13.1 Kinematics of a particle in circular motion

13.1 If a particle moves along a circle at constant rate (constant $\dot{\theta}$) following the equation

$$\vec{\boldsymbol{r}}(t) = R\cos(\dot{\theta}t)\hat{\boldsymbol{\imath}} + R\sin(\dot{\theta}t)\hat{\boldsymbol{\jmath}}$$

which of these things are true and why? If not true, explain why.

1. $\vec{\boldsymbol{v}} = \vec{\boldsymbol{0}}$
2. $\vec{\boldsymbol{v}} =$ constant
3. $|\vec{\boldsymbol{v}}| =$ constant
4. $\vec{\boldsymbol{a}} = \vec{\boldsymbol{0}}$
5. $\vec{\boldsymbol{a}} =$ constant
6. $|\vec{\boldsymbol{a}}| =$ constant
7. $\vec{\boldsymbol{v}} \perp \vec{\boldsymbol{a}}$

13.2 The motion of a particle is described by the following equations:

$$\begin{aligned} x(t) &= 1\,\mathrm{m}\cdot\cos((5\,\mathrm{rad/s})\cdot t), \\ y(t) &= 1\,\mathrm{m}\cdot\sin((5\,\mathrm{rad/s})\cdot t). \end{aligned}$$

a) Show that the speed of the particle is constant.

b) There are two points marked on the path of the particle: P with coordinates $(0, 1\,\mathrm{m})$ and Q with coordinates $(1\,\mathrm{m}, 0)$. How much time does the particle take to go from P to Q?

c) What is the acceleration of the particle at point Q?

13.3 A bead goes around a circular track of radius 1 ft at a constant speed. It makes around the track in exactly 1 s.

a) Find the speed of the bead.

b) Find the magnitude of acceleration of the bead.

13.4 A 200 mm diameter gear rotates at a constant speed of 100 rpm.

a) What is the speed of a peripheral point on the gear?

b) If no point on the gear is to exceed the centripetal acceleration of $25\,\mathrm{m/s^2}$, find the maximum allowable angular speed (in rpm) of the gear.

13.5 A particle executes circular motion in the xy-plane at a constant angular speed $\dot{\theta} = 2\,\mathrm{rad/s}$. The radius of the circular path is 0.5 m. The particle's motion is tracked from the instant when $\theta = 0$, i.e., at $t = 0$, $\theta = 0$. Find the velocity and acceleration of the particle at

a) $t = 0.5\,\mathrm{s}$ and

b) $t = 15\,\mathrm{s}$.

Draw the path and mark the position of the particle at $t = 0.5\,\mathrm{s}$ and $t = 15\,\mathrm{s}$.

13.6 A particle undergoes constant rate circular motion in the xy-plane. At some instant t_0, its velocity is $\vec{\boldsymbol{v}}(t_0) = -3\,\mathrm{m/s}\hat{\boldsymbol{\imath}} + 4\,\mathrm{m/s}\hat{\boldsymbol{\jmath}}$ and after 5 s the velocity is $v(t_0 + 5\,\mathrm{s}) = 5/\sqrt{2}\,\mathrm{m/s}(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}})$. If the particle has not yet completed one revolution between the two instants, find

a) the angular speed of the particle,

b) the distance traveled by the particle in 5 s, and

c) the acceleration of the particle at the two instants.

13.7 A bead on a circular path of radius R in the xy-plane has rate of change of angular speed $\alpha = bt^2$. The bead starts from rest at $\theta = 0$.

a) What is the bead's angular position θ (measured from the positive x-axis) and angular speed ω as a function of time ?

b) What is the angular speed as function of angular position?

13.8 A bead on a circular wire has an angular speed given by $\omega = c\theta^{1/2}$. The bead starts from rest at $\theta = 0$. What is the angular position and speed of the bead as a function of time? [Hint: this problem has more than one correct answer (one of which you can find with a quick guess.)]

13.9 Solve $\dot{\omega} = \alpha$, given $\omega(0) = \omega_0$ and α is a constant.

13.10 Solve $\ddot{\theta} = \alpha$, given $\theta(0) = \theta_0$, $\dot{\theta}(0) = \dot{\omega}_0$, and α is a constant.

13.11 Given $\dot{\omega} = \frac{d\dot{\theta}}{dt} = \alpha$ (a constant), find an expression for ω as a function of θ if $\omega(\theta = 0) = w_0$.

13.12 Given that $\ddot{\theta} - \lambda^2\theta = 0$, $\theta(0) = \pi/2$, and $\dot{\theta}(0) = 0$, find the value of θ at $t = 1\,\mathrm{s}$.

13.13 Two runners run on a circular track side-by-side at the same constant angular rate $\omega = 0.25\,\mathrm{rad/s}$ about the center of the track. The inside runner is in a lane of radius $r_i = 35\,\mathrm{m}$ and the outside runner is in a lane of radius $r_o = 37\,\mathrm{m}$. What is the velocity of the outside runner relative to the inside runner in polar coordinates?

13.14 A particle oscillates on the arc of a circle with radius R according to the equation $\theta = \theta_0\cos(\lambda t)$. What are the conditions on R, θ_0, and λ so that the maximum acceleration in this motion occurs at $\theta = 0$. "Acceleration" here means the magnitude of the acceleration vector.

13.2 Dynamics of a particle in circular motion

13.15 Force on a person standing on the equator. The total force acting on an object of mass m, moving with a constant angular speed ω on a circular path with radius r, is given by $F = m\omega^2 r$. Find the magnitude of the total force acting on a 150 lbm person standing on the equator. Neglect the motion of the earth around the sun and of the sun around the solar system, etc. The radius of the earth is 3963 mi. Give your solution in both pounds (lbf) and Newtons (N).

13.16 The sum of forces acting on a mass $m = 10\,\mathrm{lbm}$ is $\vec{\boldsymbol{F}} = 100\,\mathrm{lbf}\hat{\boldsymbol{\imath}} - 120\,\mathrm{lbf}\hat{\boldsymbol{\jmath}}$. The particle is going in circles at constant rate with $r = 18\,\mathrm{in}$ and $\hat{\boldsymbol{e}}_r = \cos 30^\circ\hat{\boldsymbol{\imath}} + \sin 30^\circ\hat{\boldsymbol{\jmath}}$. Using $\sum\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$, find v.[Note, the center of the circle is not at the origin.]

13.17 The acceleration of a particle in planar circular motion is given by $\vec{\boldsymbol{a}} = \alpha r\hat{\boldsymbol{e}}_\theta -$