)

balsa deflects 50 times less than steel