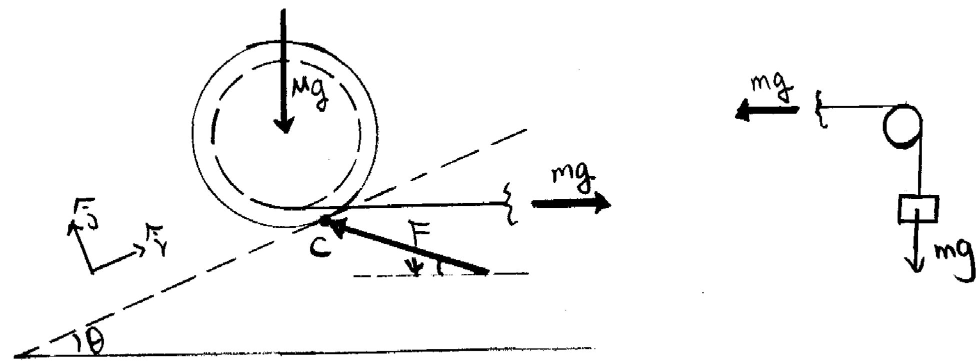
TAM 203 solns by Jing Shi

HW due Tues. March 27, 2007

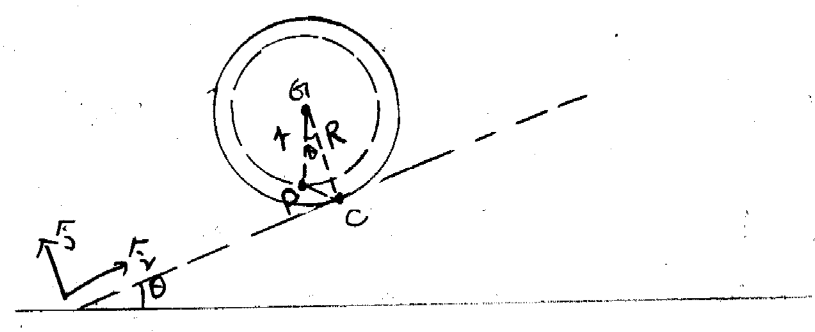
Problem: 4.69 4.72 4.97 12.16 12.18

4.69 Soln:

FBD:



Geometry



$\hat{i} \times \hat{j} = \hat{k}$

(a) The system is at rest ⇒ statics

$$\circlearrowleft \Sigma M_C = \{ \vec{r}_{G/C} \times (-Mg\hat{e}_2) + \vec{r}_{P/C} \times (mg\hat{e}_1) \} \cdot \hat{k} = 0$$

$$\vec{r}_{G/C} = R\sin\theta\,\hat{e}_1 - R\cos\theta\,\hat{e}_2$$

$$\vec{r}_{P/C} = R\sin\theta\,\hat{e}_1 - (R\cos\theta - t)\,\hat{e}_2$$

$$\Sigma M_C = \{ (R\sin\theta\,\hat{e}_1 - R\cos\theta\,\hat{e}_2) \times (-Mg\hat{e}_2) + (R\sin\theta\,\hat{e}_1 - (R\cos\theta - t)\,\hat{e}_2) \times (mg\hat{e}_1) \} \cdot \hat{k} = 0$$

$\hat{e}_1 \times \hat{e}_2 = \hat{k}$
$\hat{e}_1 \times \hat{e}_1 = 0$
$\hat{e}_2 \times \hat{e}_2 = 0$
$\hat{k} \cdot \hat{k} = 1$

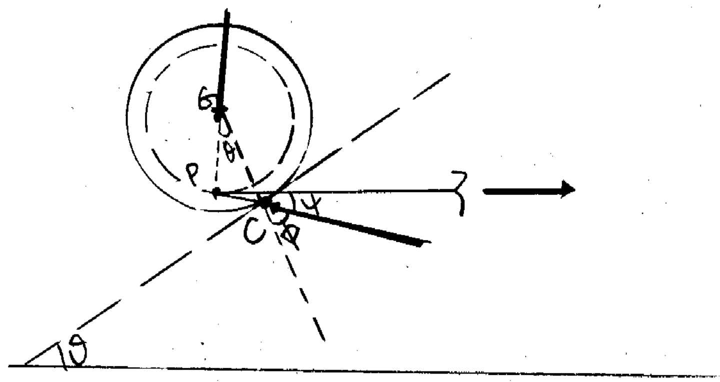
$$\therefore \Sigma M_C = \{ -MgR\sin\theta\,\hat{k} + (R\cos\theta - t)mg\,\hat{k} \} \cdot \hat{k}$$

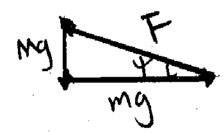
$$= -MgR\sin\theta + (R\cos\theta - t)mg = 0$$

$$\boxed{\frac{m}{M} = \frac{\sin\theta}{\cos\theta - \frac{t}{R}}}$$

a) alternative solution

The reel is a three force body so the direction of force $\vec{F}$ at C is determined: its line of action must pass through the intersection of line of actio of other two forces.





From triangle $\triangle GPC$

$$\frac{R}{\sin(\pi-\phi-\theta)} = \frac{|\vec{r}_{pc}|}{\sin\theta}$$

$$\sin(\pi-\phi-\theta) = \frac{R\sin\theta}{|\vec{r}_{pc}|}$$

$$\pi-\phi-\theta = \psi + \frac{\pi}{2}$$

$$\therefore \psi = \frac{\pi}{2}-\phi-\theta$$

$$\cos\psi = \cos\left(\frac{\pi}{2}-\phi-\theta\right) = \sin(\phi+\theta) = \sin(\pi-\phi-\theta)$$

$$\cos\psi = \frac{R\sin\theta}{|\vec{r}_{pc}|}$$

$$\therefore \frac{M}{m} = \tan\psi = \frac{\sin\psi}{\cos\psi}$$

$$= \frac{\sqrt{1-\frac{R\sin\theta}{|\vec{r}_{pc}|}}}{\frac{R\sin\theta}{|\vec{r}_{pc}|}}$$

$$= \frac{\sqrt{|\vec{r}_{pc}|^2 - R^2\sin^2\theta}}{R\sin\theta}$$

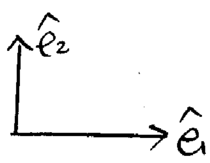
$$\left.\begin{array}{l}|\vec{r}_{pc}|^2 = R^2+r^2-2rR\cos\theta \\ r = \frac{1}{2}R\end{array}\right\} \Rightarrow \quad = \frac{\sqrt{R^2\cos^2\theta+\frac{1}{4}R^2-R^2\cos\theta}}{R\sin\theta}$$

$$= \frac{\cos\theta-\frac{1}{2}}{\sin\theta}$$

$$\boxed{\frac{m}{M} = \frac{\sin\theta}{\cos\theta-\frac{1}{2}}}$$

b) The pulley is massless

$$\boxed{T = mg = \frac{\sin\theta}{\cos\theta - \frac{1}{2}} Mg}$$

c) 

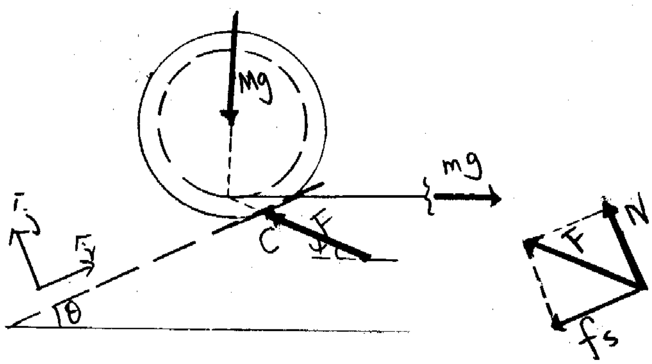
$$\Sigma F_x = 0 = mg + F_x = 0 \quad \therefore F_x = -\frac{\sin\theta}{\cos\theta - \frac{1}{2}} Mg$$

$$\Sigma F_y = 0 = -Mg + F_y = 0 \quad F_y = Mg$$

$$\therefore \vec{F} = -\frac{\sin\theta}{\cos\theta - \frac{1}{2}} Mg\hat{e}_1 + Mg\hat{e}_2$$

$$= \boxed{Mg\left(-\frac{\sin\theta}{\cos\theta - \frac{1}{2}}\hat{e}_1 + \hat{e}_2\right)}$$

d)



If the reel is about to slip

$$f_s = \mu N \qquad \mu = \frac{f_s}{N}$$

$$\Sigma \vec{F} = \vec{0}$$

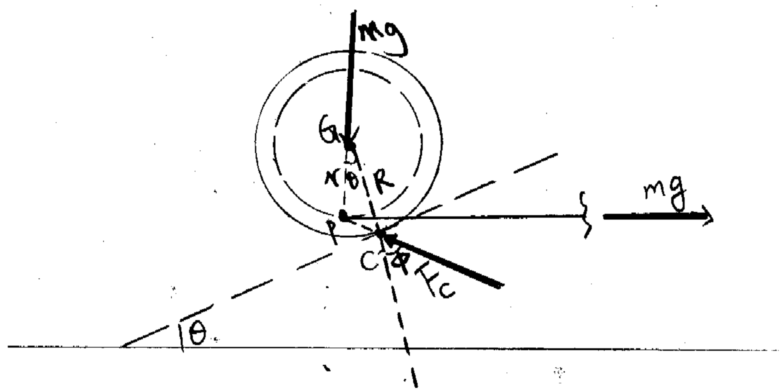
$$-Mg(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) + mg(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) - f_s\hat{i} + N\hat{j} = 0$$

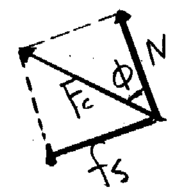
$$\therefore f_s = -Mg\sin\theta + mg\cos\theta = Mg\left(-\sin\theta + \frac{\sin\theta\cos\theta}{\cos\theta - \frac{1}{2}}\right),$$

$$N = Mg\cos\theta + mg\sin\theta = Mg\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta - \frac{1}{2}}\right)$$

$$\therefore \mu = \frac{f_s}{N} = \boxed{\frac{\frac{1}{2}\sin\theta}{1 - \frac{1}{2}\cos\theta}}$$

d) alternative solution





$\vec{F_c} = \vec{N} + \vec{f_s}$ $\quad f_s = \mu N \quad \mu = \frac{f_s}{N} = \tan\phi$

$\tan\phi$ can be calculated by the geometry of triangle ΔGPC

$$\frac{r}{\sin\phi} = \frac{|\vec{r}_{cp}|}{\sin\theta} \Rightarrow \sin\phi = \frac{r\sin\theta}{|\vec{r}_{cp}|}$$

$\because \quad |\vec{r}_{cp}|^2 = r^2 + R^2 - 2rR\cos\theta$

$$\therefore \tan\phi = \frac{\sin\phi}{\cos\phi}$$

$$= \frac{\sin\phi}{\sqrt{1-\sin^2\phi}}$$

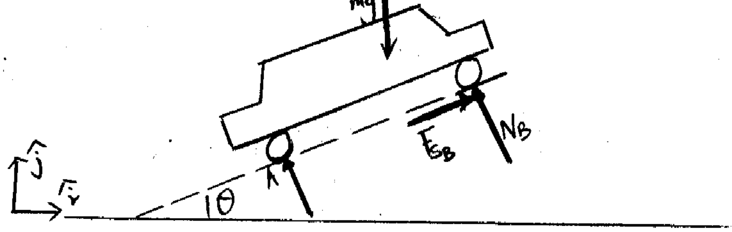
$$= \frac{\frac{r\sin\theta}{|\vec{r}_{cp}|}}{\sqrt{1-\frac{r^2\sin^2\theta}{|\vec{r}_{cp}|^2}}}$$

$$= \frac{r\sin\theta}{\sqrt{r^2+R^2-2rR\cos\theta - r^2\sin^2\theta}}$$

$$r = \tfrac{1}{2}R \Rightarrow = \frac{\frac{1}{2}\sin\theta}{\sqrt{\frac{1}{4}+1-\cos\theta-\frac{1}{4}\sin^2\theta}}$$

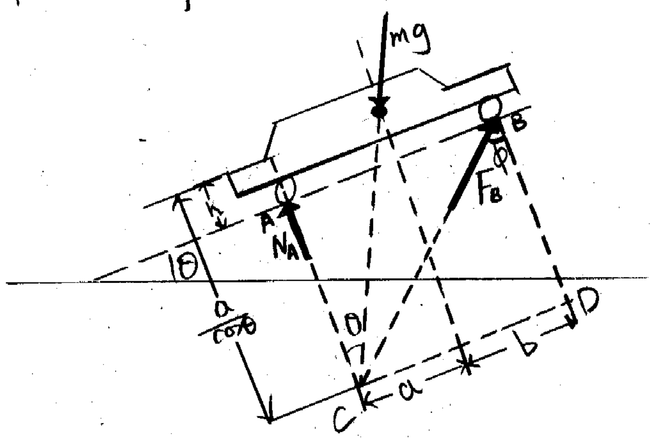
$$\boxed{= \frac{\frac{1}{2}\sin\theta}{1-\frac{1}{2}\cos\theta}}$$

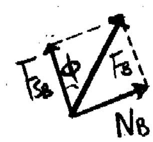
4.72 Soln:



Method 1

The car is a three force body. so the direction of the resultant force $\vec{F}_B$ is determined. Its line of action must pass through the intersection of line of action of other two forces





$\vec{F}_B = \vec{N}_B + \vec{F}_{SB}$ $\quad F_{SB} = \mu N_B \quad \mu = \frac{F_{SB}}{N_B} = \tan\phi$

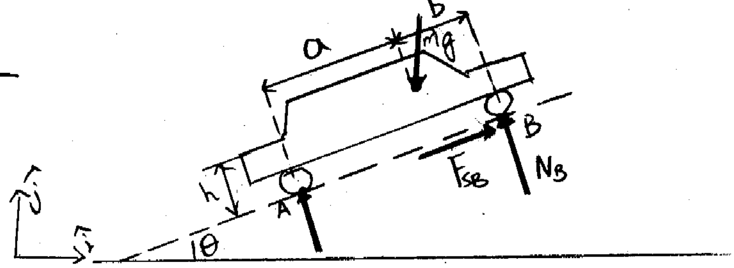
$\tan\phi$ can be calculated by the geometry of triangle $\triangle CDB$

$$|\vec{r}_{CD}| = a+b \qquad |\vec{r}_{DB}| = \frac{a}{\tan\theta} - h$$

$$\therefore \tan\phi = \frac{|\vec{r}_{CD}|}{|\vec{r}_{DB}|} = \frac{a+b}{\frac{a\cos\theta}{\sin\theta} - h}$$

$$\boxed{\mu = \frac{(a+b)\sin\theta}{a\cos\theta - h\sin\theta}}$$

Method 2



$\hat{i} \times \hat{j} = \hat{k}$

$$\sum M_A = \left\{ \vec{r}_{G/A} \times (mg\hat{j}) + \vec{r}_{B/A} \times N_B(-\sin\theta\hat{i} + \cos\theta\hat{j}) \right\} \cdot \hat{k} = 0$$

$$\vec{r}_{G/A} = (a\cos\theta - h\sin\theta)\hat{i} + (a\sin\theta + h\cos\theta)\hat{j}$$

$$\vec{r}_{B/A} = (a+b)\cos\theta\hat{i} + (a+b)\sin\theta\hat{j}$$

$$\sum M_A = \Big\{ \big((a\cos\theta - h\sin\theta)\hat{i} + (a\sin\theta + h\cos\theta)\hat{j}\big) \times (mg\hat{j}) +$$

$$\big((a+b)\cos\theta\hat{i} + (a+b)\sin\theta\hat{j}\big) \times N_B(-\sin\theta\hat{i} + \cos\theta\hat{j}) \Big\} \cdot \hat{k}$$

$\hat{i} \times \hat{i} = 0$
$\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{j} = 0$

$$= \Big\{ (a\cos\theta - h\sin\theta)mg\hat{k} + N_B(a+b)\underbrace{(\sin^2\theta + \cos^2\theta)}_{1}\hat{k} \Big\} \cdot \hat{k}$$

$$= (a\cos\theta - h\sin\theta)mg + N_B(a+b) = 0$$

$$\therefore N_B = \frac{mg(h\sin\theta - a\cos\theta)}{a+b}$$

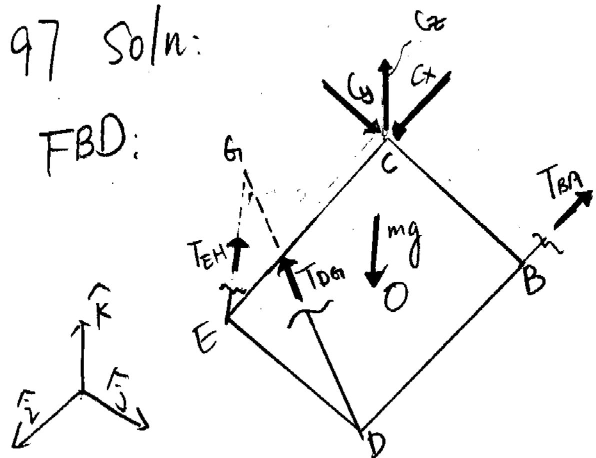
$\sum \vec{F} = 0$ $\qquad F_{SB} - mg\sin\theta = 0$ $\qquad F_{SB} = mg\sin\theta$

$N_B\mu = F_{SB}$ $\quad \therefore \mu = \dfrac{F_{SB}}{N_B}$

$$\boxed{\mu = \frac{\sin\theta(a+b)}{h\sin\theta - a\cos\theta}}$$

4.97 Soln:

a) FBD:



b) see part h) for all possible answers

c) $\sum \vec{F} = 0$

$$\sum F_x = C_x - T_{BA} = 0$$
$$\sum F_y = -T_{DG}\cos\theta + C_y = 0$$
$$\sum F_z = T_{EH} + T_{DG}\sin\theta - mg + C_z = 0$$

d) $\circlearrowleft \sum \vec{M}_C = 0$

$$\vec{r}_{CD} \times \vec{T}_{DG} + \vec{r}_{CB} \times \vec{T}_{BA} + \vec{r}_{CE} \times \vec{T}_{EH} + \vec{r}_{CO} \times (-mg\vec{k}) = 0$$

$$(\vec{i}+\vec{j}) \times T_{DG}(-\cos\theta\,\vec{j} + \sin\theta\,\vec{k}) + \vec{j} \times (-T_{BA}\,\vec{i}) + \vec{j} \times (T_{EH}\,\vec{k}) + \tfrac{1}{2}(\vec{i}+\vec{j}) \times (-mg\,\vec{k}) = 0$$

$$\circlearrowleft \sum \vec{M}_C = -T_{DG}\cos\theta\,\hat{k} - T_{DG}\sin\theta\,\hat{j} + T_{DG}\sin\theta\,\hat{i} + T_{BA}\,\hat{k} - T_{EH}\,\hat{j} + \tfrac{1}{2}mg\,\hat{j} - \tfrac{1}{2}mg\,\hat{i}$$
$$= (T_{DG}\sin\theta - \tfrac{1}{2}mg)\hat{i} + (-T_{DG}\sin\theta - T_{EH} + \tfrac{1}{2}mg)\hat{j} + (-T_{DG}\cos\theta + T_{BA})\hat{k}$$
$$= 0$$

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{i} \times \hat{k} = \hat{j}$

e) The force balance equations are in (c) above

The moment balance equations:

$$\sum M_x = 0: \quad T_{DG}\sin\theta - \tfrac{1}{2}mg = 0$$
$$\sum M_y = 0 \quad -T_{DG}\sin\theta - T_{EH} + \tfrac{1}{2}mg = 0$$
$$\sum M_z = 0 \quad -T_{DG}\cos\theta + T_{BA} = 0$$

f)

```
% Engr 203 homework solution, produced by Jing Shi
% Due Tues March 27,2007
% Problem 4.97 f)
m=5;
g=10;
mg=m*g;
theta=pi/4;

%construct the coefficient matrix

A=[1,0,0,0,0,-1;
   0,1,0,0,-cos(theta),0;
   0,0,1,1,sin(theta),0;
   0,0,0,0,sin(theta),0;
   0,0,0,-1,-sin(theta),0;
   0,0,0,0,-cos(theta),1];

b=[0;0;mg;mg/2;-mg/2;0];

% solve for the unknown forces
% X=[Cx;Cy;Cz;T_EH;T_DG;T_BA]

X=A\b;

Results:

X =

   25.0000
   25.0000
   25.0000
         0
   35.3553
   25.0000
```

g) Take moment about CD

$C_x \cdot C_y \cdot C_z \cdot T_{DG} \cdot T_{BA}$ all go through this axis

$$\Sigma M_{CD} = 0 \qquad \vec{T}_{DE} \times \vec{T}_{EH} = 0$$

$$\boxed{T_{EH} = 0}$$

h) Challenge: For how many of the reactions can you find one equation with one unknown?

① T_{EH} (see (g) above)

② T_{BA}: $\boxed{\circlearrowleft \Sigma M_{BA} = 0}$ → Take moment about BA

$$\Sigma M_{BA} = \frac{1}{\sqrt{2}}(\hat{k}+\hat{i}) \cdot \left[\frac{1}{2}(\hat{i}+\hat{j}) \times (-mg\hat{k}) + \hat{j} \times (-T_{BA}\hat{i})\right]$$

$$= \frac{1}{\sqrt{2}}(\hat{k}+\hat{i}) \cdot \left[+\frac{1}{2}mg\hat{j} - \frac{1}{2}mg\hat{i} + T_{BA}\hat{k}\right]$$

$$= \frac{1}{\sqrt{2}}\left(T_{BA} - \frac{1}{2}mg\right) = 0$$

$$\therefore \boxed{T_{BA} = \frac{1}{2}mg}$$

③ C_z $\boxed{\circlearrowleft \Sigma M_{ED} = 0}$ → Take moment about ED

$$\Sigma M_{ED} = \hat{j} \cdot \left[-\hat{i} \times (C_z\hat{k} + \frac{1}{2}(-\hat{i}+\hat{j}) \times (-mg\hat{k})\right]$$

$$= \hat{j} \cdot \left(\left(C_z - \frac{1}{2}mg\right)\hat{j} - \frac{1}{2}mg\hat{i}\right)$$

$$= C_z - \frac{1}{2}mg = 0$$

$$\boxed{C_z = \frac{1}{2}mg}$$

④ C_y Take moment about G in the $\hat{i}$ direction

$$\boxed{(\circlearrowleft \Sigma \vec{M}_G) \cdot \hat{i} = \Sigma M_{Gx} = 0}$$

$$\Sigma M_{Gx} = \hat{i} \cdot \left[-(\hat{k}+\hat{i}) \times C_y\hat{j} + (-\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} - \hat{k}) \times (-mg\hat{k})\right]$$

$$= \hat{i} \cdot \left[+C_y\hat{i} - C_y\hat{k} - \frac{1}{2}mg\hat{j} - \frac{1}{2}mg\hat{i}\right]$$

$$= +C_y - \frac{1}{2}mg = 0 \qquad \boxed{C_y = \frac{1}{2}mg}$$

⑤ T_{DG}

$\boxed{\circlearrowleft \Sigma M_{CE} = 0}$ → Take moment about CE.

$$\Sigma M_{CE} = -\hat{i} \cdot \left[\hat{j} \times (-\hat{j} + \hat{k}) \frac{T_{DG}}{\sqrt{2}} + \left(-\tfrac{1}{2}\hat{i} + \tfrac{1}{2}\hat{j}\right) \times (-mg\hat{k}) \right]$$

$$= -\hat{i} \cdot \left(+\frac{T_{DG}}{\sqrt{2}}\hat{i} - \tfrac{1}{2}mg\,\hat{j} - \tfrac{1}{2}mg\,\hat{i} \right] = \frac{-T_{DG}}{\sqrt{2}} + \tfrac{1}{2}mg = 0$$

$$\boxed{T_{DG} = \frac{\sqrt{2}}{2}mg}$$

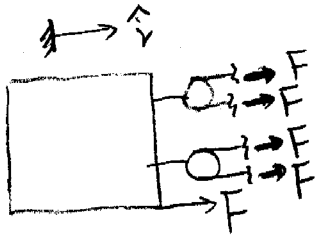
All the answers are the same as the results in part C. We find 5 of 6 using 1 equation with 1 unknown

Note: There is no single eqn. to find C_x

12.16

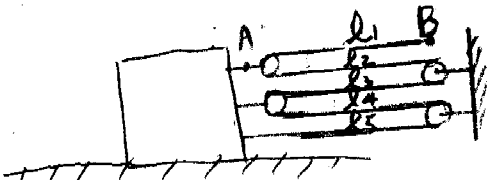
a)

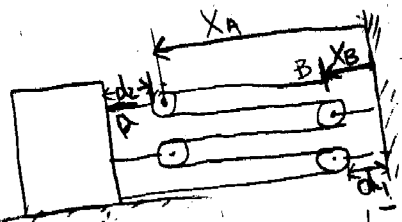
FBD:



linear momentum balance:

$5F = m a_A$ $\quad \boxed{a_A = \frac{5F}{M}}$





Note: ignore string wrapped around pulleys

$\vec{a}_A = \ddot{X}_A(-i)$

$\vec{a}_B = \ddot{X}_B(-i)$

kinematics: $l_{tot} = l_1 + l_2 + l_3 + l_4 + l_5$

$= (X_A - X_B + 4(X_A - d_1) + d_2$

$\boxed{d_1 . d_2 \text{ are const}} \Rightarrow \dot{l}_{tot} = 5\dot{X}_A - \dot{X}_B$, $a_A = -\ddot{X}_A$, $a_B = -\ddot{X}_B$

$\dot{l}_{tot} = 0 \Rightarrow 5\dot{X}_A = \dot{X}_B \Rightarrow 5a_A = a_B$

$\boxed{a_B = 5a_A = \frac{25F}{m}}$

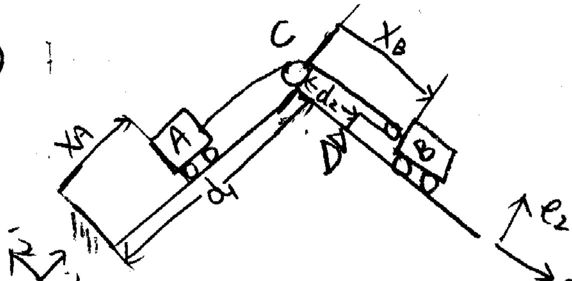
or we can use energy balance

$F\, V_B = ma \cdot V_A$

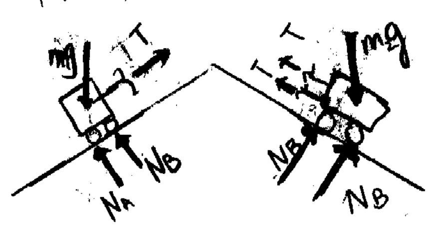
$\therefore \quad V_B = 5V_A$

$\boxed{a_B = 5a_A = \frac{25F}{m}}$

b)



FBD:



Linear momentum

$T - mg\sin 30^\circ = m_1 a_A$ ①

$mg\sin 60^\circ - 2T = m_2 a_B$ ②

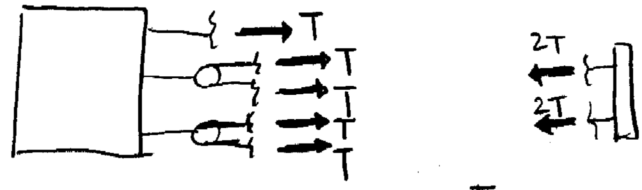
kinematics: $l_{AC} + l_{CB} + l_{BD} = l_{tot} = d_1 - X_A + X_B + X_B - d_2$

$\boxed{d_1\, d_2 \text{ are const}} \Rightarrow \dot{l}_{tot} = -\dot{X}_A + 2\dot{X}_B = 0$

$\dot{X}_A = 2\dot{X}_B$ $\quad \dot{X}_A = V_A, \dot{X}_B = V_B \Rightarrow a_A = 2a_B$ ③

① ② ③ $\Rightarrow$ $\boxed{a_A = \frac{\frac{\sqrt{3}}{2}m_2 g - m_1 g}{2m_1 + \frac{1}{2}m_2}}$ $\quad \boxed{a_B = \frac{\frac{\sqrt{3}}{2}m_2 g - mg}{4m_1 + m_2}}$

) FBD



$4T = F \quad T = \frac{F}{4}$

Linear momentum balance $5T = m a_A$ $\boxed{a_A = \frac{5}{4}\frac{F}{m}}$

Use Energy balance

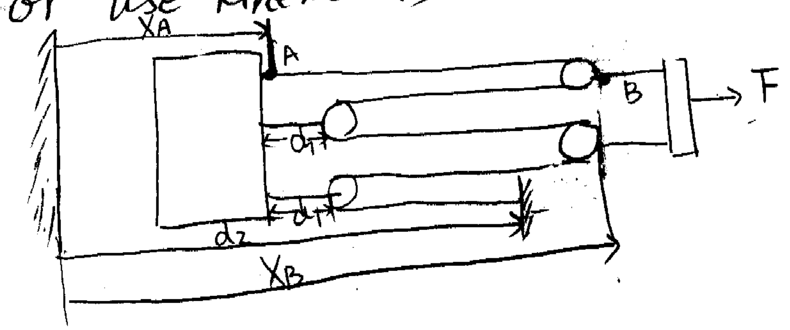
$$F V_B = m a_A V_A$$

$$V_B = \frac{5}{4} V_A$$

$$\boxed{a_B = \frac{5}{4} a_B = \frac{25}{16}\frac{F}{m}}$$

or use kinematics

Ignore the string wrapped around pulleys



$$\therefore l_{tot} = X_B - X_A + (X_B - X_A - d_1)\cdot 3 + d_2 - X_A - d_1$$

$$= 4X_B - 5X_A - 4d_1 + d_2$$

d_1, d_2 are const

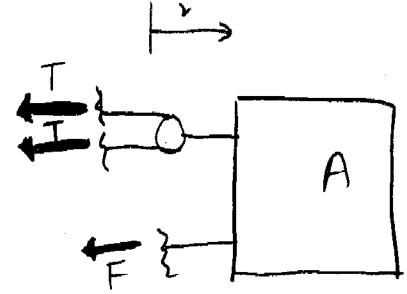
$$\therefore \dot{l}_{tot} = 4\dot{X}_B - 5\dot{X}_A \qquad \dot{X}_B = \frac{5}{4}\dot{X}_A \qquad \ddot{X}_B = \frac{5}{4}\ddot{X}_A$$

$$\ddot{X}_A = a_A, \quad \ddot{X}_B = a_B$$

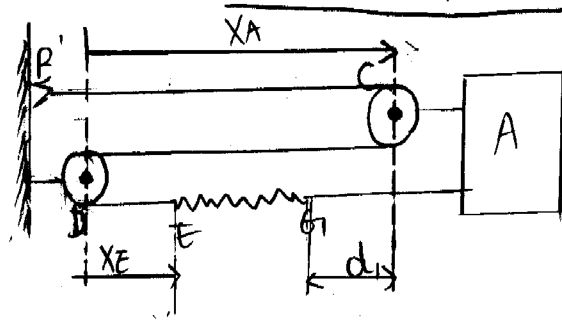
$$\boxed{a_B = \frac{5}{4} a_A = \frac{25}{16}\frac{F}{m}}$$

12.28 Soln:

a) FBD



$$\boxed{m a_A = -2T - F = -3F}$$



Ignore the string wrapped aroud pulleys.

$\therefore l_{string} = 2X_A + X_E$ $\quad \Delta l_{string} = 0 \Rightarrow 2\Delta X_A + \Delta X_E = 0$ ①

$\left.\begin{matrix} l_{spring} = X_A - d_1 - X_E \\ d_1 \text{ is const} \end{matrix}\right\} \Rightarrow \Delta l_{spring} = \Delta X_A - \Delta X_E$ ②

① . ② $\Rightarrow$

$\Delta l_{spring} = 3\Delta X_A$

$\therefore F = \Delta l_{spring} \cdot k = 3\Delta X_A k$

now $\Delta X_A = d$

$m a_A = -3F = -3 \cdot 3\Delta X_A \cdot k = -9dk$

$$\boxed{a_A = \frac{-9kd}{m}}$$

b) The energy is conserved

at $\Delta X_A = d$ $\quad E_k = 0 \quad E_p = \frac{1}{2}k(3d)^2 = \frac{9}{2}kd^2$

$\Delta X_A = 0 \quad E_p = 0 \quad E_k = \frac{1}{2}mv^2$

$\therefore \frac{1}{2}mv^2 = \frac{9}{2}kd^2$

$$\boxed{v = 3\sqrt{\frac{k}{m}}\, d}$$