9.26 (by hand)

From last HW, we have the differential equation

$$m\frac{dv}{dt} = \frac{-1}{2}c\rho_w v^2 A$$

$$\text{OR} \quad \frac{dv}{v^2} = \frac{-1}{2m}c\rho_w A\,dt$$

$$\therefore \quad \frac{-1}{v} = \frac{-1}{2m}c\rho_w At + C_0 \quad , \quad v(0) = v_0$$

$$\therefore \quad \frac{-1}{v_0} = C_0 \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{2m}c\rho_w At + \frac{1}{v_0}$$

We want to know when $v = 5\ m/s$ (stopping)

$$\frac{1}{v} = \frac{c\rho_w Atv_0 + 2m}{2mv_0} \quad \therefore \quad v(t) = \frac{2mv_0}{c\rho_w Atv_0 + 2m}$$

$$x(t) = \int v\,dt = \int \frac{2mv_0}{c\rho_w Atv_0 + 2m}\,dt$$

$$= \frac{2m\cancel{v_0}}{c\rho_w A\cancel{v_0}}\ln(c\rho_w Av_0t + 2m) + C_1 \quad , \quad C_1 = \frac{-2m\ln(2m)}{c\rho_w A}$$

$m = 0.002$ kg, $c = 1$, $\rho_w = 1000\ kg/m^3$, $A = 2.55\times10^{-5}\ m^2$, $v_0 = 400\ m/s$

$$v(t) = 5 = \frac{2(.002)(400)}{1000(2.55\times10^{-5})(400)t + 2(.002)} = \frac{1.6}{10.2t + .004} = 5$$

$$\Rightarrow \quad 51t + .02 = 1.6 \quad \therefore \quad t = 0.03098\ s$$

$$x(0.03098) = \frac{2(.002)(400)}{1000(2.55\times10^{-5})(400)}\ln\left(1000(2.55\times10^{-5})(400)(.03098) + 2(.002)\right)$$

$$-\frac{2(.002)\ln(.004)}{1000(2.55\times10^{-5})} = 0.1569\ln(0.32) + 0.8661$$

$$\therefore \quad \boxed{x = 0.687\ m}$$

9.38

Given: $F_{draw,min} = 25$ lbs. , $F_{draw,max} = 75$ lbs.

$l_{draw,min} = 24$ in. , $l_{draw,max} = 30$ in.

mass = 3/4 ance = 0.0469 lbm.

a) Using energy and work:

$F_1 = 25\text{lbs}\left(\frac{x}{l_{draw}}\right)$

$W_1 = \int F_1\,dx = \int_0^2 \frac{25x}{2}dx$

$= \frac{25x^2}{4}\Big|_0^2 = 25$ lb-ft

We know $W_1 = \frac{1}{2}mv_1^2$

$\Rightarrow v_1 = \sqrt{\frac{2W_1}{m}}$

$= \sqrt{\frac{2(25\text{lb-ft})}{0.0469\text{lbm}}} = \underline{32.7 \text{ ft/s}}$

$F_2 = 75\text{ lbs}\left(\frac{x}{l_{draw}}\right)$

$W_2 = \int_0^{2.5} \frac{75x}{2.5}dx$

$= \frac{75x^2}{5}\Big|_0^{2.5} = 93.75$ lb-ft

We know $W_2 = \frac{1}{2}mv_2^2$

$\Rightarrow v_2 = \sqrt{\frac{2W_2}{m}}$

$= \sqrt{\frac{2(93.75)}{.0469}} = \underline{63.2 \text{ ft/s}}$

$\therefore \boxed{32.7 \text{ ft/s} \le v \le 63.2 \text{ ft/s}}$

note: if you got numbers like 185 ft/s ≤ v ≤ 359 ft/s, you unnecessarily multiplied the force by g.

b) Conservation of enegy:

$\frac{1}{2}\cancel{m}v_1^2 = \cancel{m}gh_1$

$\therefore h_1 = \frac{v_1^2}{2g} = \frac{32.7^2}{2\times(32.2\text{ ft/s}^2)}$

$= 16.6$ ft.

$\frac{1}{2}\cancel{m}v_2^2 = \cancel{m}gh_2$

$\therefore h_2 = \frac{v_2^2}{2g} = \frac{63.2^2}{2(32.2)}$

$= 62.0$ ft.

$\therefore \boxed{16.6 \text{ ft.} \le h \le 62.0 \text{ ft.}}$

9.40

Given from 9.35: $m = 1000\,kg$, $g = 10\,m/s^2$, $v_1 = 60\,mph$
propulsion force $F = mg$

We know Power $P = Fv$, so we must find $v(t)$ and $v(x)$ to solve this problem.

$$m\dot{v} = mg \Rightarrow dv = g\,dt \quad \text{OR} \quad v(t) = gt$$

$$\dot{x} = v = gt \Rightarrow dx = gt\,dt \quad \text{OR} \quad x(t) = \tfrac{1}{2}gt^2$$

(assuming $v(0) = x(0) = 0$)

$$v(t) = gt, \quad x = \tfrac{1}{2}gt^2 \quad \text{OR} \quad t = \sqrt{\tfrac{2x}{g}}$$

$$\therefore \quad v(x) = g\sqrt{\tfrac{2x}{g}} = \sqrt{2gx}$$

$$v_1 = gt_1 \quad \therefore \quad t_1 = v_1/g$$

$$v_1 = \sqrt{2gx_1} \quad \therefore \quad x_1 = v_1^2/2g$$

$$\bar{P}_1 = \frac{1}{x_1}\int_0^{x_1} P(x)\,dx = \frac{1}{x_1}\int_0^{x_1} mg\sqrt{2gx}\,dx = \frac{mg}{x_1}\left(\frac{2}{3}\right)\sqrt{2g x_1^3}$$

$$= \frac{2mg}{3}\sqrt{2gx_1} = \frac{2mg}{3}\sqrt{2g\frac{v_1^2}{2g}} = \underline{\frac{2}{3}mgv_1}$$

$$\bar{P}_2 = \frac{1}{t_1}\int_0^{t_1} P(t)\,dt = \frac{1}{t_1}\int_0^{t_1} mg^2t\,dt = \frac{mg^2}{\cancel{t_1}}\left(\frac{t_1^{\cancel{2}}}{2}\right) = \frac{mg^2}{2}\left(\frac{v_1}{g}\right)$$

$$= \underline{\frac{1}{2}mgv_1}$$

$$\boxed{\begin{aligned}\bar{P}_1 &= \tfrac{2}{3}mgv_1\\ \bar{P}_2 &= \tfrac{1}{2}mgv_1\end{aligned}}$$

The two results are not equal. The definition of power though is the average work over time, so $\bar{P}_2$ is more correct.

9.43

$P = 1\ HP$ (constant) $= 550\ \frac{lbf \cdot ft}{s}$

$v_0 = 0$, $m = 150\ lbm$, $F_d = 0.006 v^2$ [lbf]

1) The peak speed is the speed at which all power is resisted by drag.

$$550\ \frac{lbf \cdot ft}{s} = F_d v = 0.006 v^3$$

$$\therefore \boxed{v_{max} = 45.1\ ft/s}$$

2) $F_d \longleftarrow (m) \longrightarrow F$

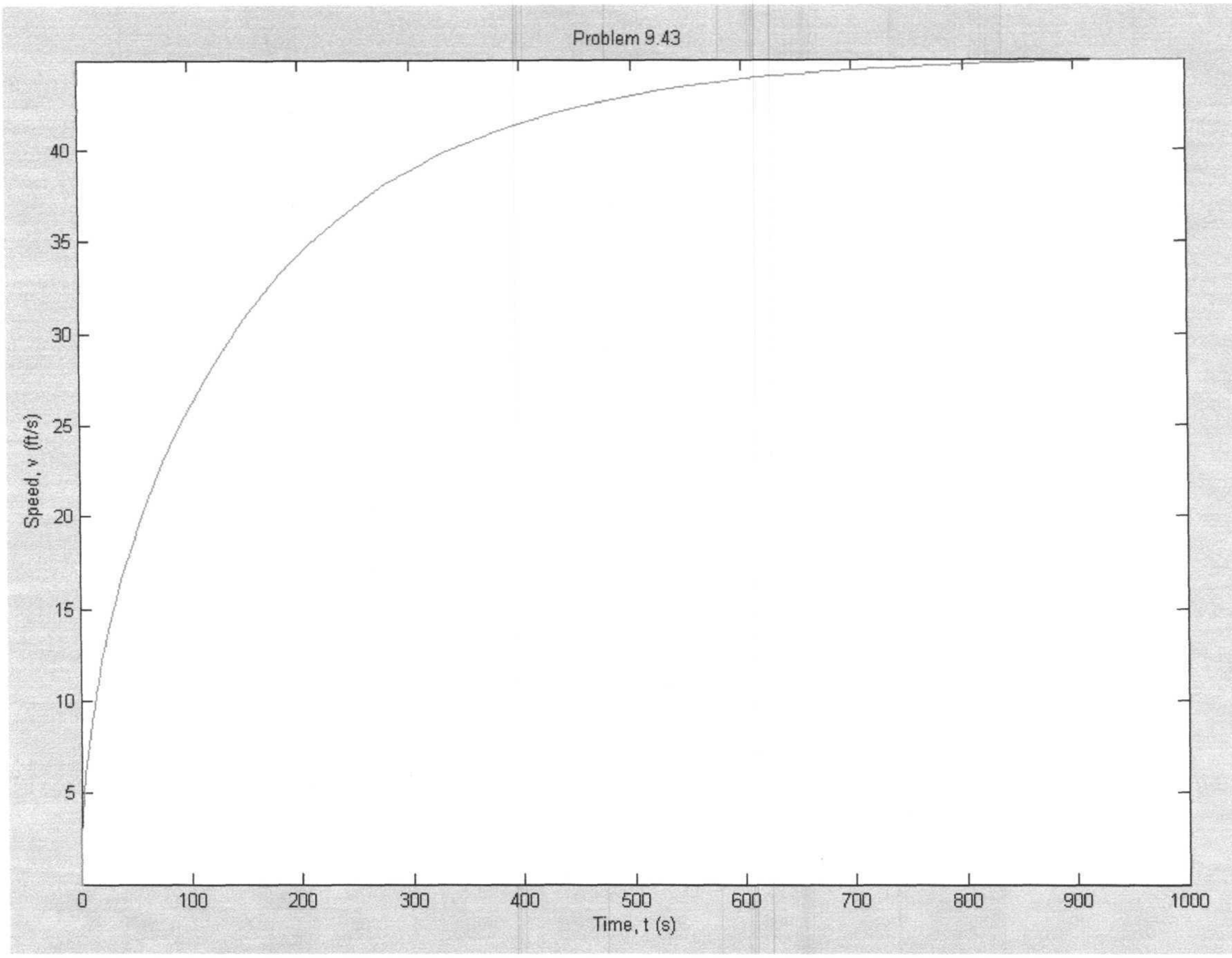
$\sum F = ma = F - F_d \quad \therefore \quad m\dot{v} = \frac{P}{v} - 0.006 v^2$

Solve numerically using Matlab.

9.43)

```
function homework943()
% Problem 9.43 Solution
% Feb 5, 2008

% CONSTANTS
P= 550  ; % power in lbf*ft/s
m= 150; % lbm
g= 32.2; % ft/s^2

% INTIAL CONDITIONS
v0= 0.001; % initial velocity, zero makes the solution explode

tspan =[0 1000]; %time interval of integration

error = 1e-4;
% Set error tolerance and use 'event detection'
options = odeset('abstol', error, 'reltol', error) ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Ask Matlab to SOLVE odes in function 'rhs'
[t v] = ode45(@rhs,tspan, v0, options, P, m, g)

%UNPACK the zarray (the solution) into sensible variables
plot (t,v)
title('Problem 9.43')
xlabel('Time, t (s)'); ylabel('Speed, v (ft/s)')
axis([0 inf -inf inf]) %inf self scales plot

end  % end of main function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%
% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function vdot = rhs(t,v,P,m,g)

vdot = P/(m*v)-0.006*v^2/m;  %  F = m a

end % end of rhs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Results from Matlab Code



3) Acceleration is the slope of the velocity on the plot above. As time goes to infinity, the acceleration goes to zero.

4) As time goes to zero, the acceleration goes to infinity. This is why the initial velocity had to be inputted as a very small number (i.e. 0.001 ft/s) instead of zero.