9.15

\[ m = 10 \text{ kg} \]
\[ v(0) = 0 \]
\[ F_0 = 10 \text{ N} \]

\[ F = ma \quad a_0 = \frac{F_0}{m_0} = \frac{10 \text{ N}}{10 \text{ kg}} = 1 \text{ m/s}^2 \]

**Force profile (a):**

@ \( t = 1 \text{ s} \):
\[ a = 1 \text{ m/s}^2 \]  
\[ v = v_0 + at = 0 + 1 \text{ m/s}^2 (1 \text{ s}) = 1 \text{ m/s} \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 = 0 + 0 + \frac{1}{2} (1 \text{ m/s}^2) (1 \text{ s})^2 = 0.5 \text{ m} \]

@ \( t = 2 \text{ s} \):
\[ a = 0 \]  
\[ v = v_0 + at = 1 \text{ m/s} + 0 = 1 \text{ m/s} \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 = 0.5 + 1(1) + 0 = 1.5 \text{ m} \]

@ \( t = 3 \text{ s} \):
\[ a = 1 \text{ m/s}^2 \]  
\[ v = v_0 + at = 1 + 1(1) = 2.0 \text{ m/s} \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 = 1.5 + 1(1) + \frac{1}{2} (1 \text{ m/s}^2) (1 \text{ s})^2 = 3.0 \text{ m} \]

**Force profile (b):**

@ \( t = 2 \text{ s} \):
\[ a = 1 \text{ m/s}^2 \]  
\[ v = v_0 + at = 0 + 1 \text{ m/s}^2 (2 \text{ s}) = 2 \text{ m/s} \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 = 0 + 0 + \frac{1}{2} (1 \text{ m/s}^2) (2 \text{ s})^2 = 2 \text{ m} \]

@ \( t = 3 \text{ s} \):
\[ a = 0 \]  
\[ v = v_0 + at = 2 + 0 = 2.0 \text{ m/s} \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 = 2 + 2(1) + 0 = 4.0 \text{ m} \]
m = 1 \, \text{kg}, \quad x = \frac{q}{g}, \quad b = - \frac{q}{g} \\
\text{(for simplicity)} \quad \frac{\text{see next page}}{} \quad \text{Assuming} \quad g = 10 \, \text{m/s}^2 \\
\text{Equation of motion:} \\
\text{\underline{Free body diagram:}}

\text{\underline{Given:} } m', h', F = \text{br} \\
\text{\underline{Hand:} } v(t), T(t), \text{v}(x) \\
\text{\underline{Solution:} } v(t), T(t), \text{v}(x)
9.23)
Given: \( F_D = \frac{1}{2} c \rho \omega v^2 A \), \( c = 1 \), \( m = \rho_k A L \), \( m = 2 \) g
\( V_0 = 400 \) m/s, \( \rho_k/\rho_\omega = 11.3 \), \( d = 5.7 \) mm

FBD: \( F_D \) \( \rightarrow \) \( x \) (only drag force)

\[ \sum F = ma \rightarrow -\frac{1}{2} c \rho v^2 A = m \frac{dv}{dt} \]

Plug into Matlab ODE solver with the above constants, and recognizing that \( \rho_\omega = 1 \times 10^6 \) g/m³.

See pages 5-6 for Matlab code, plot, and output.
9.26)

function homework926()
% Problem 9.26 Solution
% Jan 29, 2008

% VARIABLES (Assume consistent units)
% +x = displacement
% v = dx/dt
% z = [x v], z is the 'state vector'

% CONSTANTS
c= 1 ; % drag constant
pw= 1000; % density of water (kg/m^3)
m= .002; % mass of bullet (kg)
d= .0057; % diameter of bullet (m)
A= pi/4*d^2; % area of bullet (m^2)

% INITIAL CONDITIONS
v0= 400; % initial velocity
x0= 0; % initial position
z0= [x0 v0];

tspan =[0 1]; %time interval of integration

error = 1e-4;
% Set error tolerance and use 'event detection'
options = odeset('abstol', error, 'reltol',error,...
'events', @stopevent) ;

% Ask Matlab to SOLVE odes in function 'rhs'
[t zarray] = ode45(@rhs, tspan, z0, options, c, pw, m, d);
% The parameters m, c and g are passed to both
% the 'rhs' function and the 'stopevent' function
% Each row of zarray is the state z at one time.

%UNPACK the zarray (the solution) into sensible variables
x = zarray(:,1); % x is the first column of z
v = zarray(:,2); % v is the second column of z
disp(x(end));

plot (t,x)
title('Bullet Problem')
xlabel('Time, t'); ylabel('Position, x')
axis([0 inf -inf inf]) %inf self scales plot

end % end of main function
THE DIFFERENTIAL EQUATION

function zdot = rhs(t, z, c, pw, m, d)
%UNPACK state vector z into sensible variables
x = z(1); % y is the first element of z
v = z(2); % v is the second element of z

%Define Constant Area of Bullet
A = 1/4*pi*d^2;

% The equations
xdot = v; % kinematic relation between x and v
vdot = -1/2*c*pw*v^2*A/m; % F = ma

% Pack the rate of change of x, v into a rate of change
% of state:
zdot = [xdot vdot]'; % Has to be a column vector
end % end of rhs

function [value, isterminal, dir] = stopevent(t, z, c, pw, m, d)
% Have to assign numbers to value, isterminal, dir
%UNPACK z into sensible variables
x = z(1); % x is the first element of z
v = z(2); % v is the second element of z
value = v-5; % stop integrating when v=5
isterminal = 1; % 1 means stop
dir = -1; % -1 for decreasing, +1 for increasing,
% 0 for any which way.
end % end of stopevent

Output: The total penetration distance when the velocity has dropped to 5 m/s is 0.687 meters.