

9.47

$$\ddot{x} + \lambda^2 x = C_0, \quad x(0) = x_0, \quad \dot{x}(0) = 0$$

Solution consists of $x(t) = x_h(t) + x_p(t)$

$$\rightarrow \ddot{x}_h + \lambda^2 x_h = 0 \quad \therefore x_h(t) = c_1 \cos(\lambda t) + c_2 \sin(\lambda t)$$

$$x_p(t) = C_0/\lambda^2$$

$$\therefore x(t) = c_1 \cos(\lambda t) + c_2 \sin(\lambda t) + C_0/\lambda^2$$

$$\dot{x}(t) = -\lambda c_1 \sin(\lambda t) + \lambda c_2 \cos(\lambda t)$$

Using BC's: $\dot{x}(0) = \lambda c_2 = 0 \rightarrow c_2 = 0$

$$x(0) = c_1 + C_0/\lambda^2 = x_0 \rightarrow c_1 = x_0 - \frac{C_0}{\lambda^2}$$

$$\therefore x(t) = \left(x_0 - \frac{C_0}{\lambda^2}\right) \cos(\lambda t) + C_0/\lambda^2$$

Find $x\left(\frac{\pi}{\lambda}\right) = \left(x_0 - \frac{C_0}{\lambda^2}\right) \cos(\pi) + C_0/\lambda^2$

$$\therefore \boxed{x\left(\frac{\pi}{\lambda}\right) = 2C_0/\lambda^2 - x_0}$$