Your Name: "SOLUTIONS"

TA's name and Section time: TAs

T&AM 203 Prelim 3

Tuesday April 17, 2007

Draft April 17, 2007

3 problems, 25^+ points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- ⊣ reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot\theta_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

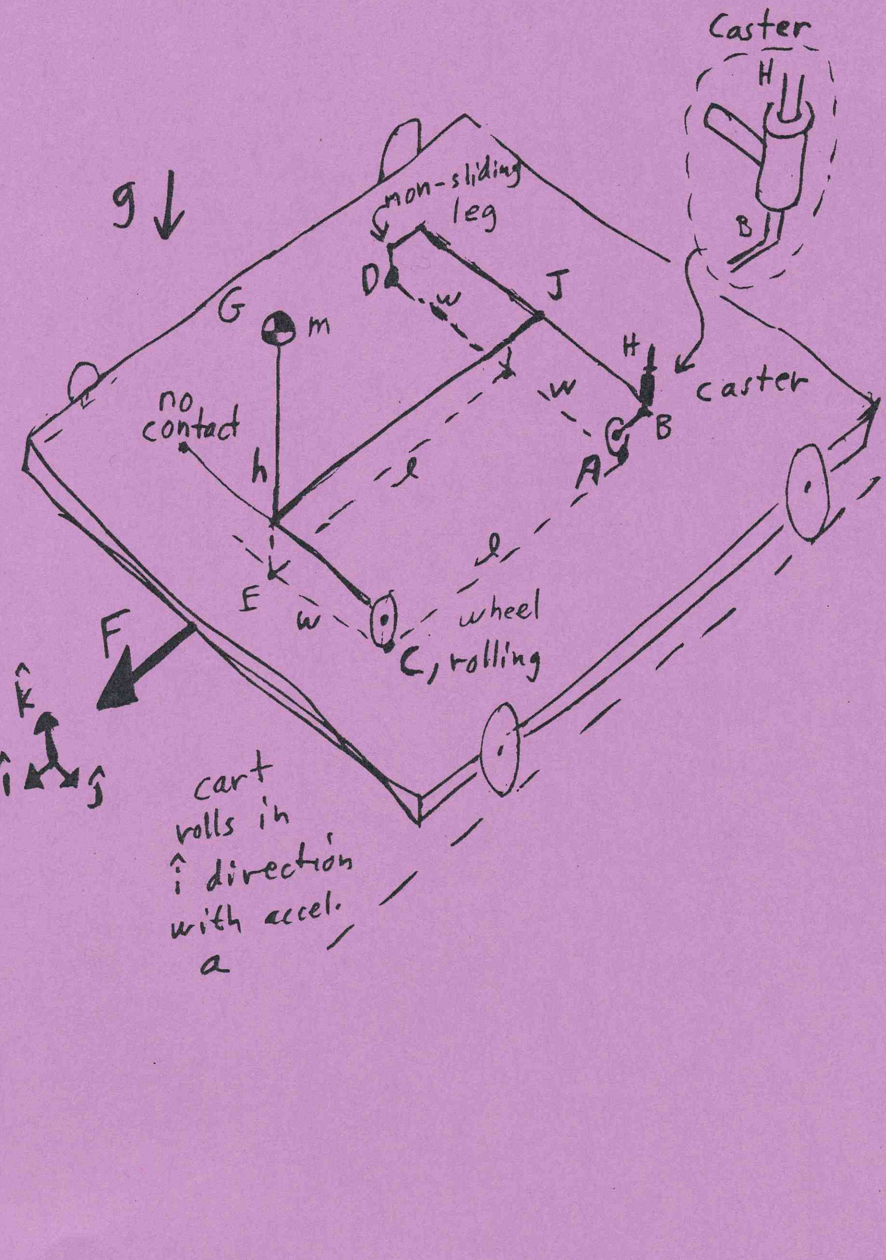
c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem 7:	/25
Problem 8:	/25
Problem 9:	/25

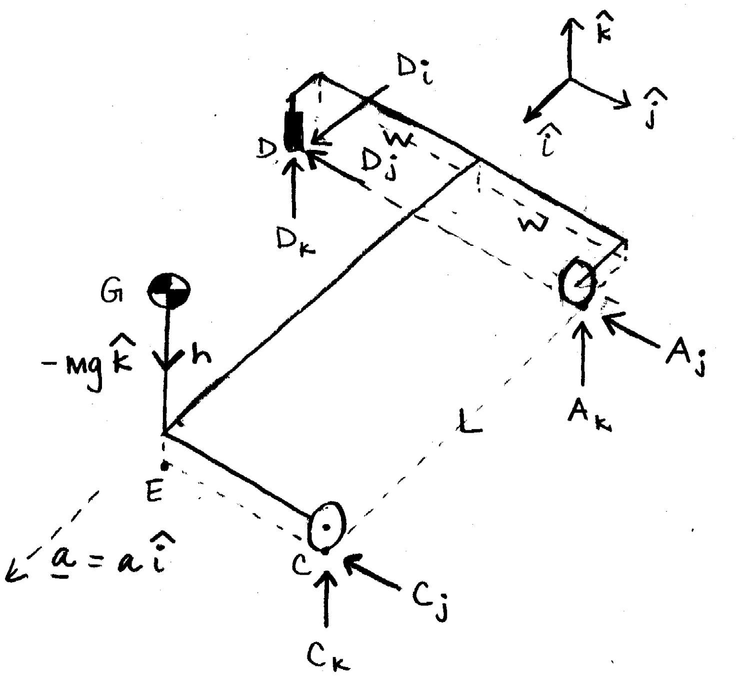
7) **(25 pt)** A funny stool CGDJHBA rides on an a level cart that accelerates with given acceleration $a\hat{\mathbf{i}}$ (Don't worry about the force F which causes this acceleration). The cart is held up at 3 points: at C there is a wheel which rolls freely in the $\hat{\mathbf{i}}$ direction; at D there is a rubber foot on a leg that can't roll or slide; at A there is a caster (ABH can rotate freely about a vertical hinge attached to the cart). The wheel at A can roll freely in the $\hat{\mathbf{i}}$ direction. The only non-negligible mass is at G. Note that if there is no acceleration ($a = 0$) the cart will fall.

a) (20 points) What is the minimum value of a needed to keep the cart from falling. Answer in terms of some or all of w, ℓ, h, m, g and base vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. Don't worry about the cart tipping if a is too big.

a) (5 points) Find the $\hat{\mathbf{j}}$ component of the reaction on the cart at A. Answer in terms of some or all of a, w, ℓ, h, m, g and base vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.



F.B.D. of Funny Stool



Note that there are 7 unknowns in this FBD – $D_i, D_j, D_k, A_j, A_k, C_j, C_k$. Thus we can't solve for all of them using just this FBD.

The stool will tip when 1 of the normal forces equals 0. This will happen first at wheel A. Thus solve for A_k, set equal to 0, and find a_{MIN}.

SOLUTION BY KEVIN

AMB/$_{DC}$:

$$\left\{ \sum \underline{M}_D \doteq \dot{\underline{H}}_D \right\} \cdot \underline{r}_{C/D}$$

$$\left\{ \begin{array}{l} (2w\hat{j} \times A_k\hat{k}) + \\ (L\hat{i} + w\hat{j} + h\hat{k}) \times -mg\hat{k} \\ + \left(\begin{array}{l}\text{contributions from other forces} \\ \text{at D, C, A which drop out}\end{array}\right) \end{array} \doteq (L\hat{i} + w\hat{j} + h\hat{k}) \times ma\hat{i} \right\} \cdot \underline{r}_{C/D}$$

$$\left\{ \begin{array}{l} 2wA_k\hat{i} + Lmg\hat{j} \\ -wmg\hat{i} + (\quad) \end{array} \doteq -wma\hat{k} + hma\hat{j} \right\} \cdot (L\hat{i} + 2w\hat{j})$$

$$-Lwmg + 2LwA_k + 2Lwmg \doteq 2whma$$

Solving the last line for A_k we get

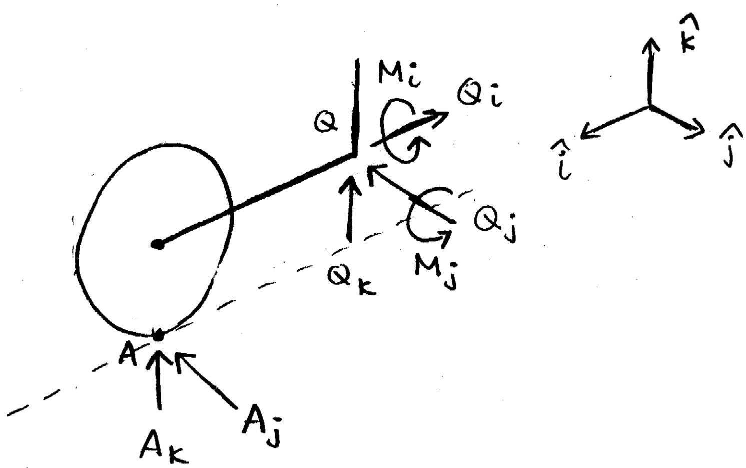
$$A_k = \frac{hma}{L} - \frac{mg}{2}$$

When $A_k = 0$, $a = a_{MIN}$

$$\Rightarrow \boxed{a_{MIN} = \frac{Lg}{2h}}$$

To solve for A_j we need to use a different FBD.

F.B.D. of Caster & Wheel

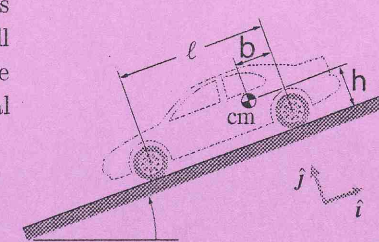


There are 5 reaction forces at Q. Only $M_k = 0$ since the wheel & caster is allowed to rotate about $\hat{k}$.

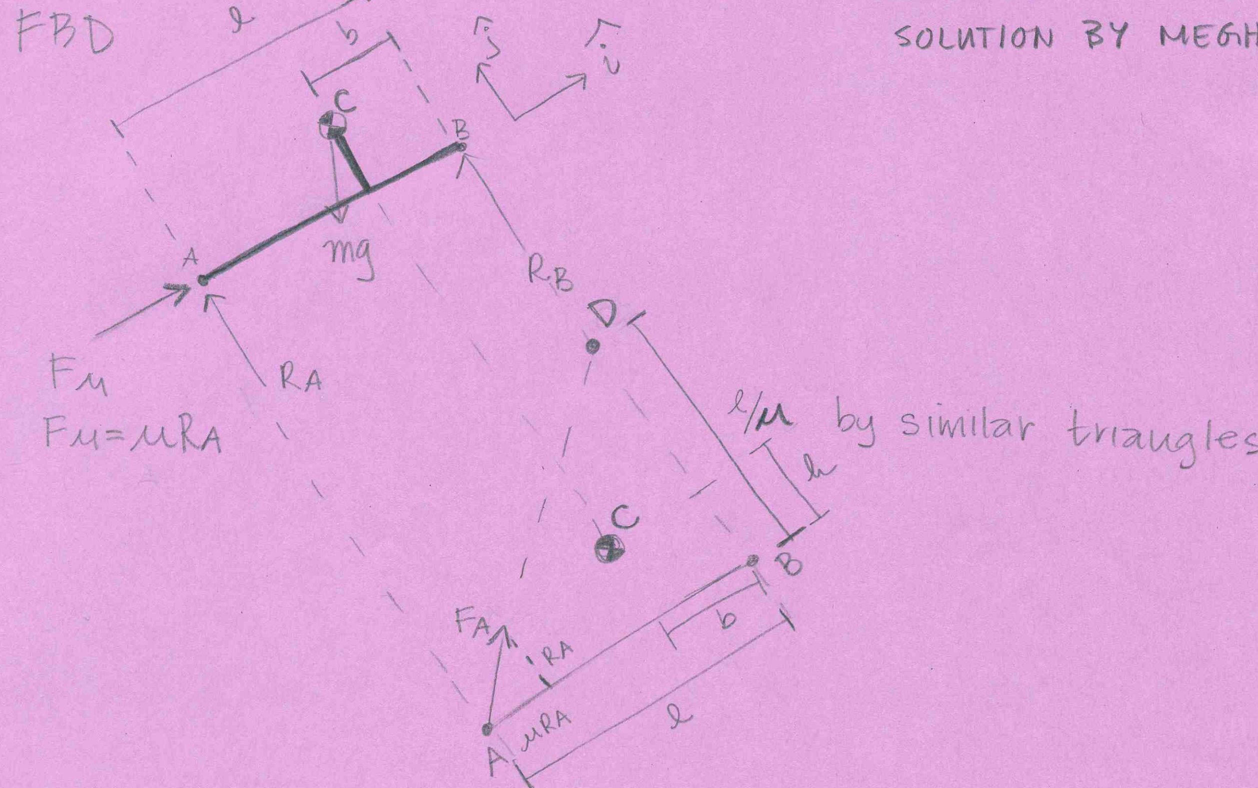
Since the wheel & caster are taken to be essentially massless

$$\left\{ \sum_1 \underline{M}_Q = \underline{0} \right\} \cdot \hat{k} \quad \Rightarrow \quad \boxed{A_j = 0}$$

8) **(25 pt)** What is the braking acceleration of the front-wheel braked car as it slides down hill. Express your answer as a function of any or all of the following variables: the slope θ of the hill, the mass m of the car, the wheel base ℓ, the height h, the distance b, the gravitational constant g and the friction coefficient μ.



SOLUTION BY MEGHAN



$$\Sigma \vec{M}_D = \vec{r}_{C/D} \times m\vec{g} = \left[-b\hat{\imath} - (\ell/\mu - h)\hat{\jmath}\right] \times mg(-\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath})$$

$$= mg\left[b\cos\theta - (\ell/\mu - h)\sin\theta\right]\hat{k}$$

$$\dot{\vec{H}}_{/D} = \vec{r}_{C/D} \times m\vec{a} = \left[-b\hat{\imath} - (\ell/\mu - h)\hat{\jmath}\right] \times ma\,\hat{\imath}$$

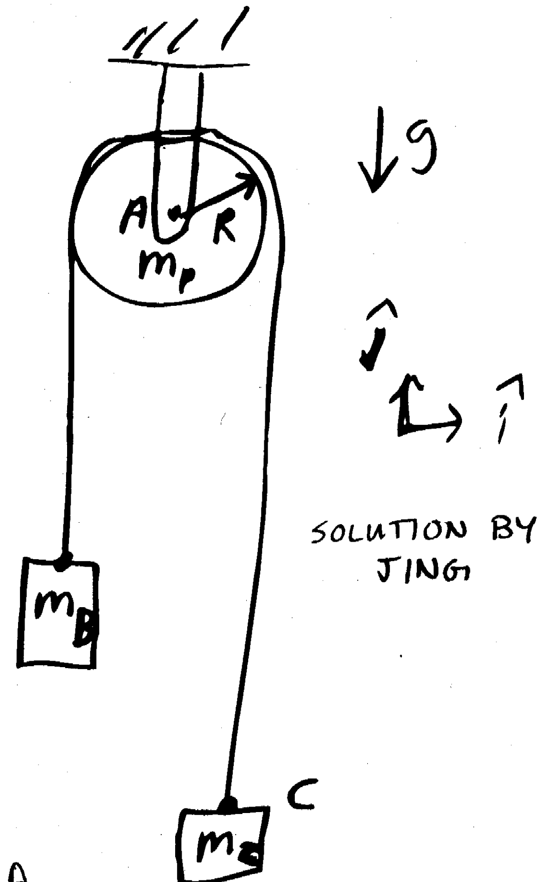
$$= (\ell/\mu - h)\,ma\,\hat{k}$$

$$\left\{\Sigma \vec{M}_{/D} = \dot{\vec{H}}_{/D}\right\}\cdot\hat{k} \Rightarrow (\ell/\mu - h)\,ma = mg\left[b\cos\theta - (\ell/\mu - h)\sin\theta\right]$$

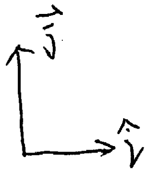
$$a = g\left[\frac{b}{\ell/\mu - h}\cos\theta - \sin\theta\right]$$

$$\boxed{a = g\left[\frac{b\mu}{\ell - \mu h}\cos\theta - \sin\theta\right]}$$

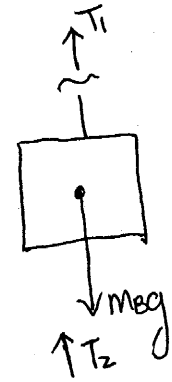
9) (25 pt) An inextensible rope doesn't slip on the round homogeneous pulley that has mass m_p and radius R. The pulley has a bearing with negligible friction. Two masses $m_C > m_B$ hang from the ends of the rope. At the instant of interest mass C is going up with velocity $v_C\,\hat{\mathbf{j}}$. Assume that the masses only move vertically. In terms of some or all of R, m_p, m_B, M_C, g, v_C and $\hat{\mathbf{j}}$ find the acceleration of B.



SOLUTION BY JING



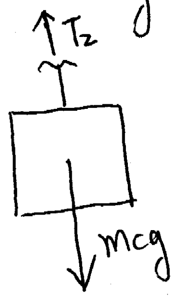
FBD of m_B



LMB of m_B:

$T_1 - m_B g = m_B a_B$ ①

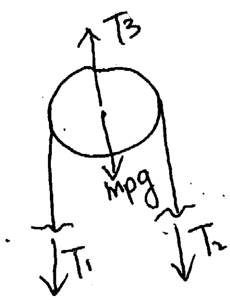
FBD of m_C



LMB of m_C

$T_2 - m_C g = m_C a_C$ ②

FBD of m_p



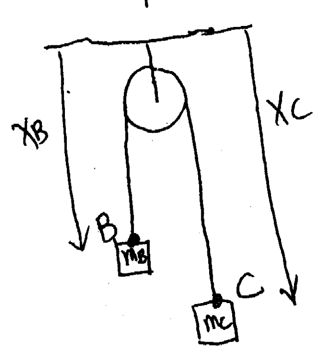
AMB of m_p about A

$(T_1 - T_2)R = I^p \alpha$ ③

where $I^p = \frac{1}{2} m_p R^2$ for the gear

By no slip condition: $\alpha = \frac{a_C}{R}$ ④

Kinematics: Ignore the string wrapped about the pulley



$X_B + X_C = l_{tot} \Rightarrow \dot{X}_B + \dot{X}_C = 0$

$\therefore \dot{X}_B = -\dot{X}_C \Rightarrow a_C = -a_B$ ⑤

Combining ①–⑤

$(m_B a_B + m_B g - m_C a_C - m_C g)R = \frac{1}{2} m_p R^2 \left(-\frac{a_B}{R}\right)$

$$\boxed{\therefore \vec{a}_B = \frac{(m_C - m_B)g}{\frac{1}{2} m_p + m_C + m_B}\,\hat{j}}$$