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Section time: _____

T&AM 203 Homework Exam**Tuesday Dec 12, 2006**

Draft December 12, 2006

4 problems, 25⁺ points each, 4 hours.

If you can do all the homework you are guaranteed a grade of C. These 4 problems are based on homework problems (see next page), or parts of homework problems, with slight changes so that memorizing answers won't help. If you can do 3 of them *fully correctly* (good work, correct answer) in 4 hours you are guaranteed a grade of at least C.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, or books allowed. You can bring a one-sided formula sheet, but not any worked out HW etc.

b) Full credit if

↖ • ↗ → free body diagrams ← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;

- correct vector notation is used, when appropriate;

↑ → any dimensions, coordinates, variables and base vectors that you add are clearly defined;

± all signs and directions are well defined with sketches and/or words;

→ | reasonable justification, enough to distinguish an informed answer from a guess, is given;

• you clearly state any reasonable assumptions if a problem seems *poorly defined*;

- work is I.) neat,
II.) clear, and
III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);

□ your answers are boxed in; and

➤ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like " $\dot{\theta}_7 = 18$ " instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

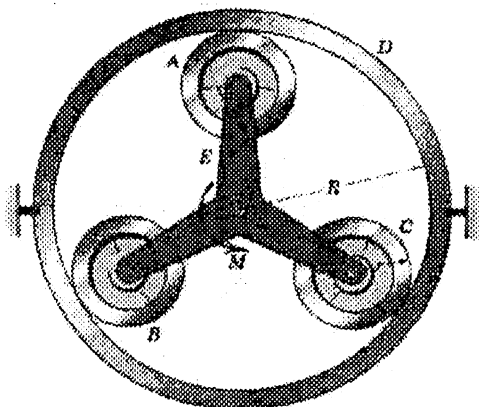
c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem 5/91: /25 Problem 6/15: /25 Problem 6/79: /25 Problem 6/168: /25

Replace the text from the book (copied with the figures on the next page) with the problem statements below. For all problems all quantities are described with letters and not numbers. Answers should be expressed with letters. Don't use the numbers given with the book problem statement. Ignore the numbers in the figures.

- 5/91 Given \dot{a} , L_{CD} , L_{AB} , a and b find \dot{b} . (Note $\mathbf{y}_A = -\dot{a}\hat{\mathbf{i}}$.)
- 6/15 The car has mass m and gravity points down with constant g . The coefficient of friction between wheels and road is μ . For rear wheel drive what is the maximum possible car acceleration? (Answer in terms of some or all of a , b , h , m , g and μ .)
- 6/79 A uniform disk with radius R and mass m rolls down a slope θ . The friction coefficient μ is large enough so the disk rolls without slipping. Gravity g points down. Find the component of the force that acts on the disk from the ramp that is tangent to the ramp. Answer in terms of some or all of R , m , g , θ and μ .
- 6/168 The angular velocity and acceleration of the spider are given as ω_s and α_s . Find the acceleration of a point on one of the planet gears that is, at the instant in question, in contact with the ring gear. Answer in terms of some or all of R , r , ω_s , α_s and unit vectors you clearly define.

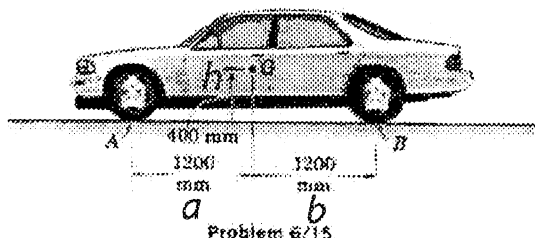
- 6/168 A planetary gear system is shown, where the gear teeth are omitted from the figure. Each of the three identical planet gears A , B , and C has a mass of 0.6 kg, a radius $r = 30$ mm, and a radius of gyration of 30 mm about its center. The spider E has a mass of 1.2 kg and a radius of gyration about O of 60 mm. The ring gear D has a radius $R = 150$ mm and is fixed. If a torque $M = 3$ N·m is applied to the shaft of the spider at O , determine the initial angular acceleration α of the spider.



Problem 6/168

- 6/15 The 1650-kg car has its mass center at G . Calculate the normal forces N_A and N_B between the road and the front and rear pairs of wheels under conditions of maximum acceleration. The mass of the wheels is small compared with the total mass of the car. The coefficient of static friction between the road and the rear driving wheels is 0.6 .

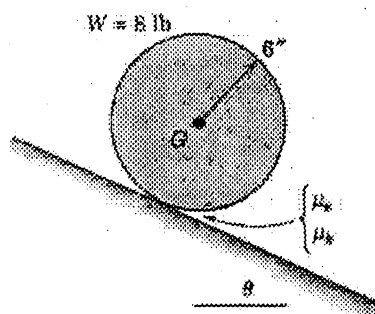
Ans. $N_A = 6.85$ kN, $N_B = 9.34$ kN



Problem 6/15

- 6/79 The solid homogeneous cylinder is released from rest on the ramp. If $\theta = 40^\circ$, $\mu_s = 0.30$, and $\mu_k = 0.20$, determine the acceleration of the mass center G and the friction force exerted by the ramp on the cylinder.

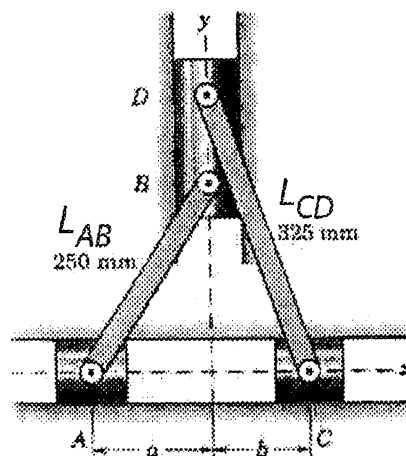
Ans. $a = 13.80$ ft/sec², $F = 1.714$ lb



Problem 6/79

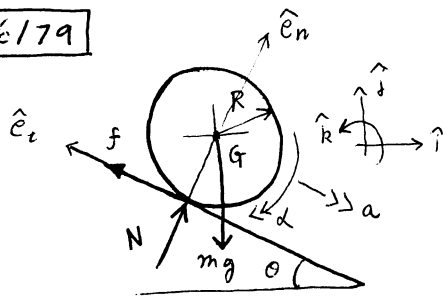
- 5/91 At the instant represented, $a = 150$ mm and $b = 125$ mm, and the distance $a + b$ between A and C is decreasing at the rate of 0.2 m/s. Determine the common velocity v of points B and D for this instant.

Ans. $v = 0.0536$ m/s



Problem 5/91

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LMB: $-mg\hat{j} + N\hat{e}_n + f\hat{e}_t = -ma\hat{e}_t$

$\{ \} \cdot \hat{e}_t \Rightarrow -mg\sin\theta + f = -ma$

$-f + mg\sin\theta = ma \quad - (1)$

$\{ \} \cdot \hat{e}_n \Rightarrow N = mg\cos\theta \quad - (2)$

AMB: about G

$-fR = -I\alpha$

$fR = \frac{1}{2}mR^2\alpha \quad - (3)$

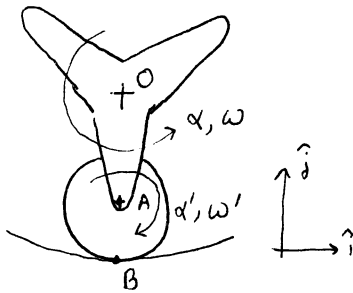
No slip condition:

$\alpha R = a \quad - (4)$

(1), (3) and (4) \Rightarrow

$f = \frac{mg\sin\theta}{3}$ Ans

6/168



$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$= \vec{a}_O + \vec{a}_{A/O} + \vec{a}_{B/A}$

$= \vec{\alpha} \times \vec{OA} + \vec{\omega} \times (\vec{\omega} \times \vec{OA}) + \vec{\alpha}' \times \vec{AB} + \vec{\omega}' \times (\vec{\omega}' \times \vec{AB})$

$= \alpha(R-r)\hat{i} + \omega^2(R-r)\hat{j} - \alpha'r\hat{i} + \omega'^2r\hat{j}$

$= [\alpha(R-r) - \alpha'r]\hat{i} + [\omega^2(R-r) + \omega'^2r]\hat{j} \quad (10)$

by no slip condition, tangential acceleration is zero.

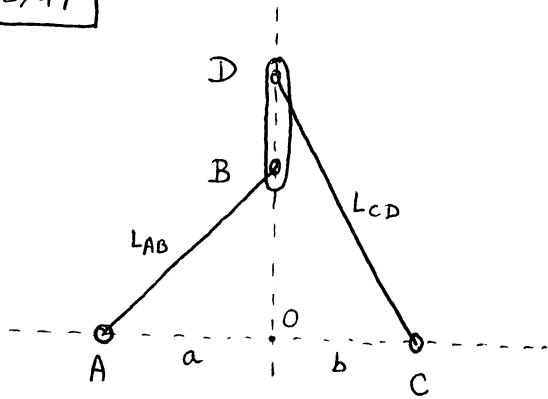
$\vec{a}_B \cdot \hat{i} = 0 \Rightarrow \alpha(R-r) - \alpha'r = 0$

$\Rightarrow \omega(R-r) - \omega'r = 0 \Rightarrow \omega' = \frac{\omega(R-r)}{r} \quad (10)$

$\Rightarrow \vec{a}_B = \left\{ \omega^2(R-r) + \left[\frac{\omega(R-r)}{r} \right]^2 r \right\} \hat{j} \quad (5)$

$= \frac{\omega^2(R-r)R}{r} \hat{j}$ Ans

5/91



\dot{a} given
to find \dot{b}

$$OB = \sqrt{L_{AB}^2 - a^2}$$

$$OD = \sqrt{L_{CD}^2 - b^2}$$

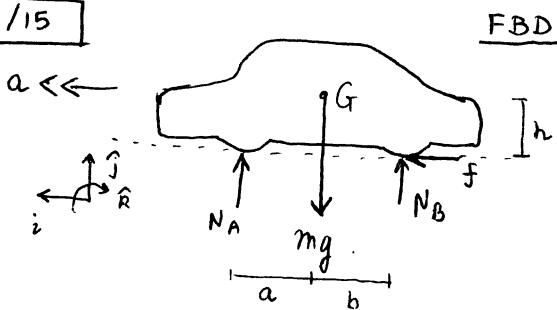
$$DB = OD - OB = \sqrt{L_{CD}^2 - b^2} - \sqrt{L_{AB}^2 - a^2}$$

$$\frac{d(DB)}{dt} = 0 \quad \text{since } DB \text{ is of fixed length}$$

$$\Rightarrow \frac{1}{2} \frac{-2b\dot{b}}{\sqrt{L_{CD}^2 - b^2}} - \frac{1}{2} \frac{-2a\dot{a}}{\sqrt{L_{AB}^2 - a^2}} = 0$$

$$\Rightarrow \boxed{\dot{b} = \frac{a}{b} \sqrt{\frac{L_{CD}^2 - b^2}{L_{AB}^2 - a^2}} \dot{a}} \quad \text{Ans}$$

6/15



FBD

$$\text{LMB: } f\hat{i} + N_A\hat{j} + N_B\hat{j} - mg\hat{j} = m a \hat{i}$$

$$\{ \} \cdot \hat{i} \quad f = ma \quad - (1)$$

$$\{ \} \cdot \hat{j} \quad N_A + N_B = mg \quad - (2)$$

$$\text{AMB: about point G}$$

$$N_A a + f h = N_B b \quad - (3)$$

acceleration is max for max f

$$\Rightarrow f = \mu N_B \quad - (4)$$

$$(2), (3) \text{ and } (4) \Rightarrow N_B = \frac{mga}{a+b-\mu h}$$

$$\Rightarrow a = \frac{f}{m} = \frac{\mu N_B}{m} = \frac{\mu ga}{a+b-\mu h} = \boxed{\frac{\mu g}{1 + \frac{b}{a} - \mu \frac{h}{a}}} \quad \text{Ans}$$