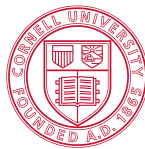
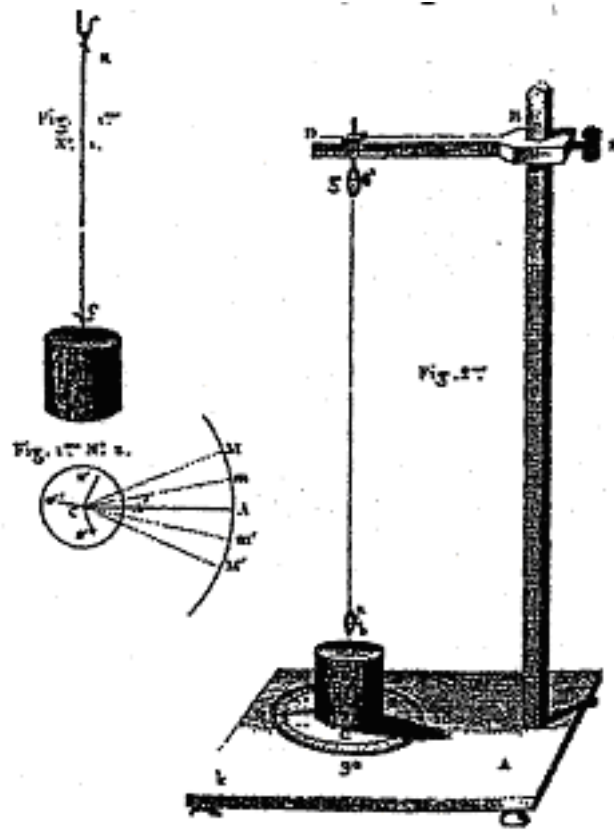


# TAM 203 Lab Manual



Cornell University  
Theoretical and Applied Mechanics

This manual has evolved over the years. Contributors in the past two decades include: Kenneth Bhalla, David Blocher, Jason Cortell, Drew Eisenberg, Jill Evensizer, Kwang Yul Kim, Richard Lance, Jamie Manos, Francis Moon, Dan Mittler, James Rice, Kevin Rompala, Andy Ruina, Bhaskar Viswanadham, and Alan Zehnder

# Contents

TAM 203 Lab Introduction	5
Lab #1 - One Degree-of-Freedom Oscillator	11
Lab #2 - Two Degrees-of-Freedom Oscillator	31
Lab #3 - Slider-Crank Lab	45
Lab #4 - Gyroscopic Motion of a Rigid Body	55

---

# TAM 203 Lab Introduction

Last Updated: January 22, 2008

---

## PURPOSE

The laboratories in dynamics are designed to complement the lectures, text, and homework. They should help you gain a physical feel for some of the basic and derived concepts in dynamics: *force*, *velocity*, *acceleration*, *natural frequency*, *resonance*, *normal modes*, and *angular momentum*. You will also get exposure to equipment and computers which you may use in the future. Some mathematics from courses you have taken recently or are now taking will be used. We hope this will help you make the connection between mathematics and physical reality that is essential to much of engineering. The labs may come either before or after you cover the relevant material in lecture. Thus, they can be either a motivation for the lecture material or an application of what you have learned depending on the timing.

## COURSE INFORMATION

There are four dynamics laboratories you will be performing during the semester:

1. One Degree-of-Freedom Oscillator
2. Two Degrees-of-Freedom Oscillator
3. Slider-Crank Mechanism
4. Gyroscopic Motion of a Rigid Body

Each of the four labs is taught for two or three weeks (depending on enrollment) in Thurston 101. You will be scheduled to attend lab during one of the weeks. The dates for your laboratory section will be posted outside Thurston 101 and on the course website. In general, you will have a lab once every two or three weeks, but be aware that this may vary due to exam and break schedules.

**NOTE: See the Administrative Assistant in Kimball 212 if you have any problems with your lab schedule. You'll need to get his or her approval for any changes so that the lab sections do not become overly full. Turning in a course change form to the registrar is not enough.**

## LABORATORY ATTENDANCE

You are expected to attend the lab section you have signed up for. In the event of an **excused** absence you must make-up the lab. **All make-up labs must be arranged with your TA.** Your options for making-up labs are

1. Attend another of your lab TA's lab sections.

2. Attend another lab TA's lab section (requires permission from both lab TAs).
3. Attend the "Lab Make-Up Section" during the final week of the semester. Information regarding the date and time of this section will be given in lecture near the end of the semester.

If you show up for lab after it is under way, your lab instructor may ask you to leave and to perform the lab another time.

### REQUIRED LABORATORY WORK

The laboratories will be done with physical equipment and some will also involve computer simulations. **It is essential that you read through the lab (especially the procedure section) before coming to lab.** It is not necessary that you understand all of the material perfectly before the lab period.

### Prelab Questions

Each lab has prelab questions to be answered before you come to lab. These questions encourage you to review necessary theory and read through the laboratory procedure before attending the lab. Answers to prelab questions are due at the beginning of lab and will not be accepted for credit later.

### Laboratory Notes

A rule of laboratory work is to keep a neat, complete record of what has been done, why it was done, how it was done, and what the result was.

The success or failure of an experiment in a research laboratory often depends critically upon the record made of the experiment. The outcome of a poorly documented experiment becomes a matter of personal recollection, which is not reliable enough to serve as a basis for further work (especially by someone else). You should take copious notes. If in doubt, **write it down**. One can ignore what is written, but one can not resurrect that which was never recorded. Similarly, **never** erase in your lab notes. If an erroneous reading was made, strike it out with a single line and record the new data. You may later decide that it was not in error.

**All lab notes, signed by your lab TA and in their original form, must be stapled to the back of your final lab report.**

### Lab Report

Your laboratory report should be typed using a word processor. This report should communicate clearly and convincingly what was demonstrated or suggested by the lab work. Your TA is looking for evidence of thought and understanding on your part. Your logic and methods are as important as results or "correct" answers. It is essential that you provide information and calculations which indicate how you arrived at your conclusions. It is permissible (and a good idea if you want a very good grade) to discuss observations and material relevant to

the lab which is not specifically asked about in the questions.

Each report must begin with a cover page containing the following (with appropriate substitutions for the words in quotes):

**“NAME OF THE LAB”**

**TAM 203**

**By:** “Your name and your signature (both partners if a joint report)”

**Performed:** “Date”

**Performed with:** “Name of person(s) with whom you performed the lab”

**Discussed lab with:** “Names of people with whom you discussed the lab, and nature of the discussions”

**TA:** “Lab TA’s name”

**TA signed the data on page:** “Page #”

It is a good idea to include an introduction, abstract, or overview of the laboratory work you performed as this will help communicate that you successfully grasp the purposes and goals of the lab. It also gives you an opportunity to review your laboratory work before answering specific questions asked in the manual. If you deviate from the procedure specified in the manual you should also state how and why you did so here.

You should concisely answer the questions that are asked and number them as they are numbered in the lab manual. Include any necessary plots, data, or calculations (make sure to include the correct dimensional units). Your answers should be self-contained and presented in an orderly fashion (i.e. the reader of the report should not have to refer back to the questions that are asked, nor should he or she have to hunt through the report to find your answers). While many questions require that you perform calculations, written explanations of what you are doing and diagrams can be very helpful. Show all calculations that you perform in arriving at your answers. If you are performing repetitive calculations you need show only one sample calculation.

Finally, at the end of your lab report you may want to include any observations, mistakes you made, or suggestions you have in a concluding section.

When answering questions, percentage difference calculations can be used to quantify how well experimental results agree with theoretical or expected values. Rather than writing “the experimental results agree very well with the theoretical calculations,” this phrase can be changed to make a quantifiable statement; “the experimental results are within 5 percent of the theoretical calculations.” Percentage difference is calculated as:

$$100\% \times (\text{Value being compared} - \text{Reference value}) / (\text{Reference value})$$

While formal error analysis can be used if it is necessary to make a point, your answers should include some discussion of the types and relative sizes of errors in your data.

All plots included with your lab report should be done on the computer using MATLAB (preferred) or Excel. Below are some guidelines for producing quality plots:

- All graphs should be titled and all axes labeled, with the appropriate units listed in parentheses.
- The independent variable should be placed on the horizontal axis.
- Numerical values on the axes should be set at reasonable intervals and scales chosen so that all of the data points can be displayed on the graphs.
- Curves should not be drawn between discrete data points unless the type of fitting used is explained and the equation of the curve given.
- On graphs with more than one curve a legend should be used to identify the curve. Data points can be enclosed by some symbol (i.e. circle, rectangle, etc.) to distinguish different data sets.

Figure 0.1 is an example of how your graphs should appear. The MATLAB code that produced the graph is given below:

```
t = linspace(0,10,1000);
x = 5*cos(2*t);
v = -10*sin(2*t);

figure(1); hold on;
plot(t,x,'b','LineWidth',2);
plot(t,v,'r--','LineWidth',2);
grid on;

plot_title = title('Plot of Position and Velocity vs. Time for Harmonic Oscillator');
x_axis_label = xlabel('Time (sec)');
plot_legend = legend('Position (m)','Velocity (m/s)');

hold off;

set(plot_title,'FontWeight','bold','FontSize',12);
set(x_axis_label,'FontWeight','bold','FontSize',12);
set(plot_legend,'FontWeight','bold','FontSize',12);
set(gca,'FontWeight','bold','FontSize',12);
```

For help with producing log-log and semi-log plots with MATLAB, type `help loglog`, `help semilogx`, or `help semilogy` in the main MATLAB window.

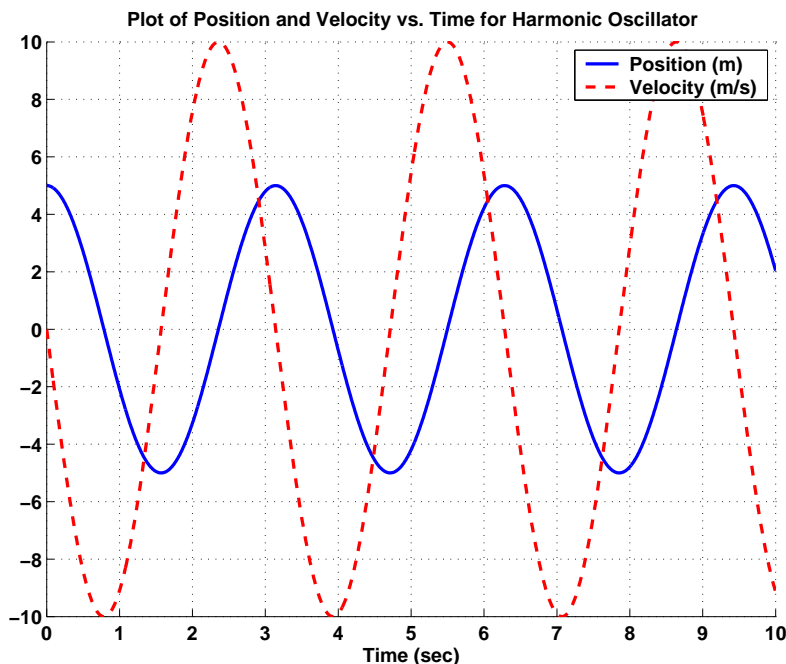


Figure 0.1: An example graph.

## CREDIT AND GRADING

Lab reports are due at 10:00 AM one week from the day you performed the lab unless your TA specifies another time. Turn in reports in the boxes in the Don Conway room, Thurston 102. Put your report in the correct box corresponding to the TA in charge of your **laboratory** section. Reports placed in incorrect boxes might not be found.

Each laboratory is graded out of 15 points. The grade breakdown for each lab report will be determined by your lab TA. This grade will be given to your recitation TA.

## ACADEMIC INTEGRITY

Your pre-lab answers and lab reports should be in your own words, based on your own understanding and your own calculations. You are encouraged to discuss the material with other students, friends, TAs, or even faculty. **Any help you receive from such discussions must be acknowledged on the cover of your lab report, including the name of the person or persons and the exact nature of the help.** Violations of this policy will be reported to the academic integrity board.

You may, however, do a joint report with your lab partners (turn in one report for your lab group). All partners get the same grade on the report but separate grades on pre-lab questions.

When you are done in the lab you must have your TA sign one of your data sheets. This sheet must include the name of your lab partners and the time and date the lab was performed.

The TA will not sign this sheet until your work station is clean and all equipment is accounted for. **No lab reports will be accepted without this signed sheet.**



---

# Lab #1 - One Degree-of-Freedom Oscillator

Last Updated: January 20, 2009

---

## INTRODUCTION

The mass-spring-dashpot is the prototype of all vibrating or oscillating systems. With varying degrees of approximation, car suspensions, violin strings, buildings responding to earthquakes, earthquake faults themselves, and vibrating machines are modeled as mass-spring-dashpot systems. This laboratory is aimed at demonstrating some of the basic concepts of the mass-spring-dashpot system. In this lab you will collect data on the motion of two different mass-spring-dashpot systems, and then use computer generated solutions of the equations of motion to determine system parameters. Phrases connected with some of the key ideas are: *natural frequency*, *resonance*, *forcing function*, and *frequency response*.

## PRELAB QUESTIONS

Read through the laboratory instructions and then answer the following questions:

1. Find the general solution to (1.3) if the forcing term is given by  $F_s(t) = 0$  and there is no damping ( $c = 0$ ), i.e.  $m\ddot{x} + kx = 0$ .
2. Repeat #1, this time numerically integrating the equation using *Matlab*. Choose  $m = 1$ ,  $k = 5$ , and integrate over the time period  $0 \leq t \leq 10$ . Assume the mass starts from rest with an initial displacement of  $x(0) = 1$ . What is the period of the oscillation? Turn in a plot and an m-file of your code.
3. Define in your own words: *natural frequency*, *damping coefficient*, *underdamped*, *overdamped*, *resonance*, and *phase-shift*.

## THE MASS-SPRING-DASHPOT SYSTEM

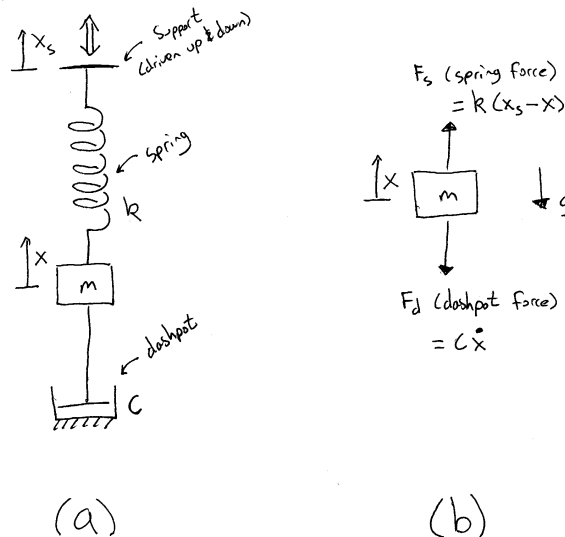
The picture in Figure 1.1a shows a mathematical model of the laboratory mass-spring-dashpot, or one degree-of-freedom oscillator. A mass is supported by a spring and is constrained to move in the  $\hat{\mathbf{e}}_x$ -direction. In this lab you will record the vertical motion of the mass both with a fixed support (*free vibration*) and with the support oscillating vertically (*forced vibration*). The spring is modeled as linear, i.e. the force it applies is proportional to its increase in length with proportionality constant  $k$ . The damping is also modeled as linear, i.e. the force transmitted by the dashpot is proportional to the rate at which it is being stretched with proportionality constant  $c$ . The vertical displacement of the mass is  $x(t)$  and the vertical displacement of the support is  $x_s(t)$ .

Pictured in Figure 1.1b is a free body diagram of the mass. Neglecting gravity (*Why can we neglect it?*), the mass has two forces acting on it in the  $\hat{\mathbf{e}}_x$ -direction:

$$F_{sp}(t) = k(x_s - x) \tag{1.1a}$$

$$F_d(t) = c\dot{x} \quad (1.1b)$$

where  $F_{sp}(t)$  is the spring force and  $F_d(t)$  is the damping force. The system is a one degree-of-freedom system because a single coordinate is sufficient to describe the complete state of the system. (The support displacement  $x_s(t)$  does not count as a degree of freedom since it is specified by the motor position and is thus considered to be a given.) From Newton's



**Figure 1.1:** Model and free body diagram of the mass-spring-dashpot system.

second law the equation of motion for this system is

$$\left\{ \sum \underline{\mathbf{F}} \right\} \cdot \hat{\mathbf{e}}_x \Rightarrow -F_d + F_{sp} = m\ddot{x} \quad (1.2)$$

Plugging in the spring term (1.1a) and a damping term (1.1b), and rearranging terms this becomes

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \quad (1.3)$$

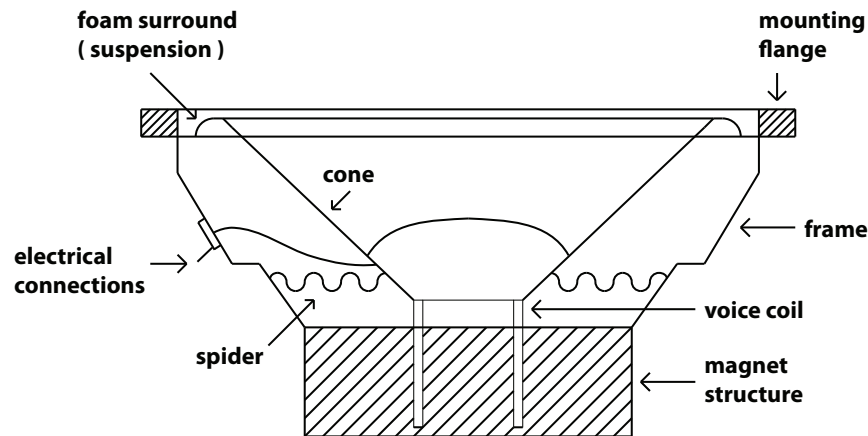
where  $F_s(t) = kx_s(t)$  is the specified “forcing function”. In this case the forcing function is the amount the spring is additionally stretched due the support motion multiplied by the spring stiffness.

In the first part of this experiment you will attempt to determine the value of the viscous damping constant  $c$  by measuring the rate at which oscillations decay towards zero, an experiment called a “ring-down test”. In addition, the system response to both *free vibration* and *forced vibration* will be observed experimentally and through computer simulation.

### A REAL-WORLD EXAMPLE: THE LOUDSPEAKER

A speaker, similar to the ones used in many home and auto speaker systems, is one of many devices which may be conveniently modeled as a one degree-of-freedom mass-spring-dashpot

system. The one you will observe in this lab is typical (see Figure 1.2). It has a plastic cone supported at the edges by a roll of plastic foam (the surround), and guided at the center by a cloth bellows (the spider). It has a large magnet structure and (not visible from outside) a coil of wire attached to the point of the cone which can slide up and down inside the magnet. When you turn on your stereo, it forces a current through the coil in time with the music, causing the coil to alternately repel and attract the magnet pushing the cone up and down in its housing. This results in the vibration of the cone which you hear as sound.



**Figure 1.2:** Cross-sectional view of a speaker.

A simplistic view is that the cone and coil provide inertia ( $m\ddot{x}$ ), the foam surround and cloth bellows act as a spring ( $kx$ ), viscous damping comes from the cone moving through the air ( $c\dot{x}$ ), and the magnet provides external forcing  $F_s(t)$ . Putting it all together we get the familiar equation of motion of a driven mass-spring-dashpot system:

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \quad (1.4)$$

Thus by appropriately choosing the parameters  $m, k, c$  and  $F_s(t)$  we can model the motion of the speaker as a mass-spring-dashpot system.

### SOLVING THE EQUATIONS OF MOTION

Our goal is to find the motion of the mass,  $x(t)$ , for a given forcing function  $F_s(t)$ . The two most important cases are *free vibration*, where  $F_s(t) = 0$ , and *sinusoidal forcing*, a specific case of *forced vibration* where  $F_s(t) = kx_s(t) = kA_{support} \cos \omega t$ , where  $A_{support}$  is the vibration amplitude of the support.

Recall that the differential equation governing the motion (1.3) is given by

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \quad (1.5)$$

From ordinary differential equation theory we can write the general solution to (1.5) as the sum of a complimentary (also referred to as the transient or homogeneous) solution  $x_c(t)$  and a particular solution,  $x_p(t)$ .

$$x(t) = x_c(t) + x_p(t) \quad (1.6)$$

The homogeneous portion  $x_c(t)$  is the solution to (1.5) with  $F(t) = 0$  (and appropriate initial conditions). In this case,  $x_c(t)$  goes to zero as  $t \rightarrow \infty$  - any initial motion of the mass will eventually be damped out if there is no external forcing. Thus the particular solution  $x_p(t)$  is what is left as  $t \rightarrow \infty$  for any initial condition and includes the information about forcing.

In this section we are concerned with unforced vibrations, so we have  $x(t) = x_c(t)$ . We will deal with  $x_p(t)$  later. As you may have seen in other courses, we posit the solution to be of the form  $x_c(t) = Ae^{\lambda t}$  (if this process seems unfamiliar to you, please review differential equations). When we insert this into (1.5), we obtain the characteristic equation,

$$m\lambda^2 + c\lambda + k = 0 \quad (1.7)$$

which has roots given by the quadratic equation as,

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} \quad (1.8)$$

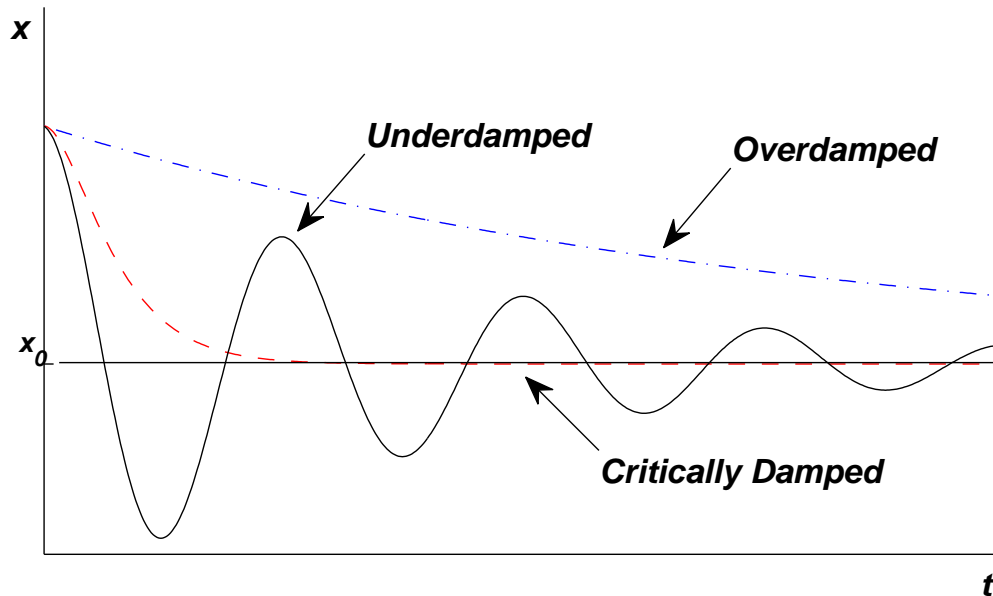
Now, depending on the values of the parameters  $c, m$ , and  $k$  (specifically the *discriminant*  $c^2 - 4mk$ ), there are three situations encountered, and thus three different behaviors of the displacement  $x(t)$ . These situations are:

- $c^2 - 4mk > 0$ : This produces two distinct real roots  $\lambda_1$  and  $\lambda_2$ , and the solution is  $x_c(t) = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t}$ . This system is called *overdamped* - the system will slowly settle down to  $x_c(t) = 0$  with no oscillations.
- $c^2 - 4mk = 0$ : This produces a repeated real root  $\lambda_1 = -c/2m$  and the solution is  $x_c(t) = C_1e^{\lambda_1 t} + C_2te^{\lambda_1 t}$ . This system is called *critically damped* - the system will quickly settle down to  $x_c(t) = 0$  with no oscillations. *Why is the decay more rapid than the overdamped case?*
- $c^2 - 4mk < 0$ : This produces a complex conjugate pair  $\alpha \pm i\beta$  with  $\alpha < 0$  and the solution is  $x_c(t) = e^{\alpha t} [C_1 \cos(\beta t) + C_2 \sin(\beta t)]$ . This system is called *underdamped* - the mass will oscillate, but the oscillations will decay with time according to the exponential factor (see Figure 1.3).

Naturally, the constants  $C_1$  and  $C_2$  will be determined from initial conditions for the speed and displacement of the mass.

A useful quantity (you will see why), termed the *natural frequency*  $\omega_n$  is defined as,

$$\omega_n = \sqrt{\frac{k}{m}} \quad (1.9)$$



**Figure 1.3:** Typical solutions for *underdamped*, *overdamped*, and *critically damped* cases.

As you will show in prelab question (1), this is the system's frequency of vibration when there is no damping ( $c = 0$ ). Additionally, instead of employing the discriminant  $c^2 - 4mk$  to describe the state of the system (over/under/critically damped), it is convenient to define a *damping factor*,  $\zeta$ , as

$$\zeta = \frac{c}{2\sqrt{mk}} \quad (1.10)$$

$\zeta$  is defined in such a way that

- $\zeta > 1$  is an overdamped system
- $\zeta = 1$  is a critically damped system
- $\zeta < 1$  is an underdamped system

Thus,  $\zeta$  is a non-dimensional measure of the amount of damping in the system. **In this lab, we will assume that both the mass-spring-dashpot system and the speaker are underdamped.** *In fact we will assume  $\zeta \ll 1$ !*

Using these definitions, we can restate the quadratic equation we found above in terms of the new variables, which yields (after some algebra)

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \quad (1.11)$$

Since we are studying the underdamped system in the lab, we take  $\zeta < 1$  and find that the roots are

$$\lambda_{1,2} = -\zeta\omega_n \pm i\omega_d \quad (1.12)$$

where we defined the *damped natural frequency* (i.e. the frequency of oscillation *with* damping) as  $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ . Thus, the solution for the underdamped system is,

$$x_c(t) = e^{-\zeta\omega_n t} [C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)] \quad (1.13)$$

which can be restated as,

$$x_c(t) = Ae^{-\zeta\omega_n t} \cos(\omega_d t - \phi) \quad (1.14)$$

where  $A = \sqrt{C_1^2 + C_2^2}$ , and  $\phi = \tan^{-1}(-C_2/C_1)$  are two constants to be determined from the initial conditions.

## THE LOGARITHMIC DECREMENT METHOD

It is often important to measure how much damping there is in an engineering system. The viscous damping constant,  $c$ , may be determined experimentally by measuring the rate of decay of unforced oscillations - this process is called a "ring down" test. We define the logarithmic decrement,  $D$ , as the natural logarithm of the ratio of any two successive amplitudes:

$$D = \ln \left( \frac{x_n}{x_{n+1}} \right) \quad (1.15)$$

where  $x_n$  and  $x_{n+1}$  are the heights of two successive peaks in the decaying oscillation (see Figure 1.4). The larger the damping, the greater will be the rate of decay of oscillations and the bigger the logarithmic decrement,  $D$ . To measure the logarithmic decrement  $D$ , we note that successive peaks occur on the decay envelope  $Ae^{-\zeta\omega_n t}$  when the periodic term,  $\cos(\omega_d t - \phi)$ , takes on its maximum value of 1 (see Figure 1.4). Writing out the equation for the decay envelope we have

$$x_{envelope}(t) = Ae^{-\zeta\omega_n t} \quad (1.16)$$

Using this equation we now write the logarithmic decrement as the ratio of successive peaks of the periodic term

$$D = \ln \left( \frac{x_{envelope}(t)}{x_{envelope}(t + \tau_d)} \right) = \ln \left( \frac{Ae^{-\zeta\omega_n t}}{Ae^{-\zeta\omega_n(t + \tau_d)}} \right) = \ln(e^{\zeta\omega_n \tau_d}) = \zeta\omega_n \tau_d \quad (1.17)$$

where  $\tau_d$  is the period of the damped oscillation, i.e.  $\tau_d = \frac{2\pi}{\omega_d}$ . We simplify this expression by substituting in (1.10) for  $\zeta$  and then solve for the damping constant  $c$ , yielding (*algebra omitted*)

$$c = \frac{2mD}{\tau_d} \quad (1.18)$$

We can also obtain an equation for  $k$  from (1.17), yielding

$$k = \frac{c^2 \left(1 + \frac{4\pi^2}{D^2}\right)}{4m} = \frac{c^2}{4m\zeta^2} \quad (1.19)$$

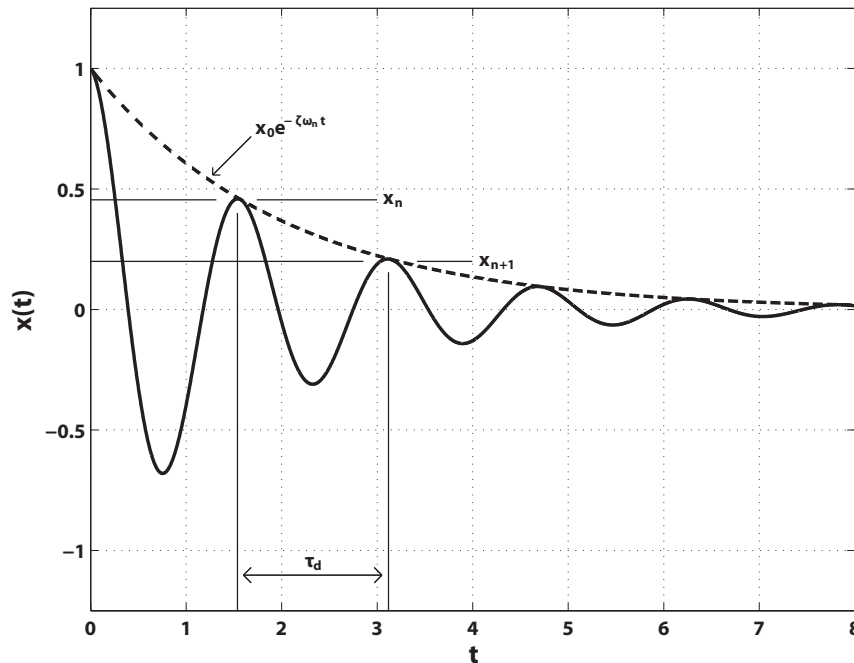


Figure 1.4: The logarithmic decrement method.

Thus, by doing a "ring-down" test we can experimentally obtain values for  $D$  and  $\tau_d$ . Then using equations (1.18) and (1.19) and given the mass  $m$ , we can find the damping constant  $c$  and spring constant  $k$  for a one degree-of-freedom oscillator.

### FORCED VIBRATIONS AND FREQUENCY RESPONSE

Often a system is periodically forced and we are interested in how it will respond, e.g. the tires on your car going over evenly spaced ruts in the road jostles the car. In this section, we will solve the equation of motion (1.3) for the forced case and examine the phase and amplitude of the response. We assume that the support is driven harmonically such that the exciting force on the mass is  $F(t) = F_{drive} \cos \omega t$  (i.e. the vibration amplitude of the support is  $A_{support} = \frac{F_{drive}}{k}$ ). The equation of motion (1.3) now becomes

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{F_{drive}}{m} \cos \omega t \quad (1.20)$$

*Note:* This is the same equation of motion from before (1.5) but in terms of the new variables from the previous section. Now we will only be interested in the steady-state oscillation that will be left after the transient response dies out, i.e.  $x_p(t)$ . We take the solution to be of the form,

$$x_p(t) = A_{response} \cos(\omega t - \phi) \quad (1.21)$$

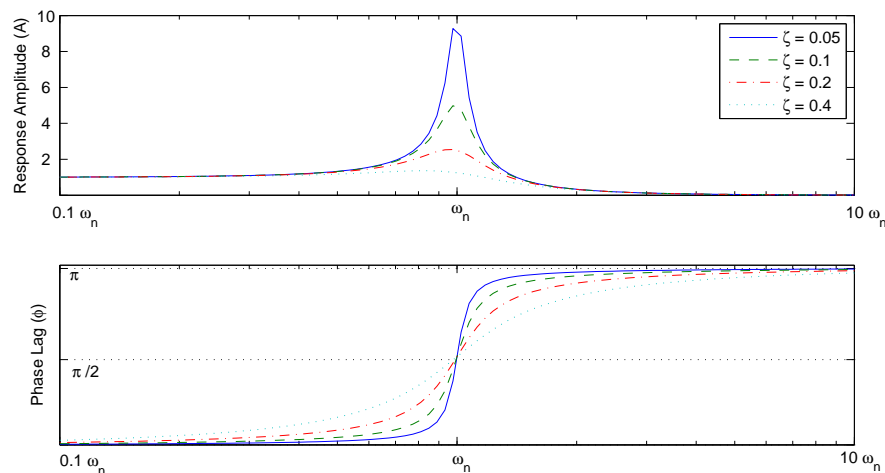
where  $A_{response}$  is the amplitude of oscillation of the mass and  $\phi$  is the phase of the displacement  $x(t)$  with respect to the exciting force  $F(t)$ . We should expect that the amplitude

$A_{response}$  and phase  $\phi$  of the response depends on how quickly ( $\omega$ ) and with what force ( $F_{drive}$ ) we drive the system. We can solve for  $\phi(\omega, F_{drive})$  and  $A_{response}(\omega, F_{drive})$  by plugging our guess at the solution (1.21) into the equation (1.20) and use the linear independence of  $\cos \omega t$  and  $\sin \omega t$ . Don't worry about the algebra, the results are given below:

$$\phi(\omega) = \arctan \frac{2\omega\omega_n\zeta}{\omega_n^2 - \omega^2}; 0 \leq \phi \leq \pi \quad (1.22)$$

$$\frac{A_{response}(\omega)}{F_{drive}} = \frac{\frac{1}{m}}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\omega^2\omega_n^2\zeta^2}} \quad (1.23)$$

Note that the phase of the response is independent of how hard we drive the system, though if we drive it twice as hard, the response amplitude is twice as big. How the system response depends on the drive frequency ( $\omega$ ) and system parameters ( $\omega_n, \zeta$ ) is a bit more complicated, and the best way to understand it is with a graph. In Figure (1.5) we assume the support oscillates with unit amplitude  $x_s(t) = \cos \omega t$ , thus  $F_{drive} = k$ , and plot the response amplitude and phase as a function of the drive frequency ( $\omega$ ) for different amounts of damping ( $\zeta$ ).



**Figure 1.5:** The system response  $x(t) = A \cos \omega t + \phi$  as a function of forcing frequency  $\omega$ , for various amounts of damping  $\zeta$

Note from the plot that:

- for very low drive frequencies ( $\omega \ll \omega_n$ ) the response is synchronized with the driving. The phase lag ( $\phi$ ) is 0, and the amplitude of vibration of the mass is the same as the amplitude of vibration of the support. *What is the physical argument for this?*
- for drive frequencies near  $\omega_n$  the response amplitude is at a maximum and the phase lag is  $\frac{\pi}{2}$ .



- for very high drive frequencies ( $\omega \gg \omega_n$ ) the response is completely out of phase with the driving and the amplitude of vibration goes to zero. *What is the physical argument for why the amplitude vanishes?*
- the less damping there is, the sharper the change in phase is, and the greater the response near  $\omega_n$ .

## RESONANCE

Resonance as defined by *Merriam-Webster* is a *vibration of large amplitude in a mechanical or electrical system caused by a relatively small periodic stimulus of the same or nearly the same period as the natural vibration period of the system*. For an undamped system, resonance occurs when we force the system at its natural frequency, i.e.  $\omega = \omega_n$ , and the response is unbounded (*note how the peaks in Figure 1.5 increase as  $\zeta \rightarrow 0$* ). However, resonance in a damped mass-spring-dashpot system does not occur when the forcing frequency is exactly the undamped natural frequency  $\omega_n$ , and the response is not unbound but simply a maximum. To find the resonant frequency,  $\omega_r$ , we maximize the response's amplitude (1.23) by differentiating  $A_{response}(\omega)$  with respect to the forcing frequency  $\omega$  (*holding everything else fixed*) and setting it equal to zero.

$$\left. \frac{dA_{response}}{d\omega} \right|_{\omega=\omega_r} = 0 \Rightarrow \omega_r = \omega_n \sqrt{1 - 2\zeta^2} \quad (1.24)$$

Note for small damping ( $\zeta \ll 1$ ) we have  $\sqrt{1 - 2\zeta^2} \sim 1$  and so the *resonant frequency*  $\omega_r$  and the *natural frequency*  $\omega_n$  are approximately equal  $\omega_r \simeq \omega_n$ .

**PHASE DIAGRAMS** In our experiment we will need a way to tell if the system is near resonance without adjusting the forcing frequency until the response is maximized. One easy way is to examine the phase lag ( $\phi$ ). It can be shown that when we force the system at its natural frequency ( $\omega_n$ ) that the phase is,  $\phi = \frac{\pi}{2}$  (Verify this by inspection (see Figure 1.5) or directly from (1.22)). The corresponding *phase diagram* will then be a circle (*If you are interested, further details on what a phase diagram is can be found in the appendix*). Note from the previous section that for small damping  $\omega_n \simeq \omega_r$ , and so for the systems in our lab when we see a circle we know we are at (or very close to) resonance.

## LABORATORY SET-UP

- **Mass-Spring-Dashpot System**

The apparatus consists of a laboratory-model mass-spring-dashpot system with displacement transducers (Linear Variable Differential Transformers or LVDTs) for measuring  $x(t)$  and  $x_s(t)$ . The output from the LVDTs is communicated to the computer via the data acquisition board. An electric motor and controller, acting through a scotch yoke, enable a sinusoidal forcing function to be applied to the system. Note that the controller dial readings are arbitrary; frequency and period data must be obtained from your computer plots.

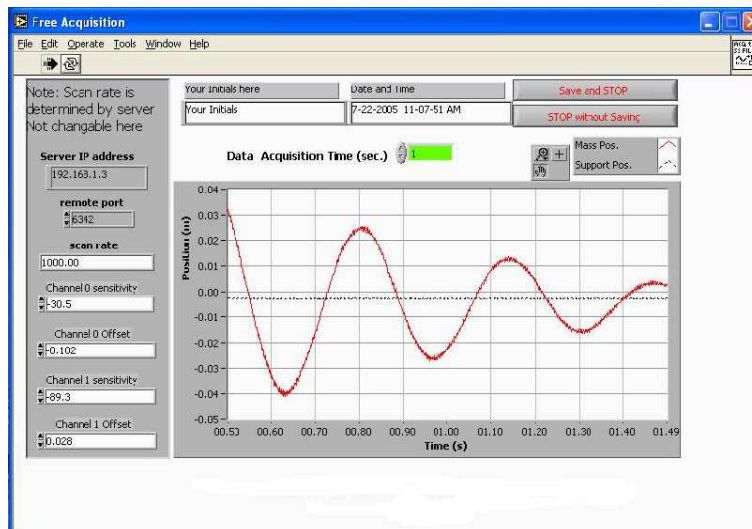
- **Loudspeaker**

The apparatus consists of a speaker on a stand with one LVDT to measure cone displacement. Waveforms are generated by the computer, amplified, and sent through a resistor to drive the speaker. The computer is also used to measure current flow through the speaker and displacement of its cone (using the attached LVDT).

**Please follow all safety precautions.** Keep long hair and loose clothing well away from the electric motor, pulleys, and other moving parts.

- **Using the *LabView* Software**

The four programs you will be using in the first part of the lab are: *FreeAcq* (Figure 1.6) for making measurements of the unforced system; *FreeSim* (Figure 1.7) for simulation of the same; *ForcedAcq* (Figure 1.8) for measurements of the system with a sinusoidal forcing function; and *ForcedSim* (Figure 1.9) which may be used for simulation of the forced system. Although somewhat different in appearance and function, the programs share many key features. The *SpeakerAcq* (Figure 1.10) program used in the second part of the lab is also similar.



**Figure 1.6:** The *FreeAcq* program.

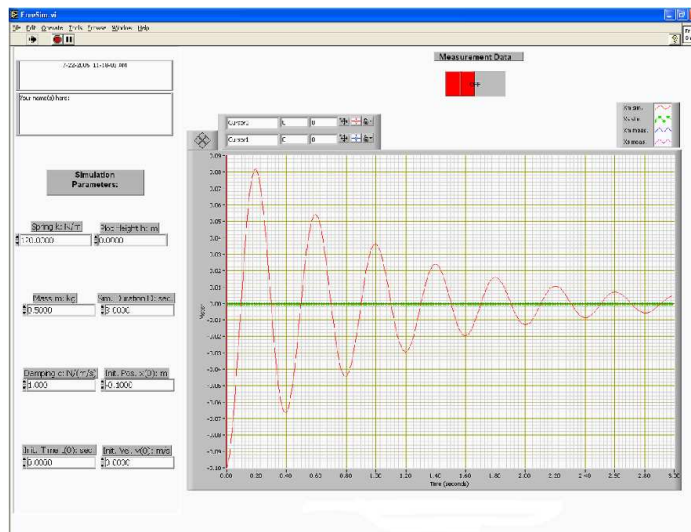


Figure 1.7: The *FreeSim* program.

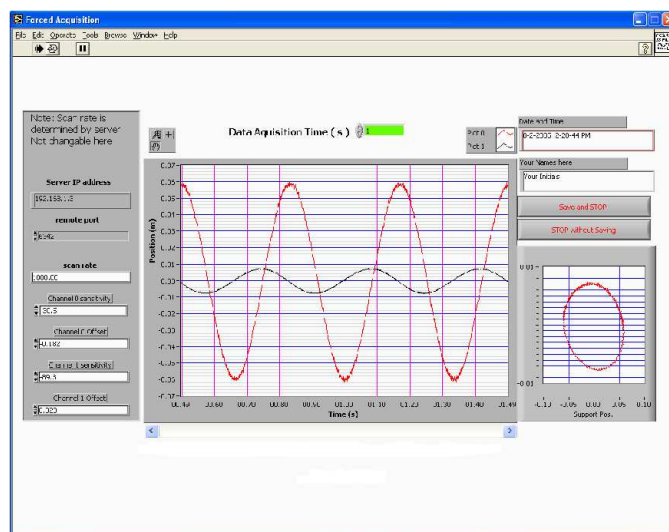


Figure 1.8: The *ForcedAcq* program.

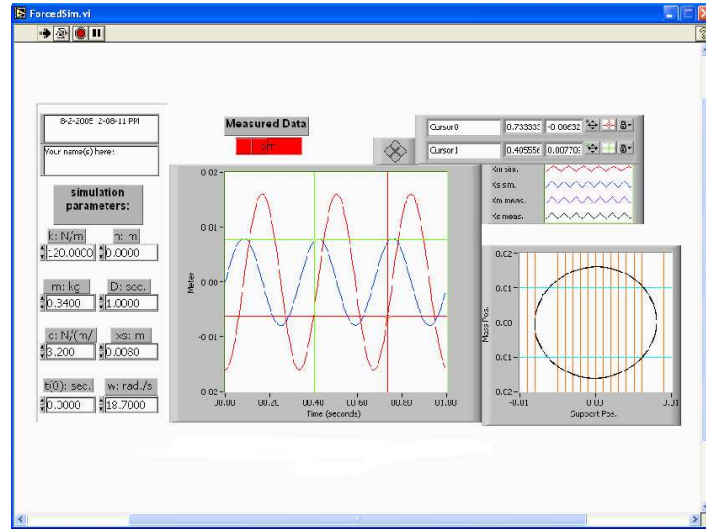


Figure 1.9: The *ForcedSim* program.

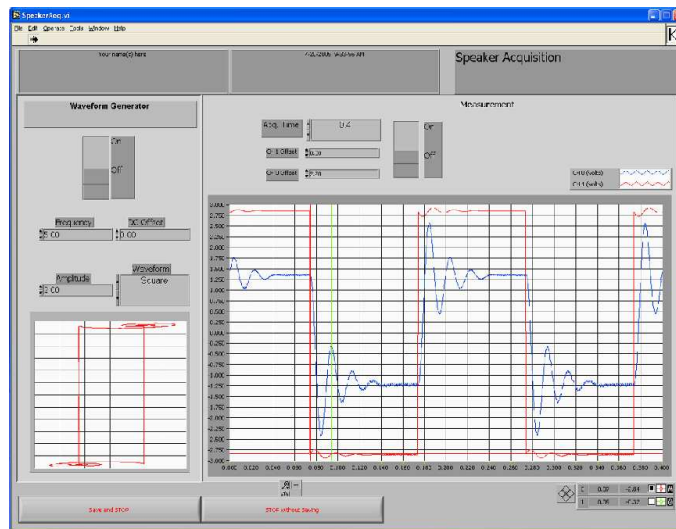


Figure 1.10: The *SpeakerAcq* program.

To run the program, you must hit the white arrow in the top left of the screen. If this arrow is black, that means that the program is already running. For the data acquisition programs, a green box on top will define the amount of time for which the program will record after hitting the arrow. To reset the data acquisition, press **STOP without Saving** and then press the white arrow to begin again.

After getting data, pressing the **Save and STOP** button stores your current data on disk. The data file is only used by the simulation programs *FreeSim* and *ForcedSim*—it is not available to the data acquisition programs.

You may find it convenient to obtain numerical data from your plots using the cursors, rather than using a ruler. Two cursors are available, one indicated by a circle and one by a square. To use a cursor, use the mouse to drag it to the point you want to measure. If your cursor has vanished off the screen, you can enter an on-screen position for it into the  $x$  and  $y$  display boxes, and it will reappear in the desired location. You can also lock the cursor to a curve by clicking the lock icon. Zoom and other features are available for the cursors and graphs; see the *LabView* manual for details.

## PROCEDURE

### • Free Vibration, Mass-Spring-Dashpot

1. First you will measure the *free vibration* of the mass.
  - Start up the *FreeAcq* program. The data acquisition programs automatically convert the voltage output of the LVDTs to meters. To do this, they need a set of conversion factors, which are on a label on the mass-spring-dashpot base board. Make sure that the sensitivity and offset values on the left hand side of the window match the values listed on a small sheet of paper in front of the apparatus, and enter your name in the box provided. Set the data acquisition time to 6 seconds.
  - Pull down the mass and hold it still, then press the white run arrow in the top left of the toolbar and immediately release the mass.
  - Repeat this procedure until you have a nice oscillation over the 6 seconds. Please note that the zero position is somewhat arbitrary. You will need to take data long enough for the mass to stop oscillating in order to have a good zero reference.
  - Save your best oscillation on disk by pressing the **Save and STOP** button.
2. Next you will measure the logarithmic decrement  $D$  and estimate the spring stiffness  $k$  and damping coefficient  $c$ .
  - Start the *FreeSim* program, and add the measured data to the graph by pressing the **Measurement Data** switch above the graph. Set  $k = 0$  to get the simulated data out of the way.

- Using the cursors, measure the logarithmic decrement  $D$  and the period of the damped oscillation  $\tau_d$  for each set of successive peaks (at least 3).
  - Using these measured values, and the mass  $m$ , calculate the damping constant  $c$  and spring stiffness  $k$  (*The mass of the weight and spring are written at the base of the setup. For your 'm' use the total of the weight and spring*).
  - Make a print-out of your curve.
3. Finally, you will simulate the *free vibration* of the mass-spring-dashpot system and verify your estimate of the system parameters. Simulate unforced motion by inputting the values of  $m$ ,  $k$ , and  $c$  that you just determined into the *FreeSim* program.
- Adjust the initial condition and viewing parameters ( $t(0), h, x(0), D$ ) to fit your data. *Don't change  $k$  or  $c$ .*
  - Make a print-out.
  - Now see if you can adjust  $k$  and  $c$  to get a better agreement. Take note of what aspects of the graph change when you change each of the parameters  $k$  and  $c$ .
  - Make another print-out.

### • Forced Vibration, Mass-Spring-Dashpot

1. Here you will be recording the motion of the mass as it undergoes sinusoidal forcing.
- Start the *ForcedAcq* program.
  - Set the acquisition time to 30 seconds, start the data acquisition and turn on the motor. Two graphs will be displayed. The left one contains two plots. One is a plot of the mass position  $x(t)$  vs. time and the second one is a plot of the spring support position  $x_s(t)$  vs. time. The right graph plots the phase diagram.
  - For at least five different forcing frequencies get nice plots of several cycles of motion (see instructions below). Make sure to save each data set to disk in order to analyze them in the *ForcedSim* program. Print-outs are not necessary but may be helpful.
  - To acquire data, set the data acquisition time to 10 seconds and run the program in order to find the desired drive frequency. Then hit **STOP without SAVING**. Reduce the data acquisition time to  $\sim 1$  second (or atleast long enough to get one whole cycle) and then run the program again. This time hit **SAVE and STOP**. Reducing the acquisition time will reduce demand on the server and save you time doing analysis. Forcing frequencies should include:
    - \* The lowest frequency for which the motor runs smoothly.
    - \* A frequency just lower than resonance.
    - \* Resonance. (*Hint: we tell from the phase diagram that it is at resonance*)

- \* A frequency just higher than resonance.
  - \* A very high frequency.
2. Next we will simulate the forced vibration of the mass-spring-dashpot system.
    - Open the *ForcedSim* program.
    - Turn on the measured data switch to view your saved data. To change the current measured data set you must close and then re-open the *ForcedSim* program. Once experimental data is loaded, make your necessary measurements (see below) using the computer cursors.
    - For each of the five frequencies you will need to measure and record the following data:
      - \* damped period ( $\tau_d$ )
      - \* forcing function amplitude ( $F$ ) (*Note: you will actually see  $\frac{F}{k}$ , the amplitude of the support oscillation*)
      - \* mass motion amplitude ( $A$ )
      - \* phase-lag between the forcing function and the resulting mass motion ( $\phi$ )

You may also want to save the data to a USB storage device or write it to a CD for later analysis. To do this just copy the text files of the desired data onto your storage device.

### • Vibration of a Speaker

- In the last part of the lab, you will experimentally fit the parameters  $k$  and  $c$  for a speaker by measuring the shift in resonance frequency due to the addition of a known mass.
  1. First you will force the loudspeaker at its resonant frequency in order to experimentally determine the mass  $m$  and spring constant  $k$  of the loudspeaker.
    - Set the **Waveform** control to **Sine** and the **Amplitude** control to 2. Leave the **DC Offset** control set to 0. Set the data acquisition time to 0.1 seconds. The **CH 0 Offset** and **CH 1 Offset** controls may be used to adjust the plots vertically if necessary.
    - Turn on the waveform generator and data acquisition switches and adjust the **Frequency** control value until you observe resonance of the speaker cone. **To change the frequency you must press STOP without Saving, enter the desired frequency and then start the program again in order to observe the new frequency.**
    - Make a print-out and record the frequency (*note the need to convert from Hz to rad/sec*).

Recall that the resonant frequency depends on both the mass  $m$  and spring stiffness  $k$ . By measuring the resonant frequency you cannot solve for both  $m$  and  $k$  uniquely. However, if you also measure the resonant frequency when the mass

is changed a known amount then you will have 2 equations ( i.e. (1.9), assume  $\omega_r \sim \omega_n$ ) for 2 unknowns ( $m, k$ ) in terms of measured data ( $\omega_{1r}, \omega_{2r}, \Delta m$ ). Now measure the mass of the rubber weight and then carefully press it onto the LVDT shaft. The best way is to spread the weight open, position it, and release it.

- Find the new resonant frequency, and record the mass of the rubber weight.
- Make a print-out and record the frequency (*note the need to convert from Hz to rad/sec*).



## LAB REPORT QUESTIONS

Please answer the following questions concerning the mass-spring-dashpot part of the lab within your lab report:

1. What is the spring constant  $k$  and damping constant  $c$  for your mass-spring-dashpot setup as calculated from your "ring-down" test? Indicate the measured data and formulas you used to calculate these values. Is the damping constant  $c$  really constant? What does this say about the air dashpot acting linearly?
2. Compare your experimental data to the simulated data for unforced motions. Comment on any similarities or differences of interest. Did you need to adjust  $k$  and  $c$  to better fit your data? How did changing  $c$  and  $k$  respectively change the simulation graph? Please attach print-outs.
3. Make a plot of the response amplitude  $A(\omega)$  using your 5 data points. Make a plot of the phase-angle  $\phi$  between  $x(t)$  and  $x_s(t)$  versus the forcing frequency  $\omega$ . Do these plots match what you expect from Figure 1.5.
4. For a typical value of damping constant  $c$  that you measured, what is the percent difference between the natural frequency  $\omega_n$  and the damped natural frequency  $\omega_d$ ? Does the addition of a dashpot to a mass-spring system increase or decrease its oscillation frequency?

Please answer the following questions concerning the loudspeaker part of the lab within your lab report:

1. Calculate  $k$  and  $m$  for the speaker, using the resonant frequencies and mass you measured in lab.
2. Find another real-world vibrating system which could be reasonably modeled as a mass-spring-dashpot. Give the system a "push" and observe its response. Try applying a forcing function of various frequencies, and look for resonance.
  - (a) Describe how you modeled your vibrating system as a mass-spring-dashpot. That is, what does the mass represent, what is the spring, and what is the dashpot? Be as specific as possible.
  - (b) Is this system typically overdamped? Underdamped? If applicable, what was the resonant frequency (approximately)?
  - (c) In what ways does the system you found most significantly differ from an ideal linear mass-spring-dashpot system?

**CALCULATIONS & NOTES**

**Appendix: Phase Diagrams** A phase diagram is a plot which contains the forcing function  $F_s(t)$  on the y-axis and the response function  $x(t)$  on the x-axis. The phase diagram is a graphical representation of the relative phase of the forcing and motion. Each point on the plot tells us both where we are in the drive cycle  $y(t)$  and on the response cycle  $x(t)$ . Time is a parameter that moves us around on the diagram. Since we are only interested in the phase, we scale each term by its amplitude. Thus on our phase diagram we would plot the parametric function

$$y(t) = \cos(\omega t) \quad (1.25)$$

$$x(t) = \cos(\omega t - \phi) \quad (1.26)$$

When we force the system at  $\omega_n \simeq$  and the phase is  $\phi = \frac{\pi}{2}$  as shown before, we have:

$$y(t) = \cos(\omega t) \quad (1.27)$$

$$x(t) = \cos(\omega t - \frac{\pi}{2}) \quad (1.28)$$

Using the following trigonometric identities:

$$\cos^2 \omega t + \sin^2 \omega t = 1 \quad (1.29)$$

$$\cos(\omega t - \frac{\pi}{2}) = \sin \omega t \quad (1.30)$$

We can establish the following relationship for our phase plot:

$$x^2(t) + y^2(t) = 1 \quad (1.31)$$

**Hopefully you will recognize this equation as the parametric form of the equation for a circle!** Thus, when we force the system at its *natural frequency* which is very close to its *resonant frequency* the phase diagram is a circle.

