Your Name: ____________________

TA's name and Section time: ____________________

T&AM 203 Prelim 1

Tuesday February 27, 2007

Draft February 27, 2007

3 problems, 25^+ points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

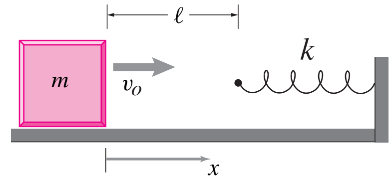
b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
II.) clear, and
III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are boxed in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ `= 18`" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/25
Problem	2:	/25
Problem	3:	/25

1) (25 pt) A mass m slides on a surface with negligible friction with initial velocity v_0 to the right. It is a distance ℓ from a spring with constant k. Give all answers in terms of some or all of m, v_0, ℓ and k.



a) (8 points) During the collision of the mass with the spring, what is the maximum deflection of the end of the spring?

b) (8 points) What is the total time of travel from the initial position back to the initial position?

c) (7 points) Make a sequence of plots that are vertically aligned and all of which have t on the 'x' axis:

i) x vs t

ii) $\dot{x}$ vs t

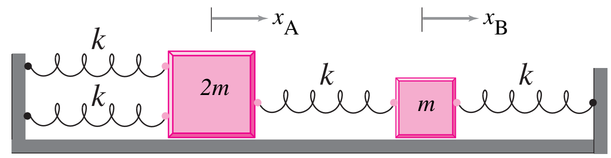
iii) $\ddot{x}$ vs t

iv) E_K vs t

v) E_P vs t

For each of the plots put values (in terms of the given variables) at key points on the plot. For example, in the plot of x vs t label the initial value of x as zero and the maximum value of x as ℓ + your answer to part (a) above.

2) (25 pt) Normal modes etc. Mass A is $2m$, mass B is m. All springs have constant k and are relaxed when $x_A = x_B = 0$. Give all answers in terms of m and k.



a) (8 points) Find the differential equations ('the equations of motion') for this system. Reduce to the simplest tidy form you can.

b) (7 points) One normal mode for this system has the two masses moving oppositely with the B moving twice as much as A. Find the period of this normal mode vibration (no need for eigen-values etc).

c) (5 points) By intuition, guessing, or whatever, find another normal mode and its period of oscillation.

d) (5 points) The minimal (and inadequately commented) Matlab code below simulates the motion of this system for the first normal mode above. Missing is the function `frhs`. Write it.

```
function normalmodes
m = 5; k = 2;
xA0 = 1; xB0 = -2;
xAdot0 = 0; xBdot0 = 0;
z0 = [ xA0 xAdot0 xB0 xBdot0 ];
tspan = linspace(0,10,1000);
[t z] = ode45(@frhs, tspan, z0, [], m,k);
xA = z(:,1);
xB = z(:,3);
plot (t,xA,t,xB); title('First normal mode')
end
```

3) (25 pt) Three balls are vertically stacked and dropped vertically a distance h on to a hard surface. The gravity constant is g. Think of a basketball m_b, a rubber ball m_r and a ping-pong ball m_p. Consider the most ideal case: all coefficients of restitution are perfectly elastic with $e = 1$, each ball is much more massive than the ball above it $m_b \gg m_r \gg m_p$, and air friction is negligible. Assume that the collisional event is actually a sequence of collisions: the big ball hits the ground and bounces up and then collides with the middle ball which then collides with the top ball. In this most ideal case, how high will the top ping-pong ball bounce up, measuring from its height at the collision point.