"SOLUTIONS"

Your Name: ANDY RUINA

TA's name and Section time: _____

T&AM 203 Prelim 3

Tuesday April 15, 2008

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3 problems, 25^+ points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

$\nwarrow\bullet\nearrow$ →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;

$\underline{\bullet}$ correct vector notation is used, when appropriate;

$\uparrow\rightarrow$ any dimensions, coordinates, variables and base vectors that you add are clearly defined;

$\pm$ all signs and directions are well defined with sketches and/or words;

$\dashv$ reasonable justification, enough to distinguish an informed answer from a guess, is given;

you clearly state any reasonable assumptions if a problem seems ***poorly defined***;

- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);

$\square$ your answers are $\boxed{\text{boxed}}$ in; and

$\gg$ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

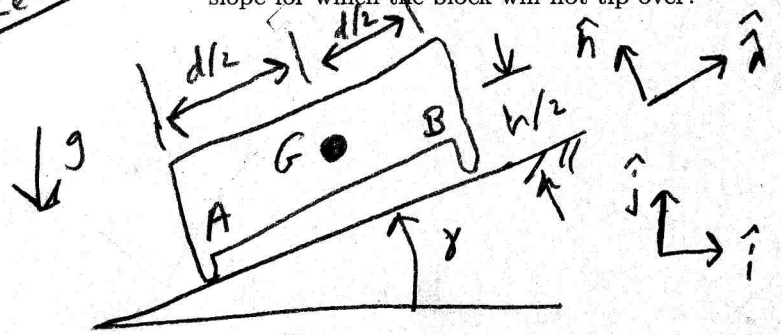
Problem	7:	/25
Problem	8:	/25
Problem	9:	/25

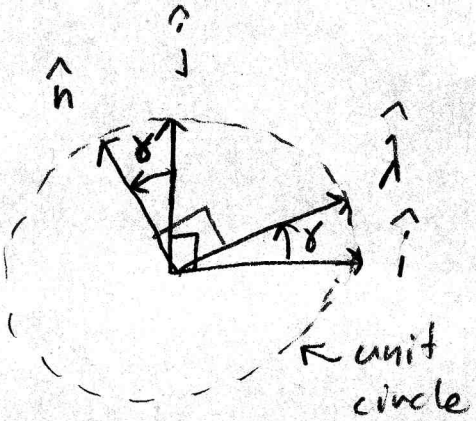
7) (25 pt) 2D problem. A uniform rectangular block with mass m, width d and height h slides down a non-level plane with slope γ ($\gamma = 0$ when the slope is level). The block only contacts the plane at its corners (on front and back edges) where the friction coefficient is μ. There is gravity.

a) (20 points) In terms of some or all of m, d, h, γ, μ and g find the magnitude of the acceleration of the block.)

b) (5 points) Given all the other parameters, what is the biggest slope for which the block will not tip over?

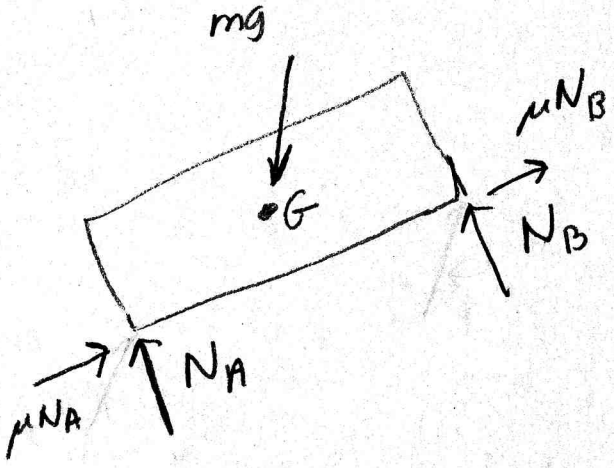
sketch





friction: $\mu = \tan\phi$

FBD



$\vec{a} = a(-\hat{\lambda})$

↑ a = accel. down the slope

LMB: $\sum \vec{F} = m\vec{a}$

$$\{(N_A + N_B)\hat{n} + \mu(N_A + N_B)\hat{\lambda} - mg\hat{j} = -ma\hat{\lambda}\}$$

$$\{\} \cdot (-\mu\hat{n} + \hat{\lambda}) \Rightarrow -(-\mu\hat{n} + \hat{\lambda}) \cdot (mg\hat{j}) = (-\mu\hat{n} + \hat{\lambda})(-ma\hat{\lambda})$$

$$(\mu\cos\gamma - \sin\gamma)g = -a$$

$$\boxed{a = g(\sin\gamma - \mu\cos\gamma)} \quad (a)$$

checks: $\mu = 0, \gamma = 0 \Rightarrow a = 0$ ✓

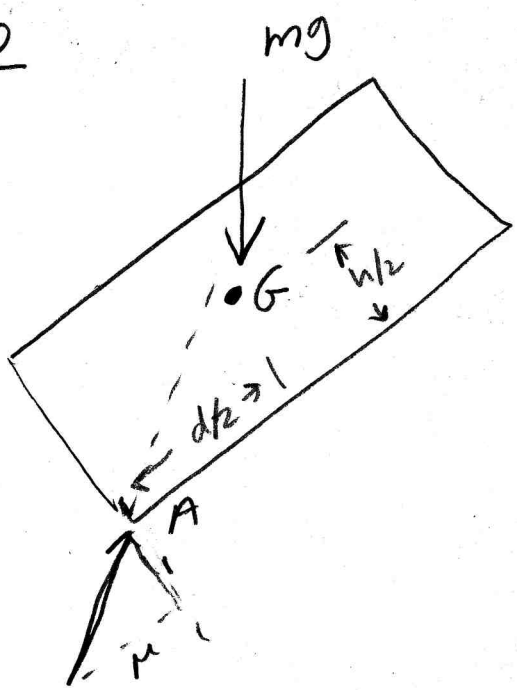
μ = anything, $\gamma = 0 \Rightarrow a = -\mu g$ ✓

μ = anything, $\gamma = \pi/2 \Rightarrow a = g$ ✓

(passes 3 sanity checks)

The boundary of tipping is when $N_B = 0$.
Simplest to assume $N_B = 0$ and no rotation & find critical condition

FBD



$$\sum \vec{M}_G = \dot{\vec{H}}_G = \vec{0}$$

(no rotation)

$\Rightarrow$ Force at A must pass through G

$$\Rightarrow \frac{\mu}{1} = \frac{d/2}{h/2}$$

$$\mu = d/h$$ critical case

$\Rightarrow$

$$\mu \leq d/h$$

for no tipping

(independent of m, g, γ!)

Tipping, or not, does not depend on slope, only on μ, d, h!

For all slopes it tips if $\mu > d/h$

For all slopes it doesn't tip if $\mu < d/h$

(b)

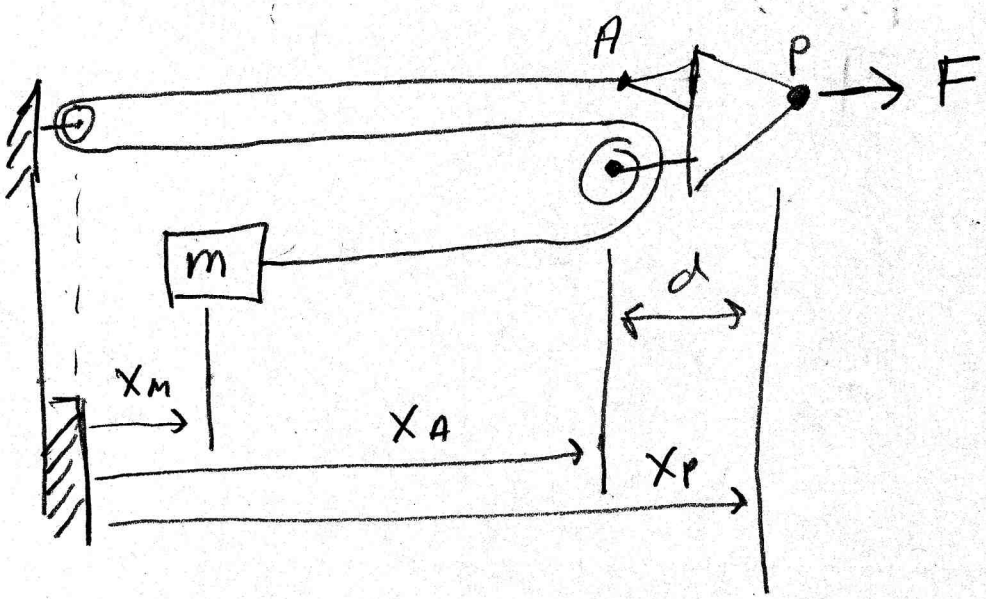
$\gamma < \pi/2$

8) (25 pt) A mass m has no forces on it but for those due to a pulley and cable system on which a force F is applied at a point on a cable P. Design the system so that the point of force application P has acceleration

$$a_P = \frac{F}{9m}.$$

Show that your system works. That is, given the system you design, calculate the acceleration of point P.)

Want force attenuated ⇒ few lines to mass, many to pt. of force application



Kinematics

$$x_A + d = x_P \Rightarrow \boxed{a_A = a_P} \quad (1)$$

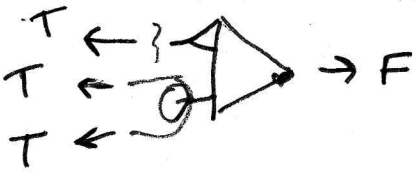
(d = const)

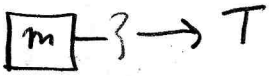
$$\{(x_A - x_m) + 2x_A + \pi R = \text{const}\}$$

$$\frac{d^2}{dt^2}\{\} \Rightarrow \boxed{3a_A = a_m} \quad (2)$$

$$(1)\ \&\ (2) \Rightarrow \boxed{a_p = a_m/3} \quad (3)$$

FBDs





$\Rightarrow \quad m a_m = T$

$\Rightarrow \quad F = 3T$

$$a_m = F/3m \qquad (4)$$

(3) & (4) $\Rightarrow$

$$a_p = \frac{F}{9m}$$

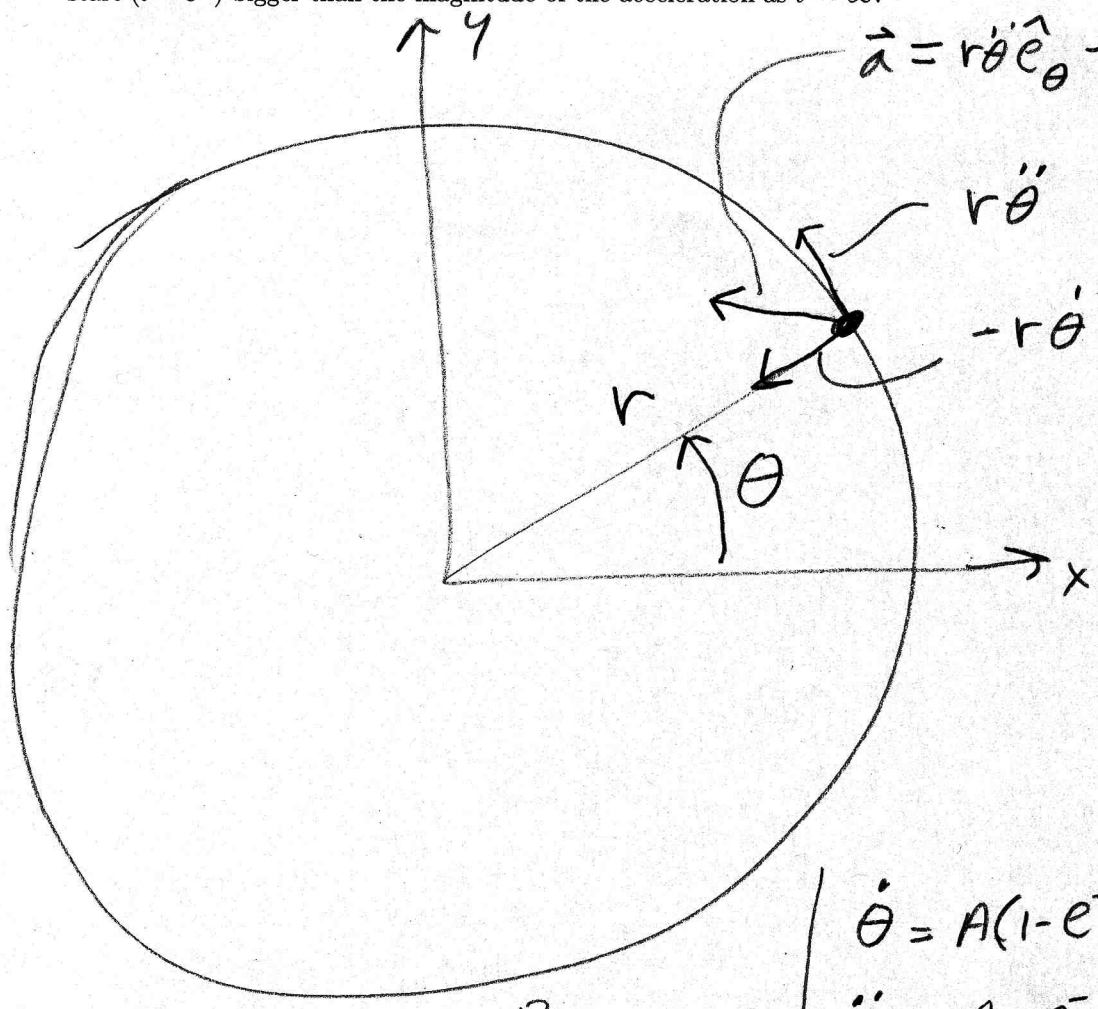
Note: $9 = 3 \times 3$

- 3: force attenuation to mass
- 3: motion attenuation from line lengths.

9) (25 pt) A particle moves on a circle of radius r according to the parametric equation

$$\dot{\theta} = A(1 - e^{-ct}) .$$

Under what situations (what values of r, A and c) is the magnitude of the acceleration at the start ($t = 0^+$) bigger than the magnitude of the acceleration as $t \to \infty$?



$$\dot{\theta} = A(1-e^{-ct}) = \begin{cases} 0, & t=0 \\ A, & t=\infty \end{cases}$$

$$\ddot{\theta} = Ace^{-ct} = \begin{cases} Ac, & t=0 \\ 0, & t=\infty \end{cases}$$

Assume $c > 0$

$$|\vec{a}|^2 = r^2\dot{\theta}^4 + r^2\ddot{\theta}^2$$

$$= \begin{cases} 0 + r^2A^2c^2, & t=0^+ \\ r^2A^4 + 0, & t=\infty \end{cases}$$

$$|\vec{a}|_{t=0} > |\vec{a}|_{t=\infty} \Rightarrow |\vec{a}|^2_{t=0} > |\vec{a}|^2_{t=\infty}$$

$$\Rightarrow r^2A^2c^2 > r^2A^4$$

$$\Rightarrow c^2 > A^2$$

$$\Rightarrow \boxed{c > |A|} \text{ and}$$