9.15

$m = 10\ kg$ $\quad F = ma \quad \therefore \quad a_0 = F_0/m_0 = \dfrac{10N}{10kg}$

$v(0) = 0 \quad = 1\ m/s^2$

$F_0 = 10\ N$

Force profile (a):

@ $t = 1s$: $a = 1\ m/s^2$, $v = v_0 + at = 0 + 1\,m/s^2(1s) = 1\ m/s$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1\,m/s^2)(1s)^2 = 0.5m$

@ $t = 2s$: $a = 0$, $v = v_0 + at = 1\ m/s + 0 = 1\ m/s$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0.5 + 1(1) + 0 = 1.5\ m$

@ $t = 3s$: $a = 1\ m/s^2$, $v = v_0 + at = 1 + 1(1) = \underline{2.0\ m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 1.5 + 1(1) + \frac{1}{2}(1)(1)^2 = \underline{3.0\ m}$

Force profile (b):

@ $t = 2s$: $a = 1\ m/s^2$, $v = v_0 + at = 0 + 1(2) = 2\ m/s$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1)(2)^2 = 2\ m$

@ $t = 3s$: $a = 0$, $v = v_0 + at = 2 + 0 = \underline{2.0\ m/s}$

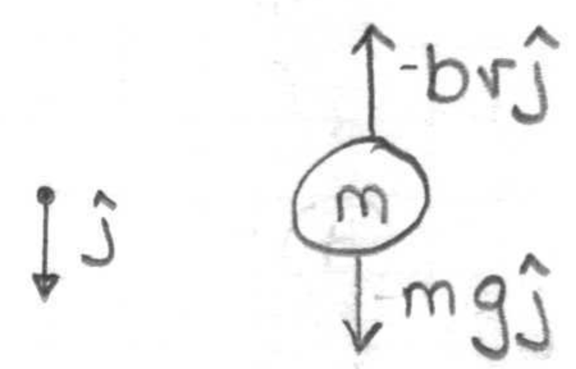
$x = x_0 + v_0 t + \frac{1}{2}at^2 = 2 + 2(1) + 0 = \underline{4.0\ m}$

9.18

Given: $m, h, F_d = bv$

Find: $v(t), x(t), v(x)$

Free body diagram:



Equation of motion:

$\sum \vec{F} = m\vec{a}$

$v = \dot{x}, \; a = \ddot{x} = \dot{v}$

$\therefore (-bv\hat{j} + mg\hat{j} = ma\hat{j}) \cdot \hat{j} \Rightarrow \dfrac{-bv + mg = m\dot{v}}{m}$

so we have $\dot{v} + \frac{b}{m}v - g = 0$ OR $\frac{dv}{dt} = g - \frac{b}{m}v$

Using separation of variables, $\dfrac{dv}{g - \frac{b}{m}v} = dt$

$\Rightarrow$ integration yields $\frac{-m}{b}\ln(g - \frac{b}{m}v) = t + c$

$v(0) = 0 \Rightarrow c = \frac{-m}{b}\ln(g)$

$\therefore \frac{-m}{b}\left[\ln(g - \frac{b}{m}v) - \ln(g)\right] = t$ OR $\ln\left(\dfrac{g - \frac{b}{m}v}{g}\right) = \dfrac{-bt}{m}$

$\Rightarrow g - \frac{b}{m}v = ge^{-\frac{bt}{m}}$ OR $\boxed{v(t) = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)}$

$x(t) = \int v(t)\,dt = \frac{mg}{b}\left(t + \frac{m}{b}e^{-\frac{bt}{m}}\right) + c$

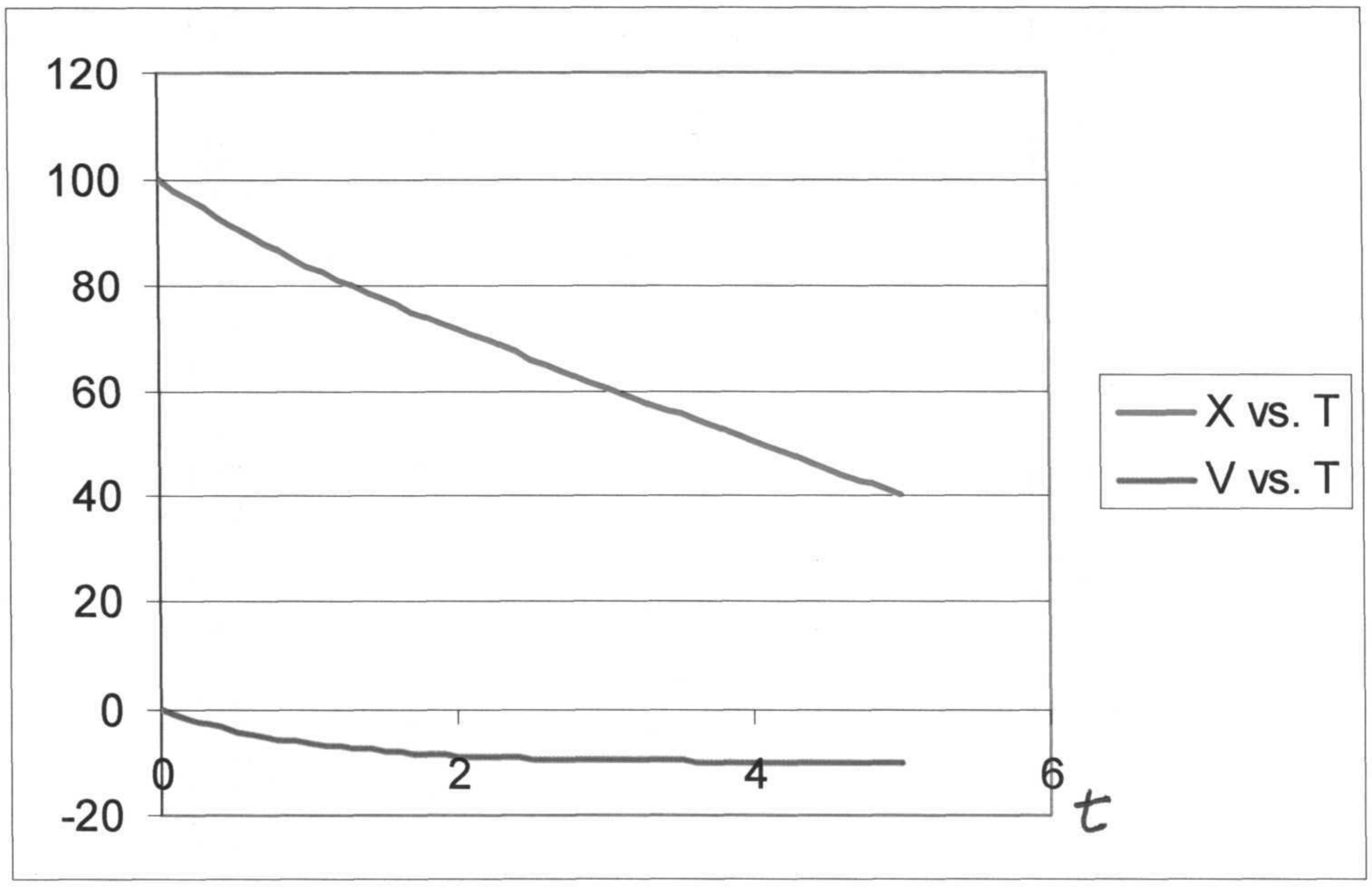
$\Rightarrow x(0) = h \rightarrow \frac{mg}{b}\left(0 + \frac{m}{b}\right) + c = h \Rightarrow c = h - \frac{m^2 g}{b^2}$

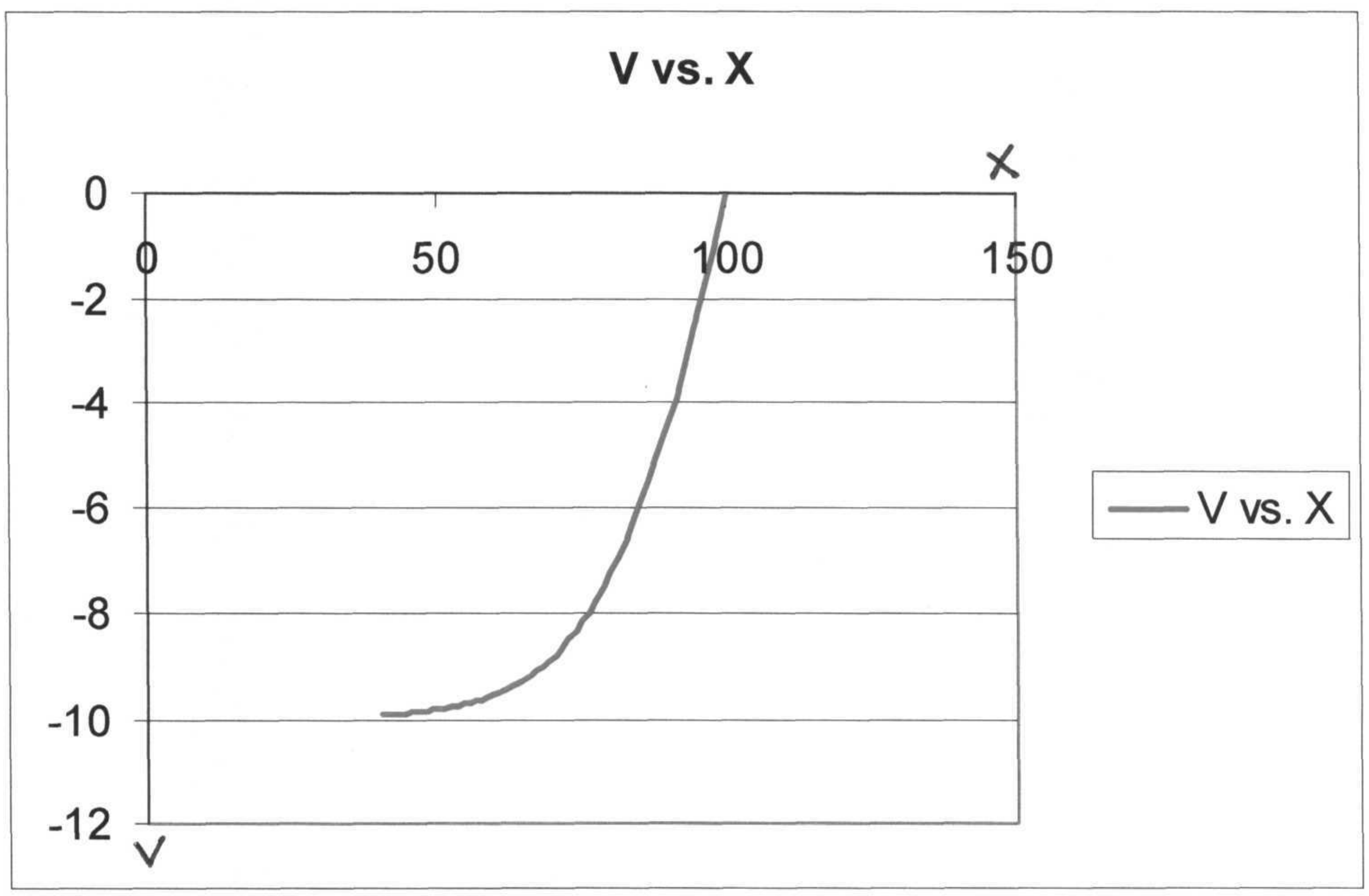
$\therefore \boxed{x(t) = \frac{mg}{b}\left(t + \frac{m}{b}e^{-\frac{bt}{m}} - \frac{m}{b}\right) + h}$

* $v(x)$ requires numerical plot — also assume $h = 100$

- see next page, assuming $g = 10\ m/s^2$, $m = 1$ kg, $b = -1$ (for simplicity)

9.23)


120
100
80
60
40
20
0
-20
0
2
4
6
t
X vs. T
V vs. T


V vs. X
0
-2
-4
-6
-8
-10
-12
0
50
100
150
x
V
V vs. X

9.26

Given: $F_D = \frac{1}{2} c \rho_w v^2 A$, $c = 1$, $m = \rho_\ell A L$, $m = 2\,g$

$V_0 = 400\ m/s$, $\rho_\ell / \rho_w = 11.3$, $d = 5.7\ mm$

FBD: $\xleftarrow{F_D} (m) \qquad \rightarrow x$ (only drag force)

$$\Sigma F = ma \Rightarrow -\frac{1}{2} c \rho_w v^2 A = m \frac{dv}{dt}$$

Plug into Matlab ODE solver with the above constants, and recognizing that $\rho_w = 1 \times 10^6\ g/m^3$.

See pages 5-6 for Matlab code, plot, and output.

9.26)

```
function homework926()
% Problem 9.26 Solution
% Jan 29, 2008

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  VARIABLES (Assume consistent units)
%  +x  = displacement
%   v  = dx/dt
%   z  = [x v],    z is the 'state vector'
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  CONSTANTS
c= 1  ;  % drag constant
pw= 1000;  % density of water (kg/m^3)
m= .002;  % mass of bullet (kg)
d= .0057; % diameter of bullet (m)

A= pi/4*d^2; % area of bullet (m^2)

% INTIAL CONDITIONS
v0= 400; % initial velocity
x0= 0; % initial position
z0= [x0 v0];

tspan =[0 1]; %time interval of integration

error = 1e-4;
% Set error tolerance and use 'event detection'
options = odeset('abstol', error, 'reltol',error,...
         'events', @stopevent) ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Ask Matlab to SOLVE odes in function 'rhs'
[t zarray] = ode45(@rhs,tspan, z0, options,c,pw,m,d);
%  The parameters m,c and g are passed to both
%  the 'rhs' function and the 'stopevent' function
% Each row of zarray is the state z at one time.

%UNPACK the zarray (the solution) into sensible variables
x = zarray(:,1);  %  x is the first  column of z
v = zarray(:,2);  %  v is the second column of z
disp(x(end));

plot (t,x)
title('Bullet Problem')
xlabel('Time, t'); ylabel('Position, x')
axis([0 inf -inf inf])  %inf self scales plot

end  % end of main function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z,c,pw,m,d)
%UNPACK state vector z into sensible variables
x = z(1);  %  y is the first  element of z
v = z(2);  %  v is the second element of z

%Define Constant Area of Bullet
A= 1/4*pi*d^2;

%The equations
xdot = v;             % kinematic relation between x and v
vdot = -1/2*c*pw*v^2*A/m;  %  F = m a

% Pack the rate of change of x, v into a rate of change
%  of state:
zdot = [xdot vdot]';  % Has to be a column vector
end % end of rhs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [value, isterminal, dir] = stopevent(t,z,c,pw,m,d)
% Have to assign numbers to value, isterminal, dir
%UNPACK z into sensible variables
x = z(1);  %  x is the first  element of z
v = z(2);  %  v is the second element of z
value      = v-5;  % stop integrating when v=5
isterminal = 1;  % 1 means stop
dir        = -1; % -1 for decreasing, +1 for increasing,
                 %  0 for any which way.
end  % end of stopevent
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Output: The total penetration distance when the velocity has dropped to 5 m/s is **0.687** meters.