Cornell TAM 2030
Prelim 3
April 14, 2009

Directions. To ease your TA's grading and to maximize your score, please:

\* \* Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.

\* Use correct **vector notation**.

\+ Be (I) neat, (II) clear and (III) well organized.

\+ TIDILY REDUCE and [box in] your answers (Don't leave simplifiable algebraic expressions).

\+ Make appropriate Matlab code clear and correct.

You can use shortcut Matlab like "\( \dot{\theta} = 18 \)" instead of, say, "\( \theta_{\text{dot}} = 18 \)."

Small syntax errors will have small penalties.

\+ Clearly **define** any needed dimensions \( (l, h, d, \ldots) \), coordinates \( (x, y, r, \theta, \ldots) \), variables \( (v, m, t, \ldots) \), base vectors \( (i, j, e_r, e_\theta, \lambda, \hat{n}, \ldots) \) and signs \( (\pm) \) with sketches, equations or words.

\+ **Justify** your results so a grader can distinguish an informed answer from a guess.

\* If a problem seems **poorly defined**, clearly state any reasonable assumptions (that do not oversimplify the problem).

\* Work for **partial credit** (from 60–100\%, depending on the problem)
  
  \- Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.

  \- Reduce the problem to a clearly defined set of equations to solve.

  \- Provide Matlab code which would generate the desired answer (and explain the nature of the output).

\+ Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.

Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 7: ___/25

Problem 8: ___/25

Problem 9: ___/25
7) A thin-walled pipe with mass $m$ and radius $r$ rolls back and forth in a trough with radius $R$. Assuming small oscillations what is the period of oscillation. Answer in terms of some or all of $r, R, g$ and $m$.

**Kinematics**

\[
\vec{V}_G = \vec{V}_C
\]

\[
\dot{\theta} = \frac{r}{R-r} \\
\dot{\omega} = -\omega = -\omega_{pipe}
\]

\[
\Rightarrow \quad \dot{\omega} = -\frac{\omega_{pipe} r}{R-r}
\]

**AMB_c**:

\[
\Sigma \vec{M}_C = \vec{H}_C
\]

\[
mgrsin\theta \hat{k} = \vec{V}_C \times m\vec{\omega}_C + I_C \omega \hat{k}
\]

\[
L(-r \hat{e}_r) \vec{a}_C = -(R-r) \dot{\theta} \hat{e}_r + (R-r) \ddot{\theta} \hat{e}_\theta
\]

\[
mgrsin\theta \hat{k} = -r(R-r) \omega \dot{\theta} \hat{k} + mr^2 \omega \hat{k}
\]

\[
\begin{cases}
g\cos\theta \hat{k} = -x(R-r) \omega \dot{\theta} \hat{k}
\end{cases}
\]

\[
\begin{align*}
\dot{\theta} + \frac{g}{2(R-r)} \theta &= 0 \\
\Rightarrow \quad \theta &= A \cos\left(\sqrt{\frac{g}{2(R-r)}} t - \phi\right)
\end{align*}
\]

**Period**

\[
T = \frac{2\pi}{\sqrt{\frac{2g}{R-r}}}
\]
8) A rectangular plate $P$ rotates with constant counter-clockwise angular velocity $\omega_p$ about the point O marked. A bug walks on the plate with constant speed $v$, relative to the plate, on the dotted circle shown (radius $r$, with center a distance $R$ from O). At the instant of interest the center of the circle and the bug are both directly to the right of O.

a) What is the velocity (a vector) of the bug at this instant?

b) What is the acceleration (a vector) of the bug at this instant?

\[ \vec{V}_b = \vec{V}_{0} + \vec{V}_p + \vec{\omega} \times \vec{V}_{10} \]

\[ \vec{V}_p = \vec{V}_{\text{bug}} \]

\[ \vec{a}_b = \vec{a}_{0} + \vec{a}_p - \omega_p^2 \vec{r}_{10} + \vec{\rho} \times \vec{V}_{10} + 2 \omega_p \times \vec{V}_p \]

\[ \vec{a}_b = \left[ -\omega_p^2 (R+r) - \frac{V^2}{r} - 2 \omega_p V \right] \hat{i} \] (b)

 Sanity check: when $R=0 \Rightarrow \vec{a}_b = -\left( \omega_p + \frac{V}{r} \right)^2 r \hat{i}$ = the net "$F"
8b) Assume \( r \) and \( \theta \) are measured in the standard way relative to an \( xy \) coordinate system. A particle motion is described with polar coordinates with

\[
\vec{r} = r_0 \cos \theta \quad \text{and} \quad \dot{\theta} = \omega = \text{constant}.
\]

We are interested in the instant that the particle passes through the \( x \) axis at \( \vec{r} = r_0 \hat{e}_x = r_0 \hat{i} \). Answer in terms of some or all of \( r_0, \omega, \hat{i} \) and \( \hat{j} \).

a) What is the velocity of the particle at this instant?
b) What is the acceleration of the particle at this instant?
c) What is the radius of curvature of the particle path at this instant?

\[
\overrightarrow{V} = \vec{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta = \left[ \begin{array}{c} r_0 \omega \hat{j} = \overrightarrow{V} (t=0) \end{array} \right] (a)
\]

\[
\hat{e}_r = \hat{i}, \hat{e}_\theta = \hat{j}, \theta = 0
\]

Take \( \theta = \omega t \) so at \( t=0 \) \( \vec{p} \) is on \( x \) axis

\[
\overrightarrow{V} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta = \left[ \begin{array}{c} r_0 \omega \hat{j} = \overrightarrow{V} (t=0) \end{array} \right] (a)
\]

\[
\dot{r} = -r_0 \sin \theta \dot{\theta}
\]

\[
\hat{a} = \left( \hat{r} - 2 \hat{\theta} \right) \hat{e}_r + \left( \hat{r} \dot{\theta} - 2 \hat{\theta} \dot{\theta} \right) \hat{e}_\theta = \left[ \begin{array}{c} -2 r_0 \omega^2 \hat{i} = \overrightarrow{a} \end{array} \right] (b)
\]

Check:

\[
|\overrightarrow{a}| = \frac{r_0^2 \omega^2}{2r_0}
\]

\[
\text{consistent with circular shape}
\]

\[
\hat{a} = \frac{\overrightarrow{V}^2}{r_0} \hat{e}_n = \left( \frac{r_0 \omega}{r} \right)^2 \left( -\hat{i} \right) (c)
\]

\[
\hat{a} = \frac{\overrightarrow{V}^2}{r_0} \Rightarrow \frac{(r_0 \omega)^2}{r_0} = 2 r_0 \omega^2 \Rightarrow \frac{r_0 \omega^2}{r_0} = 2 \frac{\omega^2}{r_0} \Rightarrow r_0 = \frac{r_0}{2}
\]
9) A rigid cart (mass $m$, moment of inertia $I^G$) with light well-lubricated wheels is rolling on level ground at constant speed $v_0$ when the front wheel suddenly gets completely stuck against a curb. Just after this collision what is the velocity of $G$? Answer in terms of some or all of $v_0$, $m$, $I^G$, $d$, $h$ and $g$.