Your Name: "SOLUTIONS"

TA's name and Section time: TAs

T&AM 203 Prelim 2

Tuesday March 27, 2007

Draft March 27, 2007

3 problems, 25^+ points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems ***poorly defined***;
- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

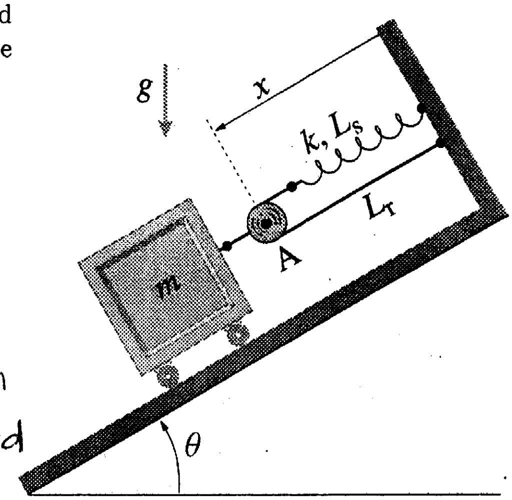
c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	4:	/25
Problem	5:	/25
Problem	6:	/25

4) **(25 pt)** A mass m slides on a surface tipped at angle θ. It is restrained by a rope with length L_R wrapped around a pulley with radius R and a spring with rest length L_s. x is the distance from the wall to the center of the pulley.

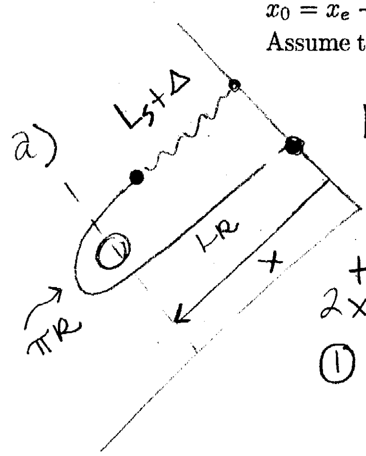
a) (13 points) Find the value of x at the equilibrium position, call it x_e, in terms of m, g, R, L_s, L_R and θ.

b) (12 points) Assuming the mass is released from rest with $x_0 = x_e + d$, find x(t). You may include x_e in your answer. Assume that d is small enough so the spring never goes slack.



SOLUTION BY: MEGHAN S.

a)



$L_s + \Delta$ = length of spring when in a stretched position

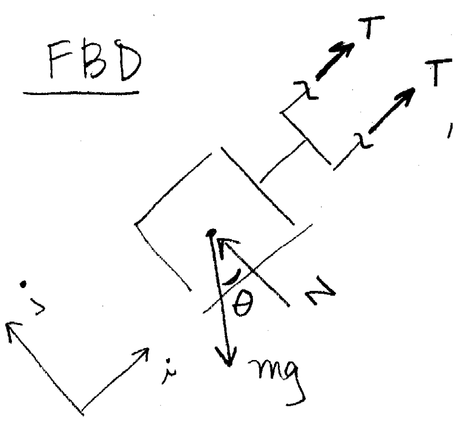
Δ = length spring stretched

total length = total length

$2x + \pi R = L_s + \Delta + L_R$

① $\Delta = 2x + \pi R - L_s - L_R$

FBD



$T = \Delta K$

at equilibrium $\ddot{x} = 0$, $x = x_e$

$\{\sum \vec{F} = m\ddot{x}\}$

$-mg(\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}) + 2T\hat{\imath} + N\hat{\jmath} = 0$

$\{\ \}\cdot\hat{\imath} \Rightarrow -mg\sin\theta + 2T = 0$

plugging in ① $-mg\sin\theta + 2K\Delta = 0$

$-mg\sin\theta + 2K(2x_e + \pi R - L_s - L_R) = 0$

$$\boxed{x_e = \frac{mg\sin\theta}{4K} + \frac{L_s + L_R - \pi R}{2}}$$

b) mass released from rest $\rightarrow \dot{x}(0) = 0$, $x(0) = x_e + d$, find $x(t)$

using FBD from (a) $\quad \vec{a} = \ddot{x}\,\hat{\imath}$

$\{\sum \vec{F} = m\vec{a}\}\cdot\hat{\imath} \rightarrow \quad mg\sin\theta - 2K(2x + \pi R - L_s - L_R) = m\ddot{x}$

$mg\sin\theta - 2K(\pi R - L_s - L_R) = m\ddot{x} + 4Kx$

$4Kx_e = m\ddot{x} + 4Kx$

$\frac{4K}{m}x_e = \ddot{x} + \frac{4K}{m}x$

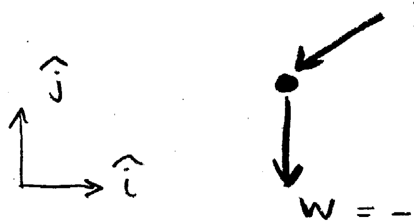
$$x = \underbrace{A\cos 2\sqrt{K/m}\,t + B\sin 2\sqrt{K/m}\,t}_{\text{homogeneous sol}} + \underbrace{x_e}_{\text{particular sol.}}$$

Apply I.C. $\dot{x}(0) = 0 \Rightarrow A = 0$, $x(0) = x_e + d \Rightarrow A = d$, $\boxed{x = d\cos 2\sqrt{\tfrac{K}{m}}\,t + x_e}$

5) **(25 pt)** A canonball with mass m is launched from the ground near the earth. Assume the x direction is horizontal and that gravity g acts in the $-y$ direction. Assume an air drag with magnitude Cv^2 acts on the cannonball and opposes its motion. In terms of any or all of the present values of x, y, v_x, v_y, m, g and C, find $\ddot{y}$.

SOLUTION BY: K. ROMPALA

F.B.D. of cannonball



$$\underline{D} = -c|\underline{v}|^2\hat{\lambda} = -c|\underline{v}|\underline{v}$$

$$\hat{\lambda} = \frac{\underline{v}}{|\underline{v}|}$$

$$\underline{W} = -mg\,\hat{j}$$

L.M.B.

$$\sum \underline{F} = m\underline{a}$$

$$\underline{W} + \underline{D} = m\underline{a}$$

$$-mg\hat{j} - c\sqrt{\dot{x}^2+\dot{y}^2}\,(\dot{x}\hat{i}+\dot{y}\hat{j}) = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

To find $\ddot{y}$

$$\{\underline{\text{L.M.B.}}\}\cdot\hat{j} \Rightarrow \boxed{\ddot{y} = -g - \frac{c}{m}V_y\sqrt{V_x^2 + V_y^2}}$$

where we have substituted
$\dot{x} = V_x \quad \dot{y} = V_y$

6) **(25 pt)** Three balls are on the plane. The only forces on them are the mutual gravitational attractions between them. Assume the ball masses are 1,2, and 3 kg, respectively. Assume $G = 1\,\mathrm{N\,m^2/kg^2}$. At the instant in question the positions and velocities of the three masses are

$$\underline{\mathbf{r}}_1 = (-2\,\hat{\mathbf{i}} + 16\,\hat{\mathbf{j}})\ \mathrm{m}$$
$$\underline{\mathbf{r}}_2 = (0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}})\ \mathrm{m}$$
$$\underline{\mathbf{r}}_3 = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\ \mathrm{m}$$

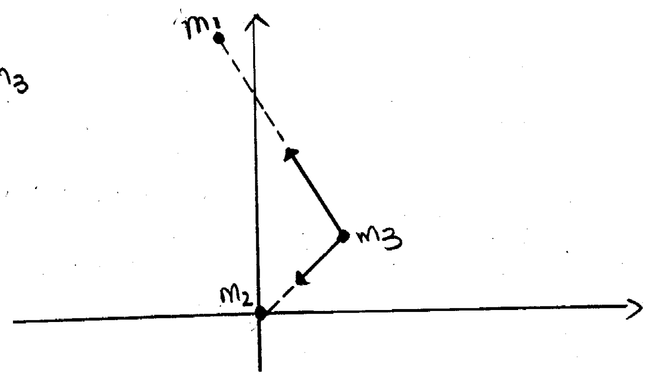
$$\underline{\mathbf{v}}_1 = (1\,\hat{\mathbf{i}} + 1\,\hat{\mathbf{j}})\,\mathrm{m/s}$$
$$\underline{\mathbf{v}}_2 = (3\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}})\,\mathrm{m/s}$$
$$\underline{\mathbf{v}}_3 = (0\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\,\mathrm{m/s}.$$

What is the x component ($\underline{\mathbf{a}} \cdot \hat{\mathbf{i}}$) of the acceleration of mass 3. A numerical answer with units is desired. (Leave fractions simplified but intact, no long division please.)

SOLUTION BY: JING SHI

Soln:

FBD for m_3



$$\vec{F}_{13} = \frac{G m_1 m_3}{|\vec{r}_1 - \vec{r}_3|^3}(\vec{r}_1 - \vec{r}_3)$$
$$= \frac{1\cdot 1\cdot 3\cdot((-2\hat{i}+16\hat{j})-(3\hat{i}+4\hat{j}))}{(\sqrt{12^2+5^2})^3}$$
$$= \frac{-15\hat{i}-36\hat{j}}{13^3}$$

$$\vec{F}_{23} = \frac{G m_2 m_3}{|\vec{r}_2 - \vec{r}_3|}(\vec{r}_2 - \vec{r}_3)$$
$$= \frac{1\cdot 2\cdot 3\,((0\hat{i}+0\hat{j})-(3\hat{i}+4\hat{j}))}{(\sqrt{3^2+4^2})^3}$$
$$= \frac{-18\hat{i}-24\hat{j}}{(5)^3}$$

$$\vec{F} = \vec{F}_{13} + \vec{F}_{23} = m_3 \cdot \vec{a} = m_3 \ddot{\vec{r}}_3$$

$$\{ \cdot\ \ddot{\vec{r}}_3 = \left(\frac{-5\hat{i}-12\hat{j}}{13^3} + \frac{-6\hat{i}-8\hat{j}}{5^3}\right) m/s^2 \}$$

$$\{\cdot\}\cdot\hat{j} \Rightarrow \boxed{\ddot{y} = \left(-\frac{12}{13^3} - \frac{8}{5^3}\right)\frac{m}{s^2}}$$